Springer Tracts in Advanced Robotics 123

Series editors

More information about this series at http://www.springer.com/series/5208

Gianluca Antonelli

Underwater Robots

Fourth Edition

Springer

Gianluca Antonelli
Dipartimento di Ingegneria Elettrica
e dell'Informazione
Università di Cassino e Lazio
Meridionale
Cassino
Italy

ISSN 1610-7438 ISSN 1610-742X (electronic)
Springer Tracts in Advanced Robotics
ISBN 978-3-319-77898-3 ISBN 978-3-319-77899-0 (eBook)
https://doi.org/10.1007/978-3-319-77899-0

Library of Congress Control Number: 2018936179

Printed on acid-free paper

This Springer imprint is published by the registered company Springer International Publishing AG part of Springer Nature
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

A “bish-bish” Andrea e “demone” Giustina

E la locomotiva sembrava fosse un mostro strano che l'uomo dominava con il pensiero e con la mano...

Francesco Guccini, La locomotiva, 1972.

Foreword

Robotics is undergoing a major transformation in scope and dimension. From a largely dominant industrial focus, robotics is rapidly expanding into human environments and vigorously engaged in its new challenges. Interacting with, assisting, serving, and exploring with humans, the emerging robots will increasingly touch people and their lives.

Beyond its impact on physical robots, the body of knowledge robotics has produced and is revealing a much wider range of applications reaching across diverse research areas and scientific disciplines, such as: biomechanics, haptics, neurosciences, virtual simulation, animation, surgery, and sensor networks, among others. In return, the challenges of the new emerging areas are proving an abundant source of stimulation and insights for the field of robotics. It is indeed at the intersection of disciplines that the most striking advances happen.

The *Springer Tracts in Advanced Robotics (STAR)* is devoted to bringing to the research community the latest advances in the robotics field on the basis of their significance and quality. Through a wide and timely dissemination of critical research developments in robotics, our objective with this series is to promote more exchanges and collaborations among the researchers in the community and contribute to further advancements in this rapidly growing field.

This classic by Gianluca Antonelli was one of the first volumes in the series and has been a bestseller through the previous three editions. Fifteen years after the publication of the first edition, the fourth edition comes to print. Further to a revision and update, the main novelty is in the revisitation of the contents which have been expanded and organized into self-contained chapters for download purposes.

A well-assessed blend of theoretical and experimental results on Underwater Vehicle-Manipulator Systems, this volume confirms to be a success in our STAR series!

Naples, Italy
January 2018

Bruno Siciliano
STAR Editor

Preface to the Fourth Edition

At the fourth edition, the Preface will be short; let me simply list the main changes.

The chapters have been reduced by merging some of the previous ones. The content is increased, however. Since the sales of the monograph can be done also by individual chapters, I decided to make them larger and, as far as possible, comprehensible without reading the others.

The core is thus made by four chapters, a modeling one, control of the sole vehicle, which includes now also the fault detection/tolerance part, control of the vehicle-manipulator system, and, finally, simulation.

All the chapters have been revised, some concepts hopefully clarified, material added, and references updated.

For the online version, color and hyper references have been included too. The reader will not find in this monograph any details on the sensorial or communication systems; moreover, if he is interested in vehicle control alone, i.e., without manipulator, I would suggest one of the Fossen's books.

Ah, and yes, I had to update also my pic with a more recent one…

Cassino, Italy
December 2017

Gianluca Antonelli

Preface to the Third Edition

Whenever possible I attend IEEE International Conference on Robotics and Automation (ICRA) conference where I usually meet Thomas, a Springer Senior Editor, that updates me on the sales of this monograph. Despite the very niche topic, underwater manipulation, sales are still interesting so that a third edition is worth.

Ten years after the first edition, I decided to work on this project by reconsidering some of the materials. First of all I have withdrawn one chapter, the one devoted to multi-vehicles coordination, since it was somehow orthogonal to the remaining part of the book. It dealt with underwater and control but was not specific to manipulation. It was also too short to cover in a decent way all the challenges of this control problem.

