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# Number-Theoretic Methods in Cryptology 

First International Conference, NuTMiC 2017 Warsaw, Poland, September 11-13, 2017 Revised Selected Papers

Springer

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## Preface

The First Number-Theoretic Methods in Cryptology (NuTMiC) Conference was held at the University of Warsaw, Poland, during September 11-13, 2017. The aim of the conference is to cross-pollinate number theory and cryptology. Besides the well-established connections between the two domains such as primality testing, factorization, elliptic curves, lattices (to mention a few), the conference endeavors to forge new ones that would encompass number theory structures and algorithms that have never been used in cryptology before. It is expected that these new connections will lead to novel, more efficient and secure cryptographic systems and protocols (such as one-way functions, pseudorandom number generators, encryption algorithms, digital signatures, etc.). The conference topics include lattice-based cryptography, elliptic curves and bilinear-based cryptography, L-functions with applications to cryptology, large sieve methods in cryptography and exponential sums over finite fields and randomness extractors.

We received 32 submissions. The review process was conducted in two phases. In the first, the papers were lightly reviewed with emphasis on helpful comments and feedback. There were 21 papers that were chosen for conference presentation. The final papers were collected after the conference for these proceedings. The papers and were subject to a rigorous review. The proceedings include 15 peer-reviewed papers and three invited talks.

We would like to thank the Program Committee members and the external reviewers for their time and effort. We also thank the local organizers who made the conference a success. In particular, Marek Janiszewski, Aleksandra Dolot, Daniel Waszkiewicz, and Marcin Tunia took care of the conference website, helped us with EasyChair, and manned the conference desk. Bartosz Źrałek helped us with e-mail communication and financial overview. We would like to express our appreciation to Springer for their support and help in the production of the conference proceedings. We thank the EasyChair team for letting us use the server.

Last but not least, we highly appreciate the support the conference received from the Faculty of Mathematics, Informatics, and Mechanics of the University of Warsaw (MIMUW) and Warsaw Center of Mathematics and Computer Science (WCMCS). In particular, the Dean of MIMUW, Professor Paweł Strzelecki, welcomed the participants and hosted the conference in his department facilities. Professor Krzysztof Barański, Director of the Institute of Mathematics, and Professor Anna Zdunik, Chair WCMCS MIMUW, supported the conference financially. We gladly acknowledge the continuous assistance of the university administration units: financial, international collaboration, and audiovisual/technical services.


The First Conference on Number-Theoretic Methods in Cryptology Warsaw University, Warsaw, Poland

September 11-13, 2017

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## Abstracts of Invited Talks

# Arithmetic Geometry: Deep Theory, Efficient Algorithms and Surprising Applications 

Gerhard Frey<br>University of Duisburg-Essen

One of the most astonishing success stories in recent mathematics is arithmetic geometry, which unifies methods from classical number theory with algebraic geometry ("schemes"). In particular, an the extremely important role is played by the Galois groups of base schemes like rings of integers of number fields or rings of holomorphic functions of curves over finite fields. These groups are the algebraic analogues of topological fundamental groups, and their representations induced by the action on divisor class groups of varieties over these domains yielded spectacular results like Serre's Conjecture for two-dimensional representations of the Galois group of $\mathbb{Q}$, which implies for example the modularity of elliptic curves over $\mathbb{Q}$ and so Fermat's Last Theorem (and much more).

At the same time the algorithmic aspect of arithmetical objects like lattices and ideal class groups of global fields became more and more important and accessible, stimulated by and stimulating the advances in theory. An outstanding result is the theorem of $\mathrm{F} . \mathrm{Heß}$ and C.Diem yielding that the addition in divisor class groups of curves of genus $g$ over finite fields $\mathbb{F}_{q}$ is (probabilistically) of polynomial complexity in $g$ (fixed) and $\log (q)$ ( $g$ fixed). So one could hope to use such groups for public key cryptography, e.g. for key exchange, as established by Diffie-Hellman for the multiplicative group of finite fields.

