Optimization Formulations for the Design of Low Embodied Energy Structures Made from Reused Elements

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Abstract. The building sector is one of the major contributors to material resource consumption, greenhouse gas emission and waste production. Load-bearing systems have a particularly large environmental impact because of their material and energy intensive manufacturing process. This paper aims to address the reduction of building structures environmental impacts through *reusing* structural elements for multiple service lives. *Reuse* avoids sourcing raw materials and requires little energy for reprocessing. However, to design a new structure reusing elements available from a stock is a challenging problem of combinatorial nature. This is because the structural system layout is a result of the available elements' mechanical and geometric properties. In this paper, structural optimization formulations are proposed to design truss systems from available stock elements. Minimization of weight, cut-off waste and embodied energy are the objective functions subject to ultimate and serviceability constraints. Case studies focusing on embodied energy minimization are presented for: 1) three roof systems with predefined geometry and topology; 2) a bridge structure whose topology is optimized using the ground structure approach; 3) a geometry optimization to better match the optimal topology from 2 and available stock element lengths. In order to benchmark the energy savings through reuse, the optimal layouts obtained with the proposed methods are compared to weight-optimized solutions made of new material. For these case studies, the methods proposed in this work enable reusing stock elements to design structures embodying up to 71 % less energy and hence having a significantly lower environmental impact with respect to structures made of new material.

Keywords: structural optimization, reuse, stock assignment, embodied energy

1 Introduction

1.1 Motivation

In many industrialized countries, the building sector is the most resource intensive sector [1]. Up to 50 % of the total material use in Europe is associated to buildings and infrastructures [2, 3]. Furthermore, a significant amount of energy is spent to manufacture building components as well as for the construction and the use of buildings and infrastructures. The building sector is responsible for about one third of all energy use and high greenhouse gas (GHG) emissions worldwide [4, 5]. With regard to buildings a distinction between two types of energy and carbon can be made:

- *Operational energy* refers to the energy consumption and associated *operational carbon* emissions during the use-phase of buildings (heating, cooling, etc.).

- *Embodied energy* quantifies the amount of energy used for the extraction of raw materials, the production and transport of building components as well as the building construction and end-of-life (EOL). *Embodied carbon* refers to the associated GHG emissions.

Technical standards (e.g. LEED, BREEAM, Passivhaus, Minergie) focus mainly on reducing the building operational energy. However, little attention has been given to the embodied energy, which contributes significantly to the total environmental impacts of buildings and infrastructures [6-7]. Load-bearing systems contribute the most to the building embodied energy share [8-10] because constructing structures is a material and energy intensive process. Another issue is the construction and demolition waste (C&DW) originating from buildings. More than a third of the European waste comes from the building sector amounting to 870 million tons annually [11].

From these observations, it follows that the design and construction of buildings and infrastructures could be improved by making a more efficient use of materials. The European Commission issued an action plan to implement circular economy principles to mitigate the exploitation of natural resources, the energy consumption and the generation of superfluous waste in the building sector [12]. The circular economy paradigm suggests a closed loop flow of materials and products including recycling and reuse.

Recycling is the common approach to treat products after their use. However, recycling requires energy to process materials (e.g. melting steel) and often results in a loss of quality (down cycling, e.g. crushed concrete for road construction). An often more sustainable option is *reuse*, which avoids sourcing raw materials and requires little energy for reprocessing (e.g. reshaping the ends of a truss member to fit a new setting). Reusing structural elements for multiple service lives is identified as an opportunity to reduce the high embodied energy of traditionally designed load-bearing systems, thus lowering the overall building environmental impacts [13-14]. A recent example where reuse was successfully implemented is the BedZED project, a large scale residential and office development in London. Up to 95 % of all structural steel could be reclaimed from local demolition sites [13]. Another example is the roof of the London 2012 Olympic Stadium, which was built reusing 2500 tons of steel tubes (20 % of the total structure) over ordered in an oil and gas pipeline project [5]. This adaption eventually reduced the embodied energy, construction time and overall costs. In order to assess the available elements' mechanical properties small specimen were cut out of each tube and tested [5]. Researchers at EPFL's Structural Xploration Lab [15] reused 210 skis for constructing a transportable, elastically bent grid-shell pavilion. It has been shown, that the structure made from reclaimed skis has a 85 % lower cumulative energy demand than an equivalent grid-shell made of new timber.

1.2 Related Work

Structural optimization is usually carried out to improve structural performances (e.g. minimize weight and compliance) under a given set of loads and boundary conditions. Optimal truss layouts can be searched varying the geometry, topology and the member cross sections [16]. Most commonly used methods to optimize truss systems are based on the ground structure approach [17]. A ground structure consist of a grillage of many members of which only a subset will be used in the optimal design. To further improve optimality, topology and geometry optimization (i.e. varying the truss node positions) have been combined. Achtziger [18] presented methods to optimize simultaneously truss geometry and topology. Alternatively, He and Gilbert [19] employ geometry optimization to improve further optimal topology truss systems.

Generally, structural optimization is carried out using continuous design variables, e.g. member cross section areas. However, in practice only a set of standardized cross sections is available (e.g. I-beam or hollow sections) thus making the design variables discrete. Rasmussen and Stolpe [20] formulated topology and discrete cross section optimization as a mixed-binary problem including stress and deformation constraints. This problem was efficiently solved to global optimality employing branch-and-cut methods [20] and the Simultaneous ANalysis and Design (SAND) approach [21]. It was key to disaggregate equilibrium and compatibility into mixed-binary constraints that can be reformulated as linear inequalities [20].

Most optimization methods work with the assumption that structural members can be fabricated according to the optimal layout (e.g. optimal member cross sections and lengths). Usually there is no constraint on the number of available elements. Conversely, when reusing structural elements from a given stock, element properties including the cross section and element length are set a-priori and the number of elements with certain properties is finite.

An example optimization of a plane frame with fixed topology and geometry is shown in [22]. From a set of standard cross sections for each member in the structure one cross section is assigned. Each stock cross section is available only once. The objective is to minimize the weight of the structure subject to stress and deflection constraints. A genetic algorithm was employed to solve the problem. However, this formulation did not consider stock element lengths.

An arch roof built from 20 unique tree forks is presented in [23]. The stock was created through 3D-scanning available tree forks. The elements were sequentially assigned to optimally match the intended arch shape employing a meta-heuristic process.

Form-fitting strategies for constructing trusses from a set of irregular timber elements are shown in [24]. The truss geometry and topology of the static determinate system is fixed and element assignment is carried out using heuristics. The objective is to minimize waste subject to element tension and compression capacity.

1.3 Outline

Generally, in conventional structural design practice the structure topology and geometry are defined a-priori or derived via optimization methods. Then, element properties (e.g. cross section and length of a truss element) are determined. Conversely, designing a structure reusing elements from a stock entails that the element mechanical and geometric properties are given prior the structural layout is known. This is a challenging problem of combinatorial nature, which has received little attention in structural optimization.

This paper proposes new structural optimization formulations to design truss systems from an available stock of elements. The main objective is to minimize the structure embodied energy subject to ultimate and serviceability limit state constraints. Case studies are presented for three roof systems and a bridge truss. Life Cycle Assessment (LCA) is carried out to benchmark structures designed with the proposed methods against weight-optimized solutions made of new material.

2 Optimization Formulation

2.1 Assumptions for Reuse and Stock.

The main underlying assumptions are that a stock of reclaimed structural elements without defects is readily available and that custom joints allow their connection. It is expected that members can be cut but not extended. A stock of structural elements is characterized by:

- Material (elasticity, strength, density, specific weight)
- Cross sections (area, area moment of inertia)
- Lengths
- Number of available elements
- Origin of the elements

Identical elements of same material, cross section size and length are grouped.

2.2 Assignment Problem.

The selection of suitable structural elements from a stock is an assignment problem of combinatorial nature. Fig. 1 shows a truss layout consisting of m = 5 members. For each of the m truss member locations one element must be assigned from s = 7 stock member groups. Each group j contains n_j identical elements.