I have obviously updated most references, mainly adding then deleting existing ones, and clarified some of the modeling parts based on the feedback from the readers. Some details on the way the Jacobians are computed, as well as all the Jacobians used in the kinematic control chapter, have been streamlined and explicitly reported.

The first edition of this monograph was based on my Ph.D. work, done during the 1997–1999, and most of the material was of *research* kind. As a matter of fact, at the time of publication some of the journal papers were still in their reviewing process. After ten years, it is interesting to notice how part of the material lost his innovative halo to acquire an educational one. For this reason, chapters containing experiments physically performed in the end of last century have been kept. On this trail is also the decision to give the simulation tool and write a small reference chapter to it together with the dynamic parameters of several models used in the testing of the algorithms. For the online version, color and hyper references have been included too. The reader will not find in this monograph any details on the sensorial or communication systems; moreover, if he is interested in vehicle control alone, i.e., without manipulator, I would suggest one of the Fossen's books.

Cassino, Italy — Gianluca Antonelli
July 2013

Preface to the Second Edition

The purpose of this Second Edition is to add material not covered in the First Edition as well as streamline and improve the previous material.

The organization of the book has been substantially modified, an introductory chapter containing the state of the art has been considered; the modeling chapter is substantially unmodified. In Chapter 3, the problem of controlling a six degrees of freedoms (DOFs) Autonomous Underwater Vehicle (AUV) is investigated. Chapter 4 is a new chapter devoted to a survey of fault detection/tolerant strategies for ROVs/AUVs, and it is mainly based on the chapter published in first edition. The following chapter (Chapter 5) reports experimental results obtained with the vehicle ODIN. The following three chapters, from Chapter 6 to Chapter 8, are devoted to presenting kinematic, dynamic, and interaction control strategies for Underwater Vehicle-Manipulator Systems (UVMSs); new material has been added thanks also to several colleagues who provided me with valuable material, I warmly thank all of them. The content of Chapter 9[1] is new in this second edition and reports preliminary results on the emerging topic of coordinated control of platoon of AUVs. Finally, the bibliography has been updated.

The reader might be interested in knowing what he/she will not find in this book. Since the core of the book is the coordinated control of manipulators mounted on underwater vehicles, control of non-holonomic vehicles is not dealt with; this is an important topic also in view of the large number of existing *torpedo*-like vehicles. Another important aspect concerns the sensorial apparatus, both from the technological point of view and from the algorithmic aspect; most of the AUVs are equipped with redundant sensorial systems required both for localization/navigation

[1] Chapter removed in the third edition.

purposes and for fault detection/tolerant capabilities. Actuation is mainly obtained by means of thrusters; those are still object of research for the modeling characteristics and might be the object of improvement in terms of dynamic response.

Cassino, Italy Gianluca Antonelli
January 2006

Preface to the First Edition

Underwater Robotics has known in the last years an increasing interest from research and industry. Currently, it is common the use of manned underwater robotics systems to accomplish missions as sea bottom and pipeline survey, cable maintenance, offshore structures' monitoring and maintenance, collect/release of biological surveys. The strong limit of the use of manned vehicles is the enormous cost and risk in working in such an hostile environment. The aim of the research is to progressively make it possible to perform such missions in a completely autonomous way.