The obtained insights play not only a constructive role but also a destructive role for the security of such systems. Algorithms for fast scalar multiplication and point counting (e.g. the algorithm of Schoof-Atkin-Elkies) make it possible to find divisor class groups in cryptographically relevant ranges but, at the same time, yield algorithms for the computation of discrete logarithms that are in many cases "too fast" for security. The good news is that there is a narrow but not empty range of candidates usable for public key cryptography and secure against all known attacks based on conventional computer algorithms: carefully chosen curves of genus 1 (elliptic curves) and hyperelliptic curves of genus $\leq 3$ over prime fields.

In the lecture we gave an overview on the methods and results for the rather satisfying situation of elliptic and hyperelliptic cryptography-as long as we restrict the algorithms to classical bit-operations. But the possibility of the existence of quantum computers in a not too far future forces to look for alternatives.

Therefore we formulated a rather abstract setting for Diffie-Hellman key exchange schemes using (closely related) categories for the exchange partners, for which push-outs exist and are computable. The DL-systems with cyclic groups are the easiest realizations (and by Shor's algorithm cracked in polynomial time), the next level are $G$-sets ( $G$ a semi group) with a commutativity condition. If $G$ is abelian (e.g. equal to $\mathbb{N}$ )
then an algorithm of Kuperberg for the hidden shift problem with subexponential complexity can be applied, for general groups no such algorithm is known (but the commutation condition is difficult to realize).

Using fundamental results of M . Deuring about isogenies of elliptic curves we described the system of Couveignes-Stolbunov for key exchange using the isogeny graph of ordinary elliptic curves with endomorphism ring $O$, which is a $G$-set with $G=\operatorname{Pic}(O)$ and so only of subexponential security under quantum computing, and the system of De Feo using supersingular elliptic curves (and nicely fitting into our categorical frame) for which no non-exponential quantum computer attack is known till now.

# A Babystep-Giantstep Method for Faster Deterministic Integer Factorization 

Markus Hittmeir<br>University of Salzburg, Hellbrunnerstraße 34, 5020 Salzburg<br>markus.hittmeir@sbg.ac.at

We consider the problem of computing the prime factorization of integers. In practice, a large variety of probabilistic and heuristic methods is used for this task. However, none of these algorithms is efficient and the problem itself is assumed to be computationally hard. The difficulty of factoring large numbers is fundamental for the security of several cryptographical systems, one of which is the public-key scheme RSA.

A more theoretical aspect of integer factorization concerns deterministic algorithms and the rigorous analysis of their runtime complexities. In the years from 1974 to 1977, Pollard and Strassen developed such a method and proved that it runs in time $\widetilde{O}\left(N^{1 / 4}\right)$. Since the seventies, the logarithmic factors in the bound have been refined and other deterministic algorithms running in $\widetilde{O}\left(N^{1 / 3}\right)$ have been found. However, the bound $\widetilde{O}\left(N^{1 / 4}\right)$ has been state of the art for the last forty years.

In this paper, we obtain an improvement by a superpolynomial factor. The runtime complexity of our algorithm is of the form

$$
\widetilde{O}\left(N^{1 / 4} \exp (-C \log N / \log \log N)\right)
$$

To describe our approach, we consider the case $N=p q$, where $p$ and $q$ are unknown prime factors and $p<q$. We will employ a refined babystep-giantstep method to solve the discrete logarithm problem $a^{X} \equiv a^{N+1} \bmod N$ for a certain $a \in \mathbb{Z}$ coprime to $N$. The purpose of this procedure is to determine $S:=p+q$. Knowing $S$ allows us to factor $N$ immediately.

Let $\Delta \leq N^{1 / 2}$ be a parameter. The scheme for our main algorithm is as follows:

1. Use the Pollard-Strassen approach to search for $p$ in the interval $[1, \Delta]$. If $p$ is found, stop. If $p$ is not found, go to Step 2.
2. Use $\Delta<p<N^{1 / 2}$ to find $S:=p+q$, the sum of the prime factors of $N$.
3. Knowing $N$ and $S$, compute $p$ and $q$.