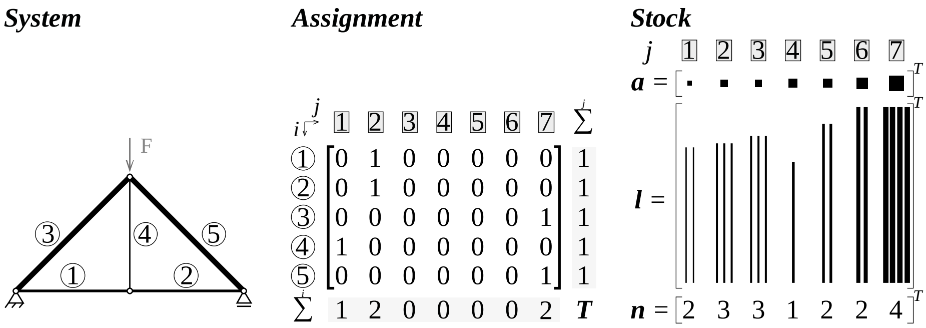

Fig. 1. Assignment of available stock elements to a structural system

The assignment is represented through a binary matrix $\boldsymbol{T} \in \{0, 1\}^{m \times s}$, whose entries are:

$$t_{i,j} = \begin{cases} 1 & \text{if a stock element of group } j \text{ is assigned to the } i\text{th bar position} \\ 0 & \text{if the stock element of group } j \text{ is not assigned to position } i \end{cases}$$

The sum of all entries in each row $\boldsymbol{t}_i$ must be one to ensure the assignment of exactly one stock element at location i in the truss layout (Eq. 1). Each column of the assignment matrix corresponds to a stock element group. The sum of all entries in each column must be smaller or equal to the number n_j of available elements in the corresponding group (Eq. 2).

Assignment
$$\sum_{j=1}^{s} t_{i,j} = 1 \;\; \forall\, i = 1 \dots m \tag{1}$$

Availability
$$\sum_{i=1}^{m} t_{i,j} \leq n_j \;\; \forall\, j = 1 \dots s \tag{2}$$

$$t_{i,j} \in \{0, 1\} \; \forall\, i, j$$

2.3 Objective and Constraints.

When topology and geometry are invariant, the design variables are the components $t_{i,j}$ of the assignment matrix $\boldsymbol{T} \in \{0, 1\}^{m \times s}$. The stock is described by column vectors of available cross section areas $\boldsymbol{a} \in \mathbb{R}^s$, Young's moduli $\boldsymbol{e} \in \mathbb{R}^s$, area moments of inertia $\boldsymbol{I} \in \mathbb{R}^s$, admissible stress in tension $\boldsymbol{\sigma}^+ \in \mathbb{R}^s$, admissible stress in compression $\boldsymbol{\sigma}^- \in \mathbb{R}^s$, element lengths $\boldsymbol{l} \in \mathbb{R}^s$, material densities $\boldsymbol{\rho} \in \mathbb{R}^s$ and specific weights $\boldsymbol{\gamma} \in \mathbb{R}^s$. The size of these vectors corresponds to the number of groups s collating stock elements with identical properties.

Most optimization formulations combine equilibrium and compatibility constraints. In this formulation instead, following [20], equilibrium and compatibility are treated separately. Different to [20], the formulation additionally includes self-weight, Euler buckling and the available member lengths and numbers (i.e. stock constraints).

The state variables are the member forces $\boldsymbol{p}^k \in \mathbb{R}^m$ and nodal displacements $\boldsymbol{u}^k \in \mathbb{R}^d$. The vector $\boldsymbol{u}^k$ has size d which is the number of unsupported degrees of freedom. For each load case k equilibrium at nodes under external forces $\boldsymbol{f}^k \in \mathbb{R}^d$ is computed via Eq. 4 where $\boldsymbol{B} \in \mathbb{R}^{d \times m}$ is the equilibrium matrix containing the element direction cosines. Self-weight is included in the external force vector adding gravity loads at the member ends through the product of matrix $\boldsymbol{D} \in \mathbb{R}^{d \times m}$ and the element specific weight (Eq. 4). Each column $\boldsymbol{d}_i$ of $\boldsymbol{D}$ contains half the member length l'_i at components corresponding to the vertical degrees of freedom of the element ends. The compatibility constraint (Eq. 5) relates element deformations, nodal displacements and member force via the transpose of the equilibrium matrix $\boldsymbol{B}^T$. Note that the assignment matrix is included in the compatibility equation because the element Young's moduli and cross-section areas are the inner product of $\boldsymbol{T}$ and the stock vectors $\boldsymbol{e}$ and $\boldsymbol{a}$. The operator $\circ$ indicates an element wise multiplication (Hadamard product).

Member forces are bounded by the admissible stress in tension and compression of assigned elements (Eq. 6). In addition, Euler buckling is considered (Eq. 7). Nodal displacements are bounded via (Eq. 8) to satisfy suitable serviceability limits. Only stock elements that are longer or equal to the structures' member lengths $\boldsymbol{l'}$ can be assigned (Eq. 9). Finally, the assignment and availability constraints introduced in previous section must be considered (Eqs. 1 and 2).

Objective	$\min f(\boldsymbol{T}, \boldsymbol{p}^k, \boldsymbol{u}^k)$	(3)
Equilibrium	$\boldsymbol{B}\boldsymbol{p}^k = \boldsymbol{f}^k - \boldsymbol{D}\boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\gamma}) \;\; \forall\, k$	(4)
Compatibility	$diag\big(\boldsymbol{T}(\boldsymbol{e} \circ \boldsymbol{a})\big)\boldsymbol{B}^T\boldsymbol{u}^k - diag(\boldsymbol{p}^k)\boldsymbol{l}' = 0 \;\; \forall\, k$	(5)
Stress capacity	$\boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\sigma}^-) \le \boldsymbol{p}^k \le \boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\sigma}^+) \;\; \forall\, k$	(6)
Buckling	$-\dfrac{\pi^2\boldsymbol{T}(\boldsymbol{e} \circ \boldsymbol{I})}{\boldsymbol{l}'^2} \le \boldsymbol{p}^k \;\; \forall\, k$	(7)
Deformation bounds	$\boldsymbol{u}^k_{min} \le \boldsymbol{u}^k \le \boldsymbol{u}^k_{max} \;\; \forall\, k$	(8)
Maximum length	$\boldsymbol{T}\boldsymbol{l} \ge \boldsymbol{l}'$	(9)

Optimization objectives can be functions of assignment variables, member forces or displacements (Eq. 3). Compliance optimization is formulated as the minimization of deformations under load (Eq. 10). If the volume is not constrained, this objective will result in selecting the strongest available stock elements that geometrically fit the design.

$$\min \sum_k \boldsymbol{f}^{k\,T} \boldsymbol{u}^k \tag{10}$$

More relevant for the case of reusing structural elements is a volume (Eq. 11) or mass (Eq. 12) minimization. Here cross section areas are minimized resulting in a better utilization of the element capacity.

$$\min V = \boldsymbol{l}'^T \boldsymbol{T} \boldsymbol{a} \tag{11}$$

$$\min M = \boldsymbol{l}'^T \boldsymbol{T} (\boldsymbol{a} \circ \boldsymbol{\rho}) \tag{12}$$

To better match the assigned stock elements with the structural layout, the total cut-off length Δl or corresponding cut-off waste ΔM can be minimized. Eq. 14 weighs the element cut-off length (Eq. 13) with the corresponding cross section area, thus cutting elements with small cross sections is encouraged.

$$\min \Delta l = \sum_{i=1}^{m} \boldsymbol{t}_i \boldsymbol{l} - l'_i \tag{13}$$

$$\min \Delta M = \sum_{i=1}^{m} \boldsymbol{t}_i (\boldsymbol{l} \circ \boldsymbol{a} \circ \boldsymbol{\rho}) - l'_i \boldsymbol{t}_i (\boldsymbol{a} \circ \boldsymbol{\rho}) \tag{14}$$

2.4 Problem Nature and Reformulation.

The formulation given in the previous section is a highly constrained mixed integer problem (MIP). For a given structure of m members and a given stock with a total number of $\hat{s}$ stock elements, the number of possible assignment combinations c is:

$$c = \frac{\hat{s}!}{(\hat{s} - m)!} \quad \text{where} \quad \hat{s} = \sum_{j=1}^{s} n_j \tag{15}$$

A full enumeration of the problem is impractical even for simple structural configurations. The objective and constraint functions are linear, except the compatibility Eq. 5. This is an equality constraint and consists of a mixed binary-continuous product of assignment variables $t_{i,j}$ and nodal displacements $\boldsymbol{u}$. A reformulation of this constraint as a set of multiple linear inequalities including additional auxiliary variables is implemented as shown in [20]. This way the optimization problem is equivalently described as a mixed integer linear program (MILP) which can be solved to global optimality employing the SAND approach. SAND implies that design variables $\boldsymbol{T}$ and state variables ($\boldsymbol{p}^k$, $\boldsymbol{u}^k$) are treated simultaneously as optimization variables in the problem. The problems for the case studies presented in this paper were solved using a branch-andbound solver [25]. The modeling of the problem was carried out with an optimization toolbox [26].