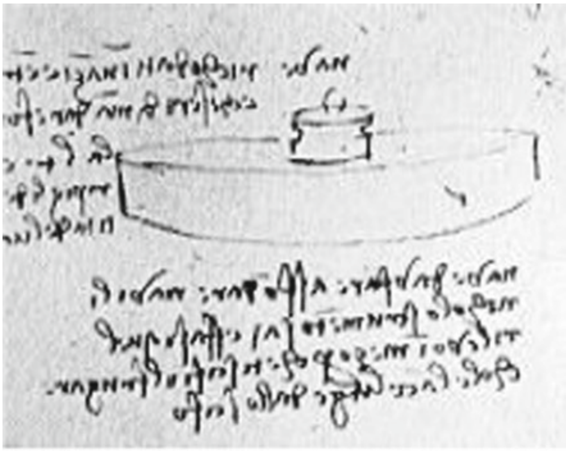
This objective is challenging from the technological as well as from the theoretical aspects since it implies a wide range of technical and research topics. Sending an autonomous vehicle in an unknown and unstructured environment, with limited online communication, requires some onboard *intelligence* and the ability of the vehicle to react in a reliable way to unexpected situations. Techniques as artificial intelligence, neural network, discrete events, fuzzy logic can be useful in this *high*-level mission control. The sensory system of the vehicle must deal with a noisy and unstructured environment; moreover, technologies as GPS are not applicable due to the impossibility to underwater electromagnetic transmission; vision-based systems are not fully reliable due to the generally poor visibility. The actuating system is usually composed of thrusters and control surfaces; all of them have a nonlinear dynamics and are strongly affected by the hydrodynamic effects.

In this framework, the use of a manipulator mounted on an autonomous vehicle plays an important role. From the control point of view, underwater robotics is much more challenging with respect to ground robotics since the former deal with unstructured environments, mobile base, significant external disturbance, low bandwidth of sensory and actuating systems, difficulty in the estimation of the dynamic parameters, highly nonlinear dynamics.

Referring to Autonomous Underwater Vehicles (AUVs), i.e., untethered, unmanned vehicles to be used mainly in survey missions, [-][2] present the state of the art of several existing AUVs and their control architecture. Currently, there are more than 46 AUV models [-], among others: *ABE* of the Woods Hole Oceanographic Institution (MA, USA), *MARIUS* developed under the Marine Science and Technology Programme of the IV framework of European Commission (Lisbon, Portugal), *ODIN* designed at the Autonomous Systems Laboratory of the University of Hawaii (Honolulu, HI, USA), *OTTER* from the Monterey Bay Acquarium and Stanford University (CA, USA), *Phoenix* and *ARIES* belonging to the Naval Postgraduate School (Monterey, CA, USA), *Twin Burgers* developed at the University of Tokyo (Tokyo, Japan), *Theseus* belonging to ISE Research Ltd (Canada). Reference [-] shows the control architecture of *VORTEX*, a vehicle developed by Inria and Ifremer (France), and *OTTER*. Focusing on the low-level motion control of AUVs, most of the proposed control schemes take into account the uncertainty in the model by resorting to an adaptive strategy [-] or a robust approach [-]. In [-] an estimation of the dynamic parameters of the vehicle NPS AUV *Phoenix* is also provided. An overview of control techniques for AUVs is reported in [-].

As a curiosity, in the Figure there is a draw of one of the first *manned* underwater vehicles. It was found in the *Codice Atlantico* (Codex Atlanticus), written by Leonardo Da Vinci between 1480 and 1518, together with the development of some diver's devices. Legends say that Leonardo worked on the idea of an underwater military machine that he further destroyed by himself the results judged too dangerous. Maybe the first idea of an underwater machine is from Aristotle; following the legend he built a machine: *skaphe andros* (boat-man) that allowed Alexander the Great to stay in deep for at least half a day during the war of Tiro in 325 B. C. This is unrealistic, of course, also considering that Archimedes's law was still to become a reality (around 250 B. C.).

[2] All the original citations have been erased. Due to reorganization in chapters, each with its own bibliography, this choice is the less stupid with respect to having a bibliography simply for this >15 years preface.



