A First Introduction to Quantum Computing and Information

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For Judith

Foreword

This book is an excellent introduction to the exciting and rapidly developing field of Quantum Computing and Information. It is well suited to senior undergraduate students, working engineers, and scientists interested in acquiring a solid understanding of this subject matter. This book is a good starting point for those wanting to develop practical experience with the conceptual and mathematical tools required for more advanced studies and for entry-level participation in related theoretical and experimental research. The book is well organized, clear, and concise, and it covers the most relevant introductory topics in Quantum Computing and Information today.

Written in easy to understand prose, and with ample mathematical depth, this book will be of great value to students who want to take the first step in mastering this subject material. Emphasis is placed on core concepts, rather than historical developments and narratives, and these concepts are developed through a series of relevant, in-text examples that allow students to work through mathematical details, step-by-step, without having to divert their attention to appendices and articles in the open literature. Each chapter is also accompanied by an equally well-designed set of problems that will allow students to test their new knowledge and incrementally hone their technical skills as they proceed from chapter to chapter. In addition, links are provided to web resources, including Mathematica notebooks, allowing readers to continue their exploration of this subject matter through numerical experimentation and visualization.

As a Ph.D. physicist and engineer working in this field, I appreciate the nature and content of this book, and the benefits students and working professionals will get by reading it. I have known Professor Bernard Zygelman for more than two decades. I have taken classes from him, have collaborated in research with him as an undergraduate, and have significantly benefited from numerous conversations with him on topics within physics. So I am thrilled he has written this book, as I am sure it will benefit others. This book successfully captures Professor Zygelman's knowledge in Quantum Computing and Information and his engaging and effective teaching style and methodology in a format that will now reach a much broader audience.

Scottsdale, AZ, USA William Clark
February 2018 Ph.D. Physics

Preface

Motivation

This monograph is an outgrowth of courseware developed and offered at UNLV to a predominantly undergraduate audience. The motivation for the course and, by extension, this textbook is twofold. First, it is a response to the explosive growth and transformative technological promise of quantum computing and information (QCI) science. Second, for an introductory treatment, the requisite mathematical background is available to most students who have successfully digested the first year of university-level calculus. In contrast, a typical undergraduate quantum curriculum requires at least some exposure to the theory of partial differential equations, complex analysis, as well as advanced vector calculus. For this reason, many universities, including UNLV, expose students to the quantum theory in their junior and senior years. Quantum science is too important and relevant for its relegation to an upper-level offering. The material in this textbook should be accessible to most sophomores, and for AP students, freshman-level, physics, computer science, and math majors. Though several excellent texts are directed to an undergraduate audience, notably N. David Mermin's *Quantum Computer Science: An Introduction*, Cambridge U. Press 2007, I have striven to make the material accessible to an entry-level cohort. To that end, the textbook includes numerous links to Wolfram Mathematica® notebooks that offer numerical demonstrations and exercises that help illuminate difficult and counterintuitive concepts. The latter help guide the student, via an interactive approach, through the written material on the page. Conceding rigor in favor of accessibility, I hope to pique the interest of students in this novel, dynamic, branch of physics and computer science. Successful completion of it should prime students for advanced study and help facilitate their exploration of leading-edge developments in the current literature.

Scope

The modern quantum theory, one of the grand achievements of twentieth century science, will soon be celebrating its 100th anniversary. Estimates show that as much as 40% of the American GDP is owed to technologies whose foundational underpinning is provided by that theory. A discipline celebrating its centennial is expected to be mature and well vetted. Indeed, I know of no experimental data that has undermined or contradicted the basic tenets of the theory codified in the mid-1920s. Arguably, it is one of the most robust and fecund scientific paradigms ever advanced. Therefore, it is surprising that it took almost three-quarters of a century of gestation for the novel insights and potential technological rewards offered by QCI to become widely disseminated and accepted in the greater community. David Kaiser presents provocative insights on that question in his engaging social science treatise, *How the Hippies Saved Physics: Science, Counterculture, and the Quantum Revival*, Norton 2011. Regardless, the days of "shut-up and calculate" are over, and it is imperative that our undergraduate quantum curriculum keeps pace with current developments.

In the first two chapters, I introduce the foundational framework for the Copenhagen interpretation of quantum mechanics as it applies to qubit systems. I discuss the Dirac bra-ket notation and the tools necessary to manipulate objects in a multi-qubit Hilbert space. Chapter 3 reviews the circuit model of computation and I introduce the first quantum gates and circuits. Chapter 4 provides a detailed discussion of the Shor and Grover algorithms. Chapters 5 and 6 are concerned with aspects of quantum information theory. Chapter 9 deals with the theory of quantum error correction; the chapters mentioned above constitute a reasonably paced 14-week semester introduction to QCI. In my experience, I found that students are also curious of recent developments in quantum hardware, the guts of a quantum computer. Unfortunately, except Nielsen and Chuang's *Quantum Computation and Quantum Information*, Cambridge U. Press (2010), few textbooks cover this aspect in detail. For that reason, I included Chap. 7 and 8 which cover developments in trapped ion, quantum cavity electrodynamics (CQED), and quantum circuit (cQED) computers. That discussion proved challenging because a comprehensive treatment requires an advanced understanding of atomic physics, quantum optics, and many-body physics. Nevertheless, I believe that I succeeded in conveying the essential features of those paradigms by using models that require only a passing familiarity with differential equations and complex analysis. In an introductory course, those chapters could be bypassed. I have also avoided a detailed discussion of computational complexity, universality, and measurement theory. The motivated student who completes the coursework will be able to access these advanced topics in Nielsen and Chuang's comprehensive treatment.

On Notation

I use a convention, common among physicist, for labeling states in multi-qubit kets that differs from that used in many QCI texts. Given ket $|abc\rangle$, I refer to the entry c as the first qubit, b the second, and a the third qubit. For the corresponding bra vectors, that order (from right to left) is inverted. This convention also leads to a notational adjustment in wire diagrams. With the convention used in this monograph (in alignment with Mermin's text), the first qubit c is placed on the lowest rung of a wire diagram, with the second and third, etc. qubit states stacked on it. I employ arrow notation to label vector quantities and use boldface for most operators. For common, such as the Pauli, operators where there is little chance of misunderstanding I use regular typeface. In most QCI literature operators are synonymous with their matrix representations, and so I make little effort to distinguish them. However, in some instances where there is a possibility for ambiguity, I explicitly use an under-bar notation to stress the matrix character of an operator.

Las Vegas, NV, USA Bernard Zygelman
February 2018

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