To speed up the application of the babystep-giantstep method in Step 2 and to optimize the value for $\Delta$, we will consider so called modular hyperbolas $\mathcal{H}_{N, m}$. They are defined as the sets of solutions $(x, y)$ to the congruence equation $N \equiv x y \bmod m$. Clearly, the corresponding set $\mathcal{L}_{N, m}$ consisting of the elements $x+y \bmod m$ for $(x, y) \in \mathcal{H}_{N, m}$ contains the residue of $S$ modulo $m$. If $r$ is prime, than the cardinality of $\mathcal{L}_{N, r}$ is about half
of the possible residue classes modulo $r$. Considering all primes up to a suitable bound $B$, we deduce significant information about $S$. For example, let $N=3823 \cdot 2069$ and $m=2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$. Then $\mathcal{L}_{N, m}$ contains only 40 elements. As a result, the residue of $S$ modulo $m$ is restricted to $40 / 2310=1.7 \%$ of all residue classes modulo $m$. The information obtained by this idea yields the main contribution to our improvement.

# A Crossbred Algorithm for Solving Boolean Polynomial Systems 

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#### Abstract

We consider the problem of solving multivariate systems of Boolean polynomial equations: starting from a system of $m$ polynomials of degree at most $d$ in $n$ variables, we want to find its solutions over $\mathbb{F}_{2}$. Except for $d=1$, the problem is known to be NP-hard, and its hardness has been used to create public cryptosystems; this motivates the search for faster algorithms to solve this problem. After reviewing the state of the art, we describe a new algorithm and show that it outperforms previously known methods in a wide range of relevant parameters. In particular, the first named author has been able to solve all the Fukuoka Type I MQ challenges, culminating with the resolution of a system of 148 quadratic equations in 74 variables in less than a day (and with a lot of luck).


## Contents

Invited Talk
A Crossbred Algorithm for Solving Boolean Polynomial Systems ..... 3
Antoine Joux and Vanessa Vitse
Elliptic Curves in Cryptography
Generation and Implementation of Cryptographically Strong Elliptic Curves ..... 25
Przemystaw Dąbrowski, Rafat Gliwa, Janusz Szmidt, and Robert Wicik
On the Possibility of Transformation of Multidimensional ECDLP into 1-Dimensional ECDLP ..... 37
Michat Wroński and Tomasz Kijko
Explicit Bound for the Prime Ideal Theorem in Residue Classes ..... 48
Maciej Grześkowiak
Public-Key Cryptography
Short Solutions to Nonlinear Systems of Equations ..... 71
Alan Szepieniec and Bart Preneel
A Novel RSA-Like Cryptosystem Based on a Generalization of the Rédei Rational Functions ..... 91
Nadir Murru and Francesco M. Saettone
Commutativity, Associativity, and Public Key Cryptography ..... 104
Jacques Patarin and Valérie Nachef
Lattices in Cryptography
Computational Differential Privacy from Lattice-Based Cryptography ..... 121
Filipp Valovich and Francesco Aldà
Explicit Formula for Gram-Schmidt Vectors in LLL with Deep Insertions and Its Applications ..... 142
Junpei Yamaguchi and Masaya Yasuda
Number Theory
Factoring $n$ and the Number of Points of Kummer Hypersurfaces $\bmod n$ ..... 163
Robert Drylo and Jacek Pomykata
Detection of Primes in the Set of Residues of Divisors of a Given Number ..... 178
Rafat Bystrzycki
Pseudorandomness
The Measures of Pseudorandomness and the NIST Tests ..... 197
László Mérai, Joël Rivat, and András Sárközy
On the Cross-Combined Measure of Families of Binary Lattices and Sequences ..... 217
Katalin Gyarmati
Algebraic Structures and Analysis
The Cube Attack on Courtois Toy Cipher ..... 241
Janusz Szmidt
Near Butson-Hadamard Matrices and Nonlinear Boolean Functions ..... 254
Sibel Kurt and Oğuz Yayla
Multi-secret Sharing Scheme for Level-Ordered Access Structures. ..... 267
Appala Naidu Tentu, Abdul Basit, K. Bhavani, and V. Ch. Venkaiah
Author Index ..... 279