Draw of the manned underwater vehicle developed by Leonardo Da Vinci

The current technology in control of underwater manipulation is limited to the use of a master/slave approach in which a skilled operator has to move a master manipulator that works as *joystick* for the slave manipulator that is performing the task [-]. The limitations of such a technique are evident: the operator must be well trained, underwater communication is hard, and a significant delay in the control is experienced. Moreover, if the task has to be performed in deep waters, a manned underwater vehicle close to the unmanned vehicle with the manipulator needs to be considered to overcome the communication problems thus leading to enormous cost increasing. Few research centers are equipped with an Autonomous Underwater Vehicle-Manipulator System. Among the others:

- *ODIN* and *OTTER* can be provided with a one/two-link manipulator to study the interaction of the manipulator and the vehicle in order to execute automatic retrieval tasks [-];
- On *VORTEX* a 7-link manipulator (*PA10*) can be mounted with a large inertia with respect to the vehicle that implies a strong interaction between them;
- *SAUVIM*, a semi-autonomous vehicle with an *Ansaldo* 7-link manipulator is under development at the Autonomous Systems Laboratory of the University of Hawaii; this vehicle, in the final version, will be able to operate at the depth of 4000 m.
- *AMADEUS*, an acronym for Advanced MAnipulation for DEep Underwater Sampling, funded by the European Commission, that involved the Heriot-Watt University (UK), the Università di Genova (Italy), CNR Istituto Automazione Navale, (Italy), the Universitat de Barcelona (Spain), the Institute of Marine Biology of Crete (Greece). The project focused on the coordinated control of two tele-operated underwater *Ansaldo* 7-link manipulators and the development of an underwater hand equipped with a slip sensor.

Focusing on the motion control of UVMSs, [-] presents a telemanipulated arm; in [-] an *intelligent* underwater manipulator prototype is experimentally validated; [-] presents some simulation results on a Composite Dynamics approach for *VORTEX/PA10*; [-] evaluates the dynamic coupling for a specific UVMS; adaptive approaches are presented in [-]. Reference [-] reports some interesting experiments of coordinated control. Very few papers investigated the redundancy resolution of UVMSs by applying inverse kinematics algorithm with different secondary tasks [-].

This book deals with the main control aspects in underwater manipulation tasks and dynamic control of AUVs. First, the mathematical model is discussed; the aspects with significant impact on the control strategy will be remarked. In Chap. 6, kinematic control for underwater manipulation is presented. Kinematic control plays a significant role in unstructured robotics where off-line trajectory planning is not a reliable approach; moreover, the vehicle-manipulator system is often kinematically redundant with respect to the most common tasks and redundancy resolution algorithms can then be applied to exploit such characteristic. Dynamic control is then discussed in Chap. 7; several motion control schemes are analyzed and presented in this book. Some experimental results with the autonomous vehicle *ODIN* (without manipulator) are presented; moreover, some theoretical results on adaptive control of AUVs are discussed. In Chap. 8, the interaction with the environment is detailed. Such kind of operation is critical in underwater manipulation for several reasons that do not allow direct implementation of the force control strategies developed for ground robotics. Finally, after having developed some conclusions, a simulation tool for multi-body systems is presented. This software package, developed for testing the control strategies studied along the book, has been designed according to modular requirements that make it possible to generate generic robotic systems in any desired environment.

Napoli, Italy
August 2002

Gianluca Antonelli

Acknowledgements

The contributions of a large number of people were determinant for the realization of this monograph: Stefano Chiaverini was my co-tutor during my Ph.D. at the Università degli Studi di Napoli Federico II, and he is now Full Professor at the Università di Cassino e Lazio Meridionale. Since the beginning of the doctoral experience up to present, most of the research that I have done is shared with him. As a matter of fact, I consider myself as a co-author of this monograph. Furthermore, Stefano was, somehow, also *responsible* of my decision to join the academic career.

Lorenzo Sciavicco developed a productive, and friendly, research environment in Napoli that was important for my professional growing. Bruno Siciliano, my tutor both in the Master's and Ph.D. thesis, to whom goes my warmest acknowledgments.

Pino Casalino contributed in creating ISME, the Italian interuniversity center aimed at clustering to do research in the marine environment. In this case too, a friendly and productive group has been built, which is not trivial.