3 Life Cycle Assessment

3.1 Environmental Impacts Associated with Reusing Structural Elements.

The aim of reusing structural elements is the reduction of building environmental impacts. To benchmark the impact of structures optimized with the method proposed here, a Life Cycle Assessment (LCA) is carried out. Fig. 2 illustrates the main assumptions taken for the LCA of structures made from reused elements:

- Reclaimed stock elements are retrieved through selective deconstruction.
- The impacts caused through storage of elements is neglected.
- Only the elements taken from the stock are transported.
- The left over stock can be used elsewhere.
- The elements are transported to a fabrication site over a distance $d_{T,S}$ = 150 km
- Cutting of elements happens at the fabrication site.
- The impacts caused by element ends adaption and fabrication of custom joints is neglected.
- The final structure or its parts are transported from the fabrication site to the building site over a distance $d_{T,F}$ = 50 km.
- Cut-off scrap is transported to a recycling facility over a distance $d_{T,EOL}$ = 20 km and disposed causing EOL impacts I_{EOL}.

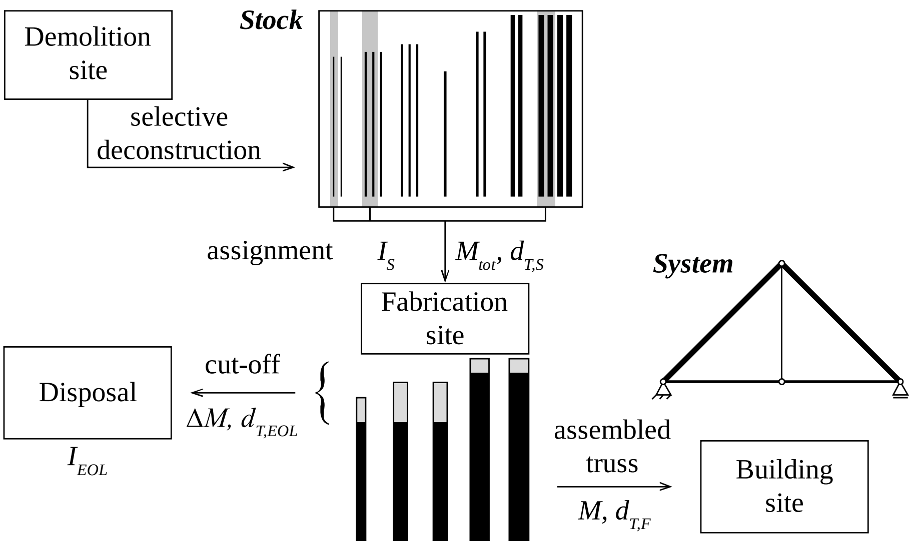

Fig. 2. Life Cycle Assessment for structures made from reused elements

The energy impacts given in [27] are considered for the calculation of impacts I_S related to sourcing structural elements for reuse. Selective deconstruction impacts I_{SD} consider a careful disassembly of a steel structure by removing connections and hoisting structural members with a mobile crane (I_C) [27]. Comparative impacts I_D caused by conventional demolition whereby all steel scrap is recycled, are reported in [27]. Because conventional demolition is assumed inevitable, for the supply of stock elements only the environmental impacts of the difference between selective and conventional demolition as shown in Table 1 are considered. The associated GHG emissions are not provided in [27] thus here converted from the combustion of diesel.

Table 1. Environmental impacts of selective deconstruction for stock element supply

Process	Energy [MJ/kg]	GHG emissions [kgCO_{2eq}/kg]	Source
I_{SD}	2.181	0.1878	[27]
I_D	0.359	0.0309	[27]
I_C	1.275	0.110	[27]
$I_S = I_{SD} - I_D + I_C$	3.097	0.2669	

For the impacts I_T related to transport of elements, the generic data of the Ökobaudat dataset 9.3.01 [28] is used as indicated in Table 2.

Table 2. Environmental impacts of transport

Process	Energy [MJ/(kg·km)]	GHG emissions [kgCO_{2eq}/(kg·km)]	Source
I_T	$0.7385 \cdot 10^{-3}$	$0.0508 \cdot 10^{-3}$	[28]

If element cutting is necessary to fit stock elements to the structural system dimensions, the off cut parts are disposed. This cut-off waste ΔM causes EOL environmental impacts in accordance Ökobaudat dataset 100.1.04 [28] which is indicated in Table 3:

Table 3. Environmental impacts of the EOL treatment of cut-off scrap

Process	Energy [MJ/kg]	GHG emissions [kgCO_{2eq}/kg]	Source
I_{EOL}	$12.072 \cdot 10^{-3}$	$0.8068 \cdot 10^{-3}$	[28]

From this information it is possible to quantify the total embodied energy and GHG emissions (Eq. 16) for structures made from reused elements:

$$I_{Reuse} = I_S M_{tot} + d_{T,S} I_T M_{tot} + d_{T,F} I_T M + I_{EOL} \Delta M + d_{T,EOL} I_T \Delta M \quad (16)$$

where the total mass of the selected stock elements is:

$$M_{tot} = M + \Delta M = \sum_{i=1}^{m} \boldsymbol{t_i} (\boldsymbol{l} \circ \boldsymbol{a} \circ \boldsymbol{\rho}) \quad (17)$$

Replacing M_{tot} in Eq. 16, the total embodied energy is expressed as:

$$E_{Reuse} = 3.245\frac{MJ}{kg}M + 3.235\frac{MJ}{kg}\Delta M \quad (18)$$

Eq. (18) includes the mass M of the structure and the cut-off mass ΔM. Since mass (Eq. 12) or cut-off (Eq. 14) minimization alone could converge to different stock element assignment solutions, Eq. (18) is used to combine both objectives.

3.2 Environmental Impacts of Structures Made of New Elements.

Reuse avoids the production of new structural elements. To benchmark the environmental savings through reuse against newly produced elements, common production methods involving primary and secondary (recycled) are considered. The impacts I_{Reuse} are compared to impacts I_{New} of weight-optimized systems. The production impacts I_P for steel MSH profiles are indicated in Table 4. This data (Ökobaudat dataset 4.1.03, [28]) is based on Environmental Product Declarations and represents today's common manufacturing processes.

Table 4. Environmental impacts for the production of new steel MSH profiles

Process	Energy [MJ/kg]	GHG emissions [$kgCO_{2eq}/kg$]	Source
I_P	13.175	0.921	[28]

It is assumed that first-hand steel members are produced with exact lengths (no waste), having a structural mass M_{new}. The transport distance to the fabrication site is assumed to be $d_{T,N}$ = 20 km. The total environmental impacts for structures with newly produced elements are calculated as:

$$I_{New} = I_P M_{New} + (d_{T,N} + d_{T,F}) I_T M_{New} \quad (19)$$

4 Applications

4.1 Optimization of Roof Trusses with Fixed Geometry and Topology

System Description. Fig. 3 shows three roof truss systems which are taken as case studies. A conventional Howe (A), Warren (B) and pitched Pratt (C) truss are considered. System topology and geometry are fixed. The span of the trusses is 12.0 m. The lateral span is assumed 6.25 m and out of plane stability is provided by other means. All member connections are pin-jointed.

