

Studies in Fuzziness and Soft Computing

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Fuzzy Dual Numbers

Theory and Applications

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Preface

The objective of this book is to introduce a particular class of fuzzy numbers, the fuzzy dual numbers, which present comparatively to classical fuzzy numbers additional attractive properties either from the point of view of theory as from the point for view of applications in the field of engineering and decision theory.

This work is the result of a collaboration between Labfuzzy Laboratosry of COPPE, Universidade Federal do Rio de Janeiro, Brazil, and MAIAA, the Applied Mathematics, Computation and Automation Laboratory of ENAC, the French Civil Aviation Institute in Toulouse, France.

This book introduces fuzzy dual numbers which are a special class of fuzzy numbers and are described in a minimal way by two real-valued parameters. This book shows that using this formalism, uncertainty can be taken explicitly into account in quantitative planning problems without implying an unacceptable computational burden. The first three chapters are devoted to introduce in a progressive way fuzzy dual numbers. Then, the next four chapters extend classical deterministic and probabilistic concepts associated with quantitative planning problems to the case in which uncertainty is represented by fuzzy dual numbers.

Chapter 1 introduces briefly fuzzy sets and their relation with real intervals, giving way to fuzzy intervals and fuzzy numbers.

Chapter 2 is about dual numbers which have been introduced in the ninetieth century by William Clifford when dealing with the theory of engines using a nilpotent operator. They have been introduced initially to study the kinematics of rigid articulated bodies, and more recently, computer tools have been made available for dual numbers calculus.

Chapter 3 is about fuzzy dual numbers which is a special class of dual numbers, associated with symmetrical triangular fuzzy numbers. Fuzzy dual vectors and fuzzy dual matrices are also introduced. Fuzzy dual numbers, vectors and matrices are introduced to handle explicitly using an extension of dual calculus, the uncertainty on the value of parameters or variables in the formalization and solution of decision problems.

Chapter 4 is devoted to fuzzy dual linear matrix inequalities (LMI). LMI formalism has received an increasing acceptance for the formulation of feasible sets for

crisp optimization problems. The new concepts proposed in this chapter are applied to the representation of fuzzy LMI domains using semi-positive definite fuzzy dual matrices.

Chapter 5 considers the fuzzy estimation of distribution matrices. Origin–destination matrices play a central role in transportation and logistics planning while many uncertainties should be taken into account when considering their estimation. An approach is proposed to combine the entropy maximization approach with fuzzy modelling to get coherent intervals for trip distribution estimates based on structured uncertainties.

In Chap. 6, fuzzy dual numbers and probabilities as well as fuzzy dual entropy are introduced. The classical entropy maximization approach for trip distribution prediction in transportation networks is reviewed, and a new formulation is proposed using the fuzzy dual formalism.

Chapter 7 considers general optimization problems with uncertain parameters and variables. The formulation of optimization problems using this new formalism is discussed. It is shown that each fuzzy dual programming problem generates a finite set of classical optimization problems, even in the case in which the feasible set is defined using fuzzy dual LMI constraints. Then, fuzzy dual dynamic programming is introduced and discussed.

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About the Book

This book is devoted to those engineers, architects and other people involved in real-life decision-making as well as in systems design to make reality more predictable and therefore more safe and efficient.

Fuzzy modelling, as introduced by **Lotfi Aliasker Zadeh** in the ninety seventieth, offered an opportunity to face the complexity of real world by quickly producing effective solutions to problems related with many fields of knowledge. In the present book, a subset of fuzzy numbers is introduced and its power to tackle different engineering problems is displayed.

Contents

1 Fuzzy Numbers	1
Introduction to Fuzzy Sets	1
Operations on Real Intervals	2
Fuzzy Intervals and Numbers	3
References.	3
2 Dual Numbers	5
Definition of Dual Numbers	5
Dual Vectors and Matrices	7
Dual Vectors.	7
References.	9
3 Fuzzy Dual Numbers	11
Definition of Fuzzy Dual Numbers	11
Fuzzy Dual Calculus	11
Fuzzy Dual Vectors.	13
Partial Orders Between Fuzzy Dual Numbers	14
Fuzzy Dual Vectors and Matrices	15
Fuzzy Dual Matrices	15
References.	16
4 Fuzzy Dual Linear Matrix Inequalities	17
Definition	17
Fuzzy Dual Linear Matrix Inequalities	19
Semi-Definite Positive Fuzzy Dual Matrices	19
Fuzzy Lmi Domains	20
Example of Lmi Domain.	21
Conclusion	22
References.	22

5 Fuzzy Estimation of Distribution Matrices	25
Introduction	25
Direct Estimation Approach	26
Fuzzy Affine Model of O-D Uncertainties	29
Fuzzy Dual Cost Sensitivity	31
Fuzzy Dual Cost Constraint	32
Sensitivity Analysis	34
Conclusion	37
References	38
6 Fuzzy Dual Entropy and Applications	39
Introduction	39
Fuzzy Dual Probabilities	40
Fuzzy Dual Entropy	41
Fuzzy Dual Trip Distribution Estimation	42
The Classical Trip Distribution Problem	42
Fuzzy Dual Trip Distribution Problem	43
Numerical Application	45
Conclusion	47
References	47
7 Fuzzy Dual Programming	49
Introduction	49
Programming With Fuzzy Dual Parameters	50
Programming With Fuzzy Dual Variables	52
Fuzzy Dual Dynamic Programming	53
References	56
General Conclusion	59
Index	61

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Prof. Felix Mora-Camino started working with fuzzy modelling applications as early as 1981 when modelling urban traffic congestion for path planning purpose (COPPE-Programa de Engenharia de Transportes). He pursued his research activities in the field of fuzzy control applied to water systems at LAAS du CNRS (Laboratoire d'Architectures et d'Analyse des Systèmes) in Toulouse, France, and more recently in the field of Fuzzy Operations Research in Air Transportation at ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse.

Prof. Carlos Alberto Nunes Cosenza developed for the last 25 years intense and sustained teaching and research activities in the field of fuzzy logic and fuzzy modelling, creating the Labfuzzy research laboratory of COPPE/UFRJ (Coordenação dos Programas de Pos-Graduação em Engenharia/Universidade Federal do Rio de Janeiro) while tackling strategic planning problems for industry and development in Brazil.