Filippo Arrichiello, currently at the Università di Cassino e Lazio Meridionale, Fabrizio Caccavale, currently at the Università della Basilicata, Giuseppe Fusco, currently at the Università di Cassino e Lazio Meridionale, Tarun Podder, currently at the Case Western Reserve University, Nilanjan Sarkar, currently at the Vanderbilt University, Luigi Villani, currently at the Università di Napoli Federico II, Michael West, currently at the Georgia Tech, Elisabetta Cataldi, Paolo Di Lillo, Daniele Di Vito, Ph.D. students and postdoc at my Institution, all of them are co-authors of the *wet* papers embedded in this monograph and deserve a lot of credit for this work.

All started during the Ph.D., when I have been visiting researcher at the Autonomous Systems Laboratory of the University of Hawaii where I carried out some experiments on dynamic control of underwater vehicles and worked on the interaction control. I would like to acknowledge Nilanjan Sarkar and Junku Yuh, my guests during the staying.

For this fourth edition, several colleagues provide me with their material; I would like to thank Massimo Caccia, Giuseppe Casalino, Wan Kyun Chung, Jeremi Gancet, Jonghui Han, Giovanni Indiveri, Konstantin Kebkal, Maarja Kruusma, Bong-Huan Jun, Tim McLain, Antonio Pascoal, Kristin Pettersen, Pere Ridao, Pedro Sanz, Hanumant Singh, Asgeir Sorensen, Alessio Turetta, Gianmarco Veruggio, Peter Weiss, and Junku Yuh.

My mother, my father, Marco, Fabrizio, Giustina, Andrea and Ettore, they all tolerated my *engineeringness*.

Contents

Acronyms and Notations

In this chapter, the main acronyms and the notations that will be used in the work are listed. In *almost all* the equations normal font indicates a scalar variable, lower case boldface font a vector and upper case boldface a matrix.

Acronyms

AUV	Autonomous Underwater Vehicle
CLIK	Closed-Loop Inverse Kinematics
DH	Denavit–Hartenberg
DOF	Degree of freedom
ECEF	Earth-Centered-Earth-Fixed frame
ECI	Earth-Centered-Inertial frame
EKF	Extended Kalman Filter
FD	Fault detection
FIS	Fuzzy Inference System
FTC	Fault Tolerant Controller
GUI	Graphical User Interface
IK	Inverse kinematics
KF	Kalman Filter
NED	North-East-Down frame
PID	Proportional Integral Derivative
ROV	Remotely Operated Vehicle
TCM	Thruster Control Matrix
UAV	Unmanned Aerial Vehicle
USV	Unmanned Surface Vehicle
UUV	Unmanned Underwater Vehicle
UVMS	Underwater Vehicle-Manipulator System