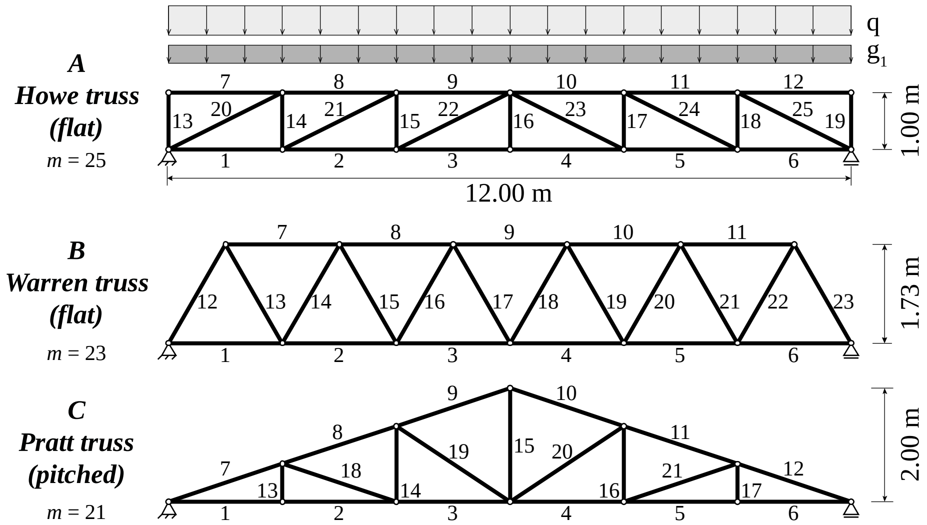

Fig. 3. Roof truss systems

Loading. The self-weight g_0 of the structure is taken into account. In addition, a superimposed dead load g_1, resulting from a roof cover and a secondary structure, is applied at the top chord nodes. Similar, a snow load q is applied over the full span. Two load combinations ($k = 2$) are considered: one for ultimate limited state (ULS) and one for serviceability limit state (SLS). For the SLS combination deflection limits are set to $l/300 = 40$ mm (Eq. 8). Table 5 summarizes load magnitudes and cases.

Table 5. Roof structures – load cases and combinations

Load case	Load magnitude	Description
g_0	from assignment	self-weight
g_1	2.50 kN/m	dead load (0.40 kN/m^2)
q	5.00 kN/m	live load (snow, 0.80 kN/m^2)
Load combination	Load factors	Description
ULS	$1.35 \cdot (g_0 + g_1) + 1.50 \cdot q$	design loads
SLS	$1.00 \cdot (g_0 + g_1) + 1.00 \cdot q$	characteristic loads

Stock. For this study, a stock of available elements is assumed. Table 6 shows the composition of $s = 7$ element groups. The sections are square MSH profiles of variable size. All elements in the stock are grade S235 steel with a yield strength of $f_{y,d} = \sigma^+ = 235$ MPa, a Young's modulus of $E = 210000$ MPa and a material density of $\rho = 7850$ kg/m^3.

Table 6. Roof structures - Stock composition

Stock group j		1	2	3	4	5	6	7
MSH		40×4	40×5	50×4	50×5	50×5	60×4	60×5
a_j	[cm^2]	5.59	6.73	7.19	8.73	8.73	8.79	10.7
I_j	[cm^4]	11.8	13.4	25.0	28.9	28.9	45.4	53.3
l_j	[m]	2.50	1.50	2.00	1.50	2.20	2.20	2.50
n_j	[-]	6	10	10	6	10	6	6

Comparison to Weight-Optimized Results. The three systems with reused elements are compared to equivalent designs made of new steel. In this case, the optimization is carried out assuming infinite availability of standard square MSH section types: 40×2.9, 40×3.2, 40×4, 40×5, 40×6.3, 40×7.1, 40×8, 50×3.2, 50×4, 50×5, 50×6.3, 50×7.1, 50×8, 50×10, 60×3.2, 60×4, 60×3.2, 60×4, 60×5, 60×6.3, 60×8, 60×10.

Optimization Results. The roof systems described in the previous section are optimized for three different objectives:

1) *min M* Mass (or volume) minimization (Eq. 12),
2) *min* ΔM Cut-off waste minimization (Eq. 14), and
3) *min E* Direct minimization of embodied energy E_{Reuse} (Eq. 18)

Fig. 4, Fig. 5 and Fig. 6 show the optimal layouts of the three case study structures. In each figure the top system (a) represents the minimum embodied energy solution from reused elements. The bottom system (b) is the corresponding minimum weight solution made of new material. The member cross section size is indicated through a corresponding line thickness.

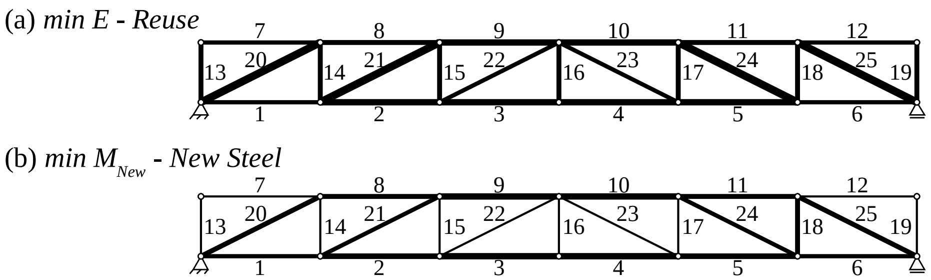

Fig. 4. System A - Optimized structures (a) made from reused elements (b) from new material

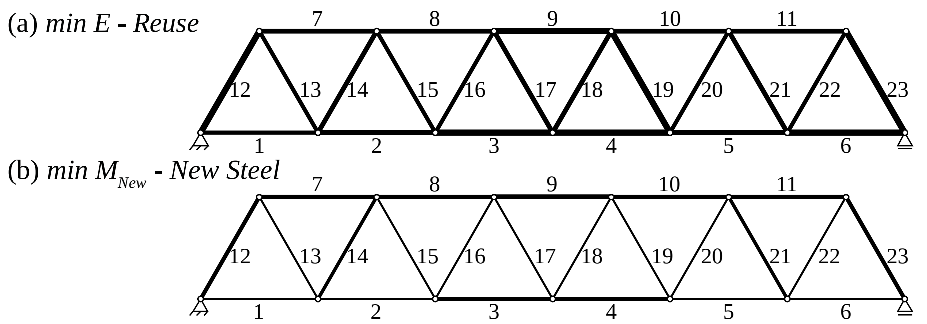

Fig. 5. System B - Optimized structures (a) made from reused elements (b) from new material

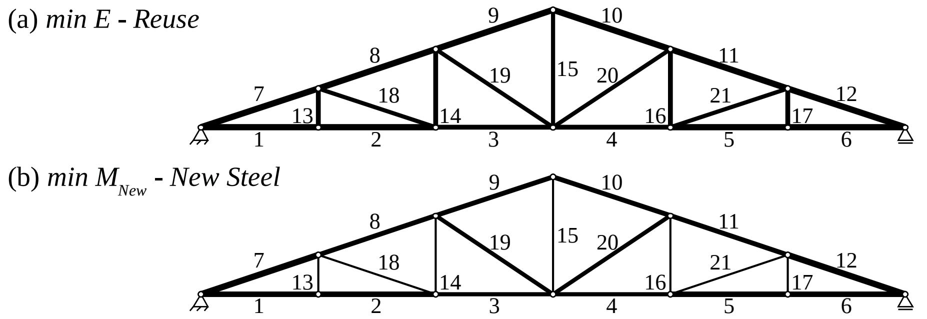

Fig. 6. System C - Optimized structures (a) made from reused elements (b) from new material

Table 7, Table 8 and Table 9 give results for reuse systems A, B and C respectively. The right most columns give results for structures made from new material. For reuse systems A and B mass minimization is equivalent to embodied energy minimization (A: 1033 MJ, B: 922 MJ). Minimizing cut-off waste results in a slightly higher embodied energy for both systems (A: 1037 MJ, B: 1003 MJ) yet it avoids 8.8 kg and 4.9 kg of additional waste respectively. Direct energy minimization of system C results in a tradeoff between mass and waste minimization (Table 9).