Notations

$\Sigma_i, O - \boldsymbol{xyz}$	Inertial frame (see Fig. 2.10)
$\Sigma_b, O_b - \boldsymbol{x_b y_b z_b}$	Body(vehicle)-fixed frame (see Fig. 2.10)
$\mathbb{R}$, $\mathbb{N}$	Real, natural numbers
$\eta_1 = [x\,y\,z]^{\mathrm{T}} \in \mathbb{R}^3$	Body(vehicle) position coordinates in the inertial frame (see Fig. 2.10)
$\eta_2 = [\phi\,\theta\,\psi]^{\mathrm{T}} \in \mathbb{R}^3$	Body(vehicle) Euler angle coordinates in the inertial frame (see Fig. 2.10)
$\mathcal{Q} = \{\varepsilon \in \mathbb{R}^{\not\models}, \eta \in \mathbb{R}\}$	Quaternion expressing the body(vehicle) orientation with respect to the inertial frame defined in Sect. 2.4.2
$\boldsymbol{\eta} = \left[\boldsymbol{\eta}_1^{\mathrm{T}}\,\boldsymbol{\eta}_2^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^6$	Body(vehicle) position/orientation defined in Eq. 2.15
$\boldsymbol{\eta}_q = \left[\boldsymbol{\eta}_1^{\mathrm{T}}\,\varepsilon^{\mathrm{T}}\,\boldsymbol{\eta}\right]^{\mathrm{T}} \in \mathbb{R}^7$	Body(vehicle) position/orientation with the orientation expressed by quaternions defined in Eq. 2.20
$\boldsymbol{v}_1 = [u\,v\,w]^{\mathrm{T}} \in \mathbb{R}^3$	Vector representing the linear velocity of the origin of the body(vehicle)-fixed frame with respect to the origin of the inertial frame expressed in the body (vehicle)-fixed frame (see Fig. 2.10)
$\boldsymbol{v}_2 = [p\,q\,r]^{\mathrm{T}} \in \mathbb{R}^3$	Vector representing the angular velocity of the body (vehicle)-fixed frame with respect to the inertial frame expressed in the body(vehicle)-fixed frame (see Fig. 2.10)
$\boldsymbol{v} = [\boldsymbol{v}_1^{\mathrm{T}}\,\boldsymbol{v}_2^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^6$	Vector representing the linear/angular velocity in the body(vehicle)-fixed frame defined in Eq. 2.16
$\boldsymbol{R}_\alpha^\beta \in \mathbb{R}^{3\times 3}$	Rotation matrix expressing the transformation from frame α to frame β
$\boldsymbol{J}_{k,o}(\boldsymbol{\eta}_2) \in \mathbb{R}^{3\times 3}$	Jacobian matrix defined in Eq. 2.9
$\boldsymbol{J}_{k,oq}(\mathcal{Q}) \in \mathbb{R}^{4\times 3}$	Jacobian matrix defined in Eq. 2.14
$\boldsymbol{J}_e(\boldsymbol{\eta}_2) \in \mathbb{R}^{6\times 6}$	Jacobian matrix defined in Eq. 2.17
$\boldsymbol{J}_{e,q}(\mathcal{Q}) \in \mathbb{R}^{7\times 6}$	Jacobian matrix defined in Eq. 2.21
$\tau_1 = [X\,Y\,Z]^{\mathrm{T}} \in \mathbb{R}^3$	Vector representing the resultant forces acting on the rigid body(vehicle) expressed in the body(vehicle)-fixed frame defined in Eq. 2.29
$\tau_2 = [K\,M\,N]^{\mathrm{T}} \in \mathbb{R}^3$	Vector representing the resultant moment acting on the rigid body(vehicle) expressed in the body(vehicle)-fixed frame to the pole O_b defined in Eq. 2.36
$\tau_v = [\tau_1^{\mathrm{T}}\,\tau_2^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^6$	Generalized forces: forces and moments acting on the vehicle defined in Eq. 2.38
$\tau_\upsilon^* \in \mathbb{R}^6$	Generalized forces in the earth-fixed-frame-based model defined in Eq. 2.60
n	Degrees of freedom of the manipulator
$\boldsymbol{q} \in \mathbb{R}^n$	Joint positions defined in Sect. 2.13