Table 7. System A – Howe truss – optimization results

System A		Reuse - Objective *min M*	*min ΔM*	*min E*	New steel *min M_{new}*
M_{tot}	[kg]	318.5	319.8	318.5	-
M	[kg]	271.8	281.9	271.8	217.3
ΔM	[kg]	46.7	37.9	46.7	-
E	[MJ]	1033	1037	1033	2874
u	[mm]	28.6	28.0	28.6	31.1
util.	[%]	51.1	48.0	51.1	84.6

Table 8. System B – Warren truss – optimization results.

System B		Reuse - Objective *min M*	*min ΔM*	*min E*	New steel *min M_{new}*
M_{tot}	[kg]	284.2	309.2	284.2	-
M	[kg]	261.5	291.3	261.5	177.9
Δ*M*	[kg]	22.7	17.8	22.7	-
E	[MJ]	922	1003	922	2354
u	[mm]	11.7	11.6	11.7	15.7
util.	[%]	43.9	37.7	43.9	80.0

Table 9. System C – Pratt truss – optimization results.

System C		Reuse - Objective *min M*	*min ΔM*	*min E*	New steel *min M_{new}*
M_{tot}	[kg]	263.3	268.8	260.3	-
M	[kg]	232.8	246.9	234.0	200.5
Δ*M*	[kg]	30.5	21.9	26.3	-
E	[MJ]	854	872	844	2652
u	[mm]	22.7	22.6	22.7	25.7
util.	[%]	64.8	55.9	61.2	86.9

The structures made from reused elements have a higher mass (up to +30 %, +64 %, +23 % for each system respectively) than the corresponding systems made of new sections. This is mainly due to a limited availability of small cross sections in the stock (see also Fig. 10 to Fig. 12) and results in a better mean element capacity utilization for the structures made of new material, compared to the reuse cases (Table 7 to Table 9). A comparison of the assigned cross sections for both, reuse and new material systems, are shown as bar charts in Fig. 7, Fig. 8 and Fig. 9.

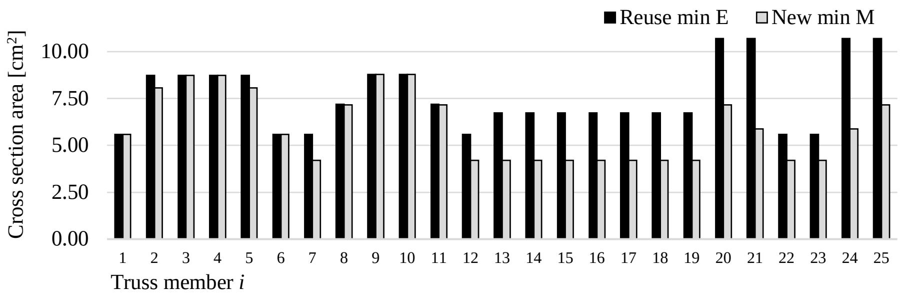

Fig. 7. System A – Howe truss – cross-section areas – reuse (*min E*) and new material system

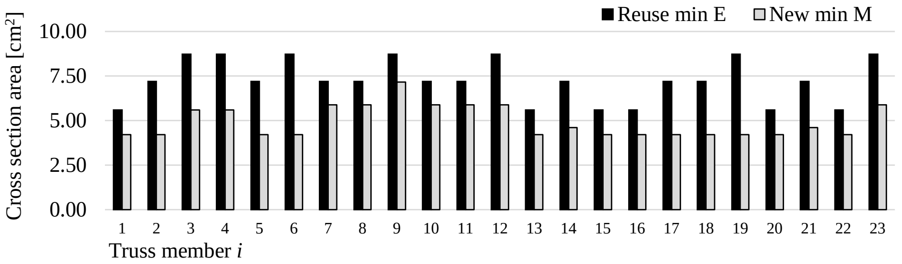

Fig. 8. System B – Warren truss – cross-section areas – reuse (*min E*) and new material system

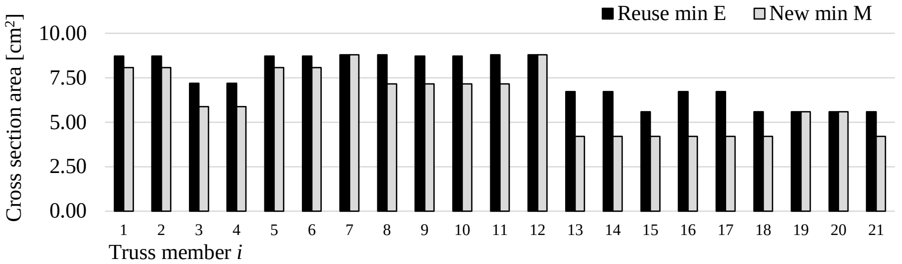

Fig. 9. System C – Pratt truss - cross-section areas – reuse (*min E*) and new material system

Fig. 10, Fig. 11 and Fig. 12 show the available stock elements represented as grey bars. The elements assigned from the stock to the structures are represented by black bars superimposed on the grey bars. This way, the remaining grey part of the superimposed bars represents the length, which is cut off to fit a stock element into the structure. For each truss, it was possible to assign elements matching exactly the system dimensions. Elements of stock group 4 (cross-section area: 8.73 cm^2, length: 1.5 m) are not used in any structure since the elements with 6.73 cm^2 and equal length (stock group 2) have sufficient capacity or because longer element lengths were required (stock group 5).

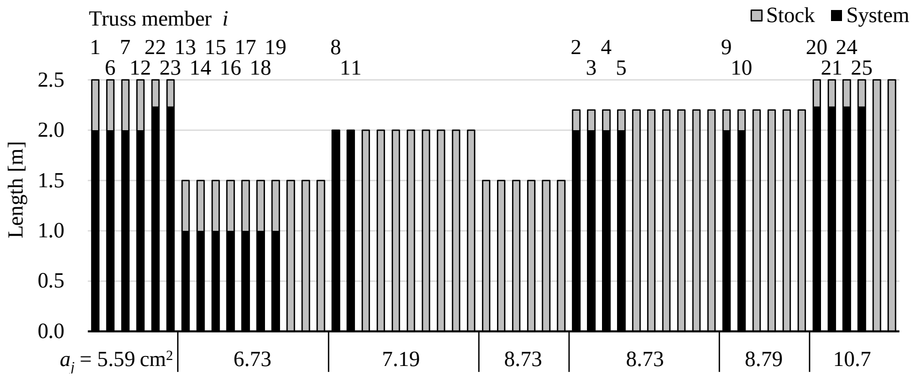

Fig. 10. System A – Howe truss – stock assignment – *min E*

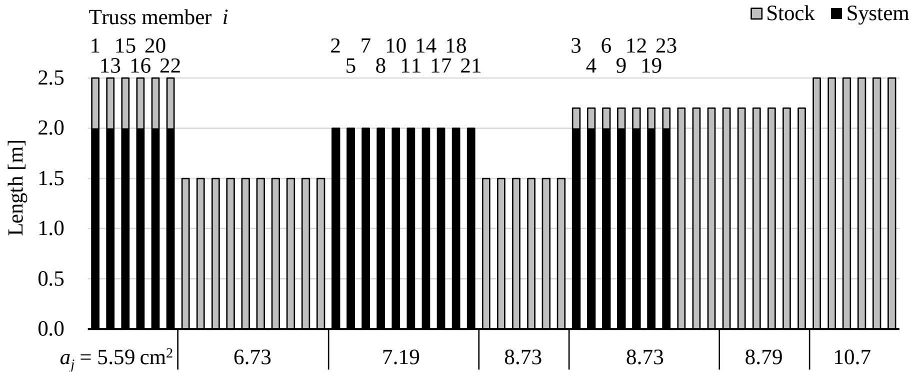

Fig. 11. System B – Warren truss – stock assignment – *min E*

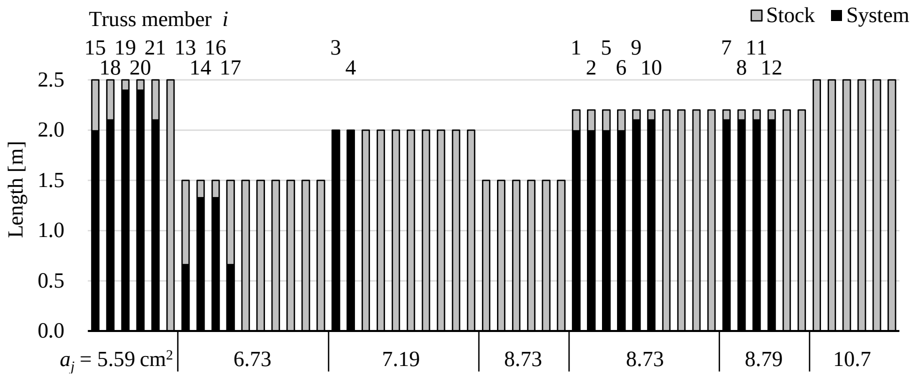

Fig. 12. System C – Pratt truss – stock assignment – *min E*

The bar chart in Fig. 13 shows the embodied energy (left axis) and carbon (right axis) of the optimized structures for all cases.

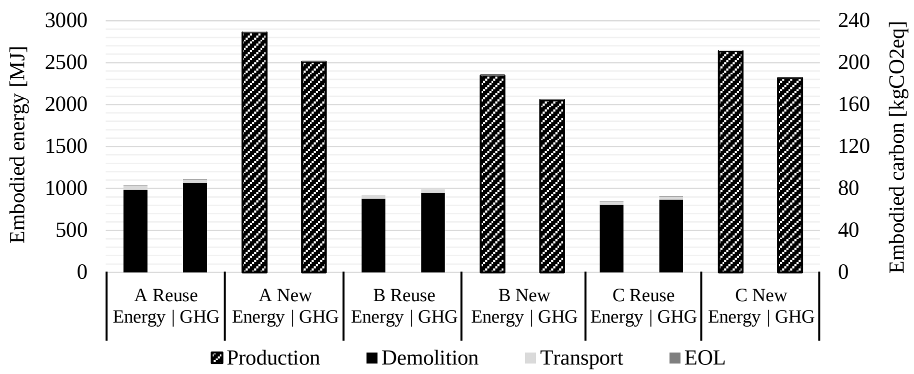

Fig. 13. Comparison of weight-optimized and reused elements structures

Transport energy and emissions are negligible compared to the production energy of new steel sections. For structures made from reused elements the selective deconstruction related energy is the biggest part of the total embodied energy. While the structures made from reused elements have a higher mass than the corresponding weight-optimized systems made of new material, they embody significantly lower energy and cause lower GHG emissions (Fig. 13). The embodied energy for reuse systems A, B and C is 36 %, 39 % and 32 % of the corresponding weight-optimized systems.

4.2 Ground Structure Approach.

Topology Optimization. In previous case studies the system topology was invariant. The ground structure approach [17] allows optimizing the topology of structures by defining a possible grillage of many bars of which only a subset will be assigned as the final structural system. To include topology optimization, the formulation is extended by adding a zero entry to each stock vector (except to the element availability $\boldsymbol{n}$), e.g.:

$$\boldsymbol{a} = \left[0, a_1, a_2, \dots, a_j\right]^T, \boldsymbol{l} = \left[0, l_1, l_2, \dots, l_j\right]^T, etc. \tag{20}$$

The assignment matrix $\boldsymbol{T}$ is extended by one more column accordingly. This way, it is possible to assign a *zero-element* at position i in the structure, if $t_{i,0} = 1$. This *zero-element* is $n_0 < m$ times available in the stock. When a *zero-element* is assigned, equations including stock element lengths have to be modified, such that no cut-off is added to the objective (Eq. 21) and that the length constraint (Eq. 22) vanishes.

$$\min \Delta l = \sum_{i=1}^{m} \boldsymbol{t}_i \boldsymbol{l} - (1 - t_{i,0}) l'_i \tag{21}$$

$$\left(1 - t_{i,0}\right) l'_i \leq \boldsymbol{t}_i \boldsymbol{l} \ \ \forall\, i = 1..m \tag{22}$$

The ground structure approach might result in unstable topologies (i.e. mechanisms might arise from the assignment of a *zero-element*). It is possible to enforce the presence of members at certain locations to avoid such unstable solutions. The following constraint (Eq. 23) e.g. requires that exactly one member has to be present at either position i_A or i_B in the structure:

$$t_{i_A,0} + t_{i_B,0} = 1 \tag{23}$$

Case Study. A two span bridge truss as shown in Fig. 14 is taken here as a case study. At each side of the 4,00 m wide bridge one truss is located. Each bay of the system measures 1.80 m in length. The ground structure consist of m = 41 candidate bars. The number and size of the bays is chosen such that feasible assignment solutions for the given stock exist. Equation (23) is used to ensure the existence of only one diagonal in each bay, thus crossing members and unstable solutions are avoided.

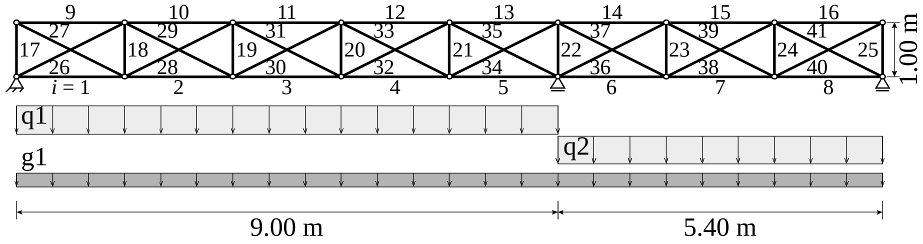

Fig. 14. Bridge truss system – ground structure

To simulate a realistic scenario, the stock of elements is assumed to originate from the selective deconstruction of the new steel Pratt truss (section 4.1). Three of such Pratt trusses make up the available stock for the optimization of one bridge truss considered in this case study. The stock composition is summarized in Table 10:

Table 10. Bridge truss - Stock composition

Stock grp. *j* MSH		1 40×2.9	2 40×2.9	3 40×2.9	4 40×2.9	5 40×4	6 50×3.2	7 60×3.2	8 40×6.3	9 60×4
a_j	[cm^2]	4.21	4.21	4.21	4.21	5.59	5.88	7.16	8.07	8.79
I_j	[cm^4]	9.59	9.59	9.59	9.59	11.8	21.2	38.2	14.7	45.4
l_j	[m]	0.67	1.33	2.00	2.11	2.40	2.00	2.11	2.00	2.11
n_j	[-]	6	6	3	6	6	6	12	12	6

Loading. In addition to self-weight g_o, dead load $g1$ is applied to the bottom chord (deck). Live loads q_1 and q_2 are applied to the bottom chord of the first and second span respectively (Fig. 14). Table 11 and Table 12 summarize all load cases and combinations taken into consideration.