$\boldsymbol{\eta}_{ee1} = [x_E\, y_E\, z_E]^{\mathrm{T}} \in \mathbb{R}^3$	Position of the end-effector in the inertial frame defined in Eq. 2.65 (denoted with $x = [x_E\, y_E\, z_E]^{\mathrm{T}}$ in the interaction control sections)
$\eta_{ee2} = [\phi_E\; \theta_E\; \psi_E]^{\mathrm{T}} \in \mathbb{R}^3$	Orientation of the end-effector in the inertial frame expressed by Euler angles defined in Eq. 2.65
$\eta_{ee} = [\eta_{ee1}^{\mathrm{T}}\; \eta_{ee2}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^6$	End-effector position/orientation expressed in the inertial frame
$\zeta = \left[v_1^{\mathrm{T}} v_2^{\mathrm{T}} \dot{q}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{6+n}$	System velocity defined in Eq. 2.66
$v_{ee1} \in \mathbb{R}^3$	End-effector linear velocity with respect to the inertial frame expressed in the end-effector frame defined in Eq. 2.73
$v_{ee2} \in \mathbb{R}^3$	End-effector angular velocity with respect to the inertial frame expressed in the end-effector frame defined in Eq. 2.74
$\boldsymbol{J}_k(\boldsymbol{R}_B^I) \in \mathbb{R}^{(6+n)\times(6+n)}$	Jacobian matrix defined in Eq. 2.67
$\boldsymbol{J}_k(\boldsymbol{R}_B^I, q) \in \mathbb{R}^{(6+n)\times(6+n)}$	Jacobian matrix defined in Eq. 2.77
$\tau_q \in \mathbb{R}^p$	Joint torques defined in Eq. 2.79
$\tau = \left[\tau_v^{\mathrm{T}} \tau_q^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{6+n}$	Generalized forces: vehicle forces and moments and joint torques defined in Eq. 2.80
$\boldsymbol{u} \in \mathbb{R}^p$	Control inputs, $\tau = \boldsymbol{Bu}$ (see Eq. 2.82)
$\Phi \in R^{(6+n)\times n_\theta}$	UVMS regressor defined in Eq. (2.83)
$\theta \in R^{n_\theta}$	Vector of the dynamic parameters of the UVMS regressor defined in Eq. 2.83
$\boldsymbol{\Phi}_\upsilon \in R^{6\times n_{\theta,v}}$	Vehicle regressor defined in Eq. 2.61
$\theta_\upsilon \in R^{n_{\theta,v}}$	Vector of the dynamic parameters of the vehicle regressor defined in Eq. 2.61
$\boldsymbol{h}_i^i = [\boldsymbol{f}_i^{i\mathrm{T}}\; \boldsymbol{\mu}_i^{i\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^6$	Forces and moments exerted by body $i-1$ on body i (see Fig. 2.27)
$\boldsymbol{h}_e = [\boldsymbol{f}_e^{\mathrm{T}}\; \boldsymbol{\mu}_e^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^6$	Forces and moments at the end-effector (see Fig. 2.29)
$t \in \mathbb{R}$	Time
$\lambda_{\min(\max)}(\boldsymbol{X})$	Smallest(largest) eigenvalue of matrix $\boldsymbol{X}$
$\mathrm{diag}\{x_1, \ldots, x_n\}$	Diagonal matrix filled with x_i in the i row, i column and zero in any other place
$\mathrm{blockdiag}\{\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n\}$	Block diagonal matrix filled with matrices $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ in the main diagonal and zero in any other place
$\mathcal{R}(\boldsymbol{X})$	Range of matrix $\boldsymbol{X}$
$\dot{x}$	Time derivative of the variable x
$\lVert\boldsymbol{x}\rVert$	2-norm of the vector x
$\hat{x}(\hat{\boldsymbol{X}})$	Estimate of the vector $\boldsymbol{x}$ (matrix $\boldsymbol{X}$)
x_{d}	Desired value of the variable x
$\tilde{x}$	Error variable defined as $\tilde{x} = x_{\mathrm{d}} - x$
$x^{\mathrm{T}}(\boldsymbol{X}^{\mathrm{T}})$	Transpose of the vector $\boldsymbol{x}$ (matrix $\boldsymbol{X}$)
x_i	ith element of the vector $\boldsymbol{x}$

$X_{i,j}$	Element at row i , column j of the matrix $\boldsymbol{X}$
$X^{\dagger}$	Moore–Penrose inversion (pseudoinversion) of matrix $\boldsymbol{X}$
$\boldsymbol{I}_r$	$(r \times r)$ identity matrix
$\boldsymbol{O}_{r_1 \times r_2}$	$(r_1 \times r_2)$ null matrix
$\boldsymbol{S}(\cdot) \in \mathbb{R}^{3\times 3}$	Matrix performing the cross product between two (3×1) vectors defined in Eq. 2.82
ρ^3	Water density
μ	Fluid dynamic viscosity
R_n	Reynolds number
g^I	Gravity acceleration expressed in the inertial frame