Table 11. Bridge truss – load cases

Load case	Load magnitude	Description
g_0	from assignment	self-weight
g_1	2.00 kN/m	dead load 1.00 kN/m^2
q_1	10.00 kN/m	live load 5.00 kN/m^2
q_2	10.00 kN/m	live load 5.00 kN/m^2

Table 12. Bridge truss – load combinations

Load combination	Load factors	Description
ULS_1	$1.35 \cdot (g_0 + g_1) + 1.50 \cdot q_1$	design loads, left span
SLS_1	$1.00 \cdot (g_0 + g_1) + 1.00 \cdot q_1$	characteristic loads, left span
ULS_2	$1.35 \cdot (g_0 + g_1) + 1.50 \cdot q_2$	design loads, right span
SLS_2	$1.00 \cdot (g_0 + g_1) + 1.00 \cdot q_2$	characteristic loads, right span
ULS_q	$1.35 \cdot (g_0 + g_1) + 1.50 \cdot (q_{1+} q_2)$	design loads, both spans
SLS_q	$1.00 \cdot (g_0 + g_1) + 1.00 \cdot (q_{1+} q_2)$	characteristic loads, both spans

Results. Table 13 compares the optimized structures made from reused elements against a weight-optimized structure made of new material. With regard to the structure made of new material, the same set of possible cross sections as in example 4.1 are considered. Referring to the optimization integrating reused elements, the weight-optimized truss has a mass of 189.1 kg, which is only slightly lower than that of the minimum energy solution. The structures made from reused elements are only marginally heavier (up to 16.1 kg for the minimum cut-off waste solution) than the structure made of new material. However, through the reuse of structural elements the minimum energy solution embodies only 29 % of the energy used for the new material structure.

Table 13. Bridge truss – assignment optimization

System C		Reuse - Objective *min M*	*min ΔM*	*min E*	New steel *min M_{new}*
M_{tot}	[kg]	219.4	216.0	210.6	-
M	[kg]	189.1	195.9	189.9	179.8
ΔM	[kg]	30.3	20.1	20.7	-
E	[MJ]	711.6	700.6	683.2	2378.6
u	[mm]	18.3	17.1	17.8	19.0
util.	[%]	64.4	56.7	59.0	64.9

Fig. 15 (a) shows the layout for the minimum embodied energy solution. Because the topology was optimized, the total number of bars was reduced from 41 in the ground structure to 26 members. Fig. 15 (b) shows the layout for the minimum weight solution obtained from new material. Both layouts use elements with larger cross section in the top chord of the left span. However, the topology differs in proximity of the central support. Fig. 16 shows a bar chart comparing the used element cross sections.

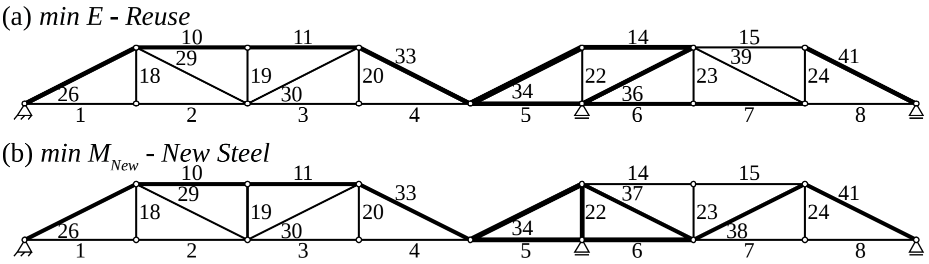

Fig. 15. Bridge truss – optimization results – final topologies

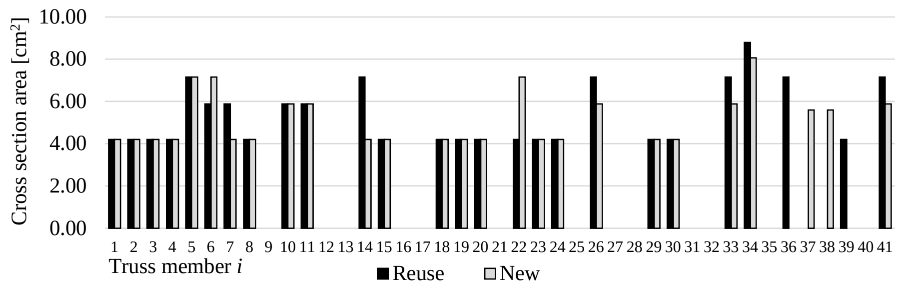

Fig. 16. Bridge truss – cross-section areas – reuse (*min E*) and new material system

The bar chart in Fig. 17 shows the stock element assignment and indicates that, for instance, some of the elements with the smallest cross-section area could not be assigned because their length was too short to fit into the structure.

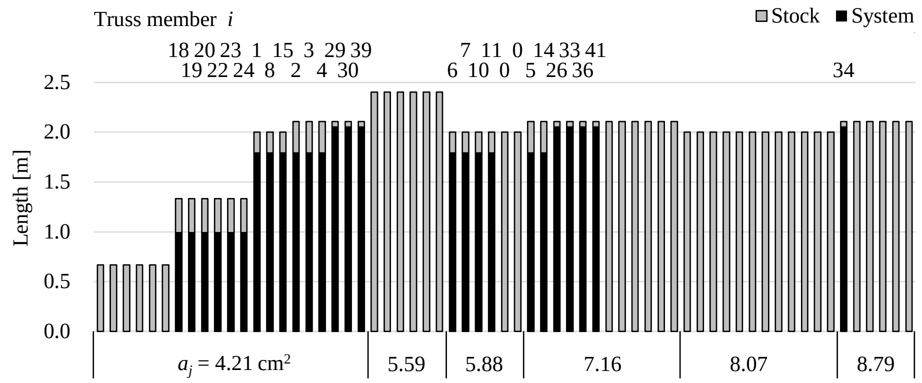

Fig. 17. Bridge truss - stock assignment – *min E*

4.3 Geometry Optimization

Formulation. In the previous case studies, the geometry of the structures was invariant. This results in most stock elements being cut to fit into the structure. However, as shown in Fig. 17 several elements assigned from the stock require only a small length adjustment. For these elements cutting could be avoided, if the structure geometry was changed, allowing the node coordinates to shift. When the node coordinates $\boldsymbol{x}$ are included in the design variables, the equilibrium matrix, self-weight and element length are functions of the nodal coordinates. All other variables remain equivalent to those introduced in section 2.3.The optimization problem is given as:

Objective	$\min f(\boldsymbol{x}, \boldsymbol{T}, \boldsymbol{p}^k, \boldsymbol{u}^k)$	(24)
Equilibrium	$\boldsymbol{B}(\boldsymbol{x})\boldsymbol{p}^k = \boldsymbol{f}^k - \boldsymbol{D}(\boldsymbol{x})\boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\gamma}) \ \forall\, k$	(25)
Compatibility	$diag\big(\boldsymbol{T}(\boldsymbol{e} \circ \boldsymbol{a})\big)\boldsymbol{B}(\boldsymbol{x})^T\boldsymbol{u}^k - diag(\boldsymbol{p}^k)\boldsymbol{l}'(\boldsymbol{x}) = 0 \ \forall\, k$	(26)
Stress capacity	$\boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\sigma}^-) \le \boldsymbol{p}^k \le \boldsymbol{T}(\boldsymbol{a} \circ \boldsymbol{\sigma}^+) \ \forall\, k$	(27)
Buckling	$-\dfrac{\pi^2\boldsymbol{T}(\boldsymbol{e} \circ \boldsymbol{I})}{\boldsymbol{l}'(x)^2} \le \boldsymbol{p}^k \ \forall\, k$	(28)
Deformation bounds	$\boldsymbol{u}^k_{min} \le \boldsymbol{u}^k \le \boldsymbol{u}^k_{max} \ \forall\, k$	(29)
Maximum length	$\big(1 - t_{i,0}\big)l(\boldsymbol{x})'_i \le \boldsymbol{t_i l} \ \forall\, i = 1 \dots m$	(30)

Eqs. 24 to 30 state a non-convex, mixed-integer nonlinear problem (MINLP) for the simultaneous optimization of stock element assignment, topology and geometry of the structure. To reduce the problem complexity, a sequential approach is proposed. The outcome from the MILP formulation for stock element assignment and topology optimization is further optimized changing nodal coordinates $\boldsymbol{x}$ (i.e. geometry optimization). Geometry optimization is implemented via Sequential Quadratic Programming. Fig. 18 illustrates the proposed scheme. The objective for stock element assignment with topology optimization is minimizing the embodied energy. The objective for the geometry optimization is the minimization of cut-off waste $\Delta M(\boldsymbol{x})$.

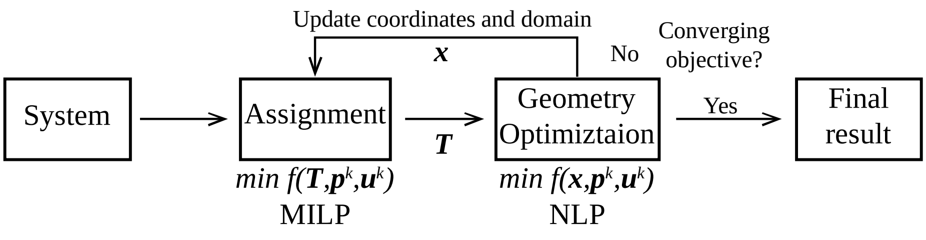

Fig. 18. Iterative assignment and geometry optimization scheme

Case Study. The same system with equivalent stock composition and load cases as shown in section 4.2 is used as case study for the proposed geometry optimization. Each iteration, the coordinates of the top chord nodes are allowed to vary in a domain of ± 80 cm in horizontal and vertical direction (Fig. 19). The bottom chord node positions are fixed to ensure an evenly distributed spacing and horizontal bridge deck.

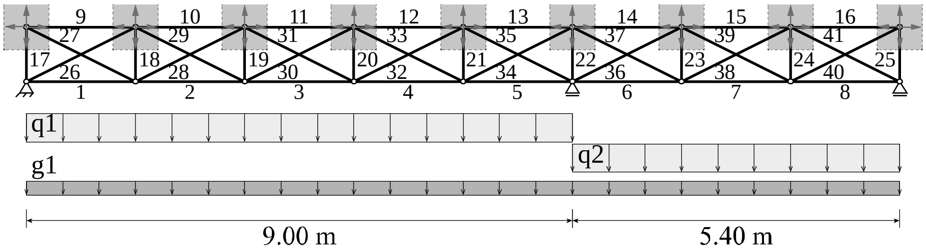

Fig. 19. System - geometry optimization

Results. Table 14 gives optimization metrics for this case study to evaluate the proposed sequential approach. The values in the column titled *1st Assignment* are those of the structure optimized in section 4.2 (stock assignment and topology optimization). Geometry optimization successfully reduces cut-off waste by 7.1 kg or 34 % respectively (see also Fig. 22). In addition, the embodied energy is reduced by 30 MJ (4.4 %).

Table 14. Bridge truss – assignment and geometry optimization

System		1st Assignment *min E*	Geometry Optimization *min ΔM*
M_{tot}	[kg]	210.6	201.4
M	[kg]	189.9	187.8
ΔM	[kg]	20.7	13.6
E	[MJ]	683.2	653.2
u	[mm]	17.8	14.9
util.	[%]	59.0	55.8

Fig. 20 shows the optimized layout. Compared to the previous solution (Fig. 15 (a), section 4.2) the top chord in the left span has changed to an arch-like shape. The right span top chord nodes are shifted upwards to accommodate the assigned member lengths. The increased height at mid-length of both spans reduces the maximum deflection compared to the previous solution without geometry optimization (Table 14).

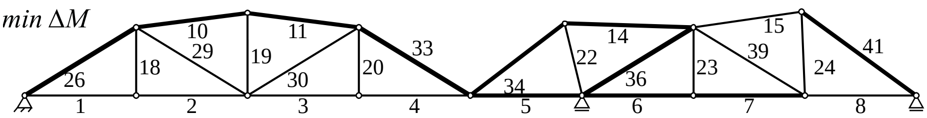

Fig. 20. Results – geometry optimization

From Fig. 21 it can be seen that via geometry optimization nine members (19, 24, 29, 30, 39, 26, 33, 34, 36) exactly match the available stock element lengths. The variation in geometry allows the assignment of thinner cross sections, e.g. member 34 has a cross section area of 7.16 cm^2 compared to 8.79 cm^2 before (Fig. 17, section 4.2).

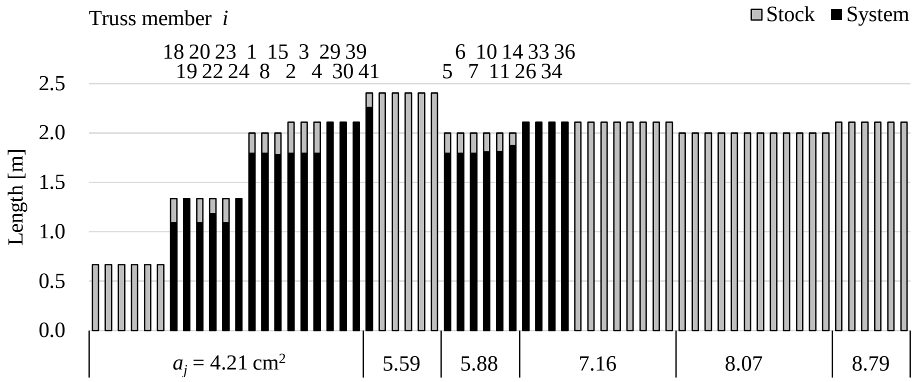

Fig. 21. Geometry optimized bridge truss - stock assignment

Fig. 22 shows the variation of the embodied energy, system mass and cut-off waste for successive assignment and geometry optimization. The first geometry optimization increases the weight because elements are generally lengthened. However, cut-off waste is drastically reduced by 32 %. Then, the 2nd assignment carried out on the optimized geometry results in a reduction of the embodied energy and system mass. The 2nd geometry optimization slightly decreases the cut-off waste. Further assignments and geometry optimizations converge to the same solution.

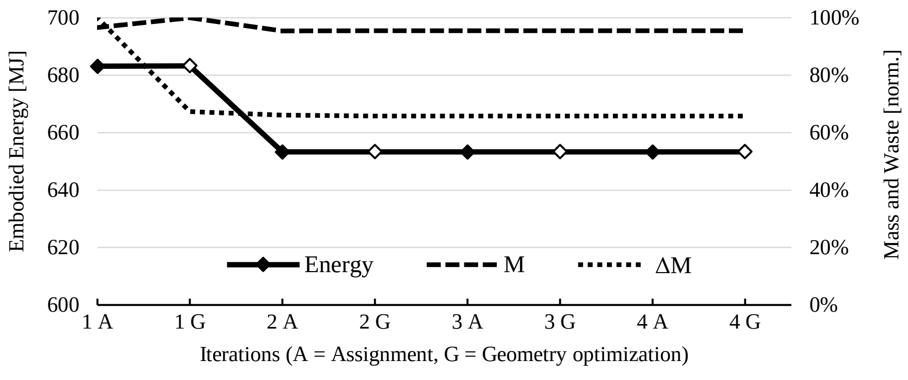

Fig. 22. Iteration history – successive assignment and geometry optimization

5 Conclusion

Reuse of structural elements offers an opportunity to reduce environmental impacts of building structures because it avoids the production of new components. This paper presented new structural optimization formulations for the design of truss structures reusing stock elements.

The selection of stock elements for a truss layout was formulated as an assignment problem, including ultimate limit state and serviceability constraints and presents a new application of discrete structural optimization problems. A sequential approach was proposed: optimal stock element assignments and topology are obtained as the solution of a mixed-integer linear program (MILP), geometry optimization is then carried out to further minimize cut-off waste. To the authors' knowledge, deterministic assignment optimization of stock elements in combination with topology and geometry optimization has not been applied yet.

A Life Cycle Assessment of the case studies taken under consideration showed that the optimized systems from reused elements embody up to 71 % less energy compared to corresponding weight optimized structures made of new material. It was identified that selective deconstruction impacts contribute the most to the total impacts of structures made from reused elements.

The optimization formulation was given for truss structures. Future work could look into extending the assignment formulation to bending systems (frames, beam-column systems), which are frequently used in practice.

The proposed sequential approach for geometry optimization might result in a local optimum compared to a simultaneous assignment and geometry optimization. Simultaneous optimization of stock element assignment, topology and geometry could be investigated in next steps.

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