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Procurement decisions in multi-period supply chain

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Abstract. Pricing and ordering decision in multi-period supply chain environments is not explored comprehensively. We consider three pragmatic procurement scenarios where the retailer can procure products (i) by maintaining strategic inventory, (ii) in bulk in first-period and distribute them in forthcoming selling period, and (iii) without maintaining any inventory. The results suggest that conventional single period planning exhibit sub-optimal characteristics. Build-up strategic inventory is not always profitable for the retailer. The retailer can also earn more profits by employing a bulk procurement strategy.

Keywords: Multi-period supply chain · Inventory · Game theory.

1. Introduction

Efficient inventory management is one of the key issues in retailing. Retailers maintain inventory to reduce transportation cost, take advantage of quantity discounts, ensure continuity of selling activities, evade variations in wholesale price and demand etc. ([4], [8], [9]). However, Anand et al. [1] reported that retailer's decision to maintain inventory in multi-period supply chain interactions under manufacturer-stackelberg game can reduce the degree of double marginalization. They found that the retailer can force the manufacturer to reduce the wholesale price of forthcoming periods by maintaining surplus order quantities as strategic inventory. Arya and Mittendorf [3] proved that the manufacturer can curtail advantage of the retailer in building strategic inventory by introducing consumer rebate. Consumer rebate prevents the retailer to maintain high amounts of SI. Arya et al. [2] extended this enticing stream of research and compare the effect of SI in the presence of multiple retail outlets. Hartwig et al. [5] conducted empirical investment to explore the effect of SI and found that the retailer can immensely induce differentiated wholesale pricing behaviour by building up SI. Mantin and Jiang [6] explored the impact of the product quality deterioration in the presence of SI. Moon et al. [7] analyzed the impact of SI in perspective of supply chain coordination. They found that the optimal supply chain profit cannot be achieved by implementing quadratic quantity discount contract mechanism. All the above cited contributions consider multi-period interaction among supply chain member to explore the consequences of SI.

In the existing literature on supply chain models, it is assumed that the retailer procures products to satisfy demand in each selling period. However, in practice, the retailer maintains SI to satisfy future demand. But, to the best of the authors' knowledge, the advantage of SI is not fully explored in current state. We consider three procurement decision for the retailer and explore the pricing and ordering behaviour under five consecutive selling period. It is found that the pricing behaviour is correlated with procurement decision. The single-period procurement decision always leads to suboptimal solution. The supply chain members can receive higher profit if the retailer maintains SI or procures in bulk.

2. Problem description

We explore the interaction in a serial supply chain with one retailer and one supplier under price-sensitive demand in a fifth-period game. The retailer in the supply chain has a downstream retail monopoly and rely solely on the upstream supplier for the retailed good. Three procurement strategies are considered. In first procurement strategy (WSI), the retailer may maintain SI in between two-consecutive selling period. In Second procurement strategy (BP), the retailer procures in bulk for the first selling period and distribute those in forthcoming periods. Third procurement strategy (BM) is similar to the conventional literature, where the retailer procures products to satisfy demand for each period. We consider linear price sensitive demand and derive optimal solution. For feasibility of the optimal solution, it is assumed that the retail (p_t) and wholesale prices (w_t) at each period satisfy the following relations $p_t > w_t > 0$, $\forall t = 1, ..., 5$. The unit holding cost for the retailer is h. All the parameters related to market demand are common knowledge between supply chain members [5].

2.1 Optimal decision in the presence of SI

At the beginning of each period ($t = 1, \cdots, 5$), the supplier determines a wholesale price (w_t^{wsi}). The retailer then procures (Q_t^{wsi}) amounts of product and sets retail price (p_t^{wsi}) to satisfy market demand ($q_t^{wsi} = a - bp_t^{wsi}$). If the procured quantity at each period is larger than the quantity sold in the that period (i.e., if $Q_t^{wsi} > q_t^{wsi}$), then the retailer builds up SI ($I_t^{wsi} = Q_t^{wsi} - q_t^{wsi}$) to be sold in the immediate period and invests hI_t^{wsi} as holding cost. The profit functions for the supplier and retailer are obtained as follows:

$$\pi_{r5}^{wsi} = p_5^{wsi}(a - bp_5^{wsi}) - w_5^{wsi}(a - bp_5^{wsi} - I_4^{wsi})$$
$$\pi_{m5}^{wsi} = w_5^{wsi}(a - bp_5^{wsi} - I_4^{wsi})$$
$$\pi_{r4}^{wsi} = p_4^{wsi}(a - bp_4^{wsi}) - w_4^{wsi}(a - bp_4^{wsi} + I_4^{wsi} - I_3^{wsi}) - hI_4^{wsi} + \pi_{r5}^{wsi}$$
$$\pi_{m4}^{wsi} = w_4^{wsi}(a - bp_4^{wsi} + I_4^{wsi} - I_3^{wsi}) + \pi_{m5}^{wsi}$$
$$\pi_{r3}^{wsi} = p_3^{wsi}(a - bp_3^{wsi}) - w_3^{wsi}(a - bp_3^{wsi} + I_3^{wsi} - I_2^{wsi}) - hI_3^{wsi} + \pi_{r4}^{wsi}$$
$$\pi_{m3}^{wsi} = w_3^{wsi}(a - bp_3^{wsi} + I_3^{wsi} - I2^{wsi}) + \pi_{m4}^{wsi}$$
$$\pi_{r2}^{wsi} = p_2^{wsi}(a - bp_2^{wsi}) - w_2^{wsi}(a - bp_2^{wsi} - I_1^{wsi} + I_2^{wsi}) + \pi_{r3}^{wsi} - hI_2^{wsi}$$
$$\pi_{m2}^{wsi} = w_2^{wsi}(a - bp_2^{wsi} - I_1^{wsi} + I_2^{wsi}) + \pi_{m3}^{wsi}$$
$$\pi_{r1}^{wsi} = p_1^{wsi}(a - bp_1^{wsi}) - w_1^{wsi}(a - bp_1^{wsi} + I_1^{wsi}) - hI_1^{wsi} + \pi_{r2}^{wsi}$$
$$\pi_{m1}^{wsi} = w_1^{wsi}(a - bp_1^{wsi} + I_1^{wsi}) + \pi_{m2}^{wsi}$$

The optimal solution for the retailer fifth-period optimization problem presented in the first equation is obtained by solving $\frac{d\pi_{r5}^{wsi}}{dp_5^{wsi}} = 0$. On simplification, we have $p_5^{wsi} = \frac{a+bw_5^{wsi}}{2b}$. The optimal solution for the supplier fifth-period optimization problem presented in the second equation is obtained by solving $\frac{\partial\pi_{m5}^{wsi}}{\partial w_5^{wsi}} = 0$. On simplification, one can obtain $w_5^{wsi} = \frac{a-2I_4^{wsi}}{2b}$. The profit function for the retailer and supplier in fifth-period is concave because $\frac{d^2\pi_{r5}^{wsi}}{dp_5^{wsi2}} = -2b < 0$ and $\frac{d^2\pi_{m5}^{wsi}}{dw_5^{wsi2}} = -b < 0$, respectively.

Substituting the optimal response obtained in fifth-period, profit function for

the retailer in fourth-period is obtained as follows:
$\pi_{r4}^{wsi} = \frac{a^2+12aI_4^{wsi}-12I_4^{wsi2}}{16b} + p_4^{wsi}(a - bp_4^{wsi}) - (a - I_3^{wsi} - bp_4^{wsi})w_4^{wsi} - hI_4^{wsi}$
The optimal solution for the above problem is obtained by solving $\frac{\partial \pi_{r4}^{wsi}}{\partial p_4^{wsi}} = 0$ and $\frac{\partial \pi_{r4}^{wsi}}{\partial I_4^{wsi}} = 0$. On simplification, $p_4^{wsi} = \frac{a+bw_4^{wsi}}{2b}$ and $I_4^{wsi} = \frac{3a-4b(h+w_4^{wsi}))}{6}$. Substituting optimal response, the profit function for the supplier is obtained as $\pi_{m4}^{wsi} = (a-I_3^{wsi})w_4^{wsi} + \frac{b(4h^2-4hw_4^{wsi}-17w_4^{wsi2})}{18}$. After solving first order condition, the wholesale price for the fourth period is obtained as $w_4^{wsi} = \frac{9a-2bh-9I_3^{wsi}}{17b}$. Because $\frac{\partial^2 \pi_{r4}^{wsi}}{\partial p_4^{wsi2}} = -2b < 0$ and $\frac{\partial^2 \pi_{r4}^{wsi}}{\partial p_4^{wsi2}} \frac{\partial^2 \pi_{r4}^{wsi}}{\partial I_4^{wsi2}} - \left(\frac{\partial^2 \pi_{r4}^{wsi}}{\partial I_4^{wsi} \partial p_4^{wsi}}\right)^2 = 3 > 0$; and $\frac{\partial^2 \pi_{m4}^{wsi}}{\partial w_4^{wsi2}} = -\frac{17b}{9} < 0$, the profit function of the retailer and supplier are concave. Similarly, the profit function for the retailer in third-period is obtained as follows:

$$\pi_{r3}^{wsi} = \frac{155a^2-118abh+304b^2h^2+846aI_3^{wsi}-460bhI_3^{wsi}-423I_3^{wsi2}}{1156b}$$
$$+p_3^{wsi}(a - bp_3^{wsi}) - (a - I_2^{wsi} + I_3^{wsi} - bp_3^{wsi})w_3^{wsi} - hI_3^{wsi}$$

Corresponding optimal retail price and SI are $p_3^{wsi} = \frac{a+bw_3^{wsi}}{2b}$ and $I_3^{wsi} = \frac{423a-2b(404h+289w_3^{wsi})}{423}$, respectively. Substituting optimal response for the retailer, profit function for the supplier in third-period is obtained as follows:

$$\pi_{m3}^{wsi} = \frac{3aw_3^{wsi}-2I_2^{wsi}w_3^{wsi}}{2} + \frac{b(38824h^2-27400hw_3^{wsi}-54561w_3^{wsi2})}{39762}$$

and corresponding wholesale price is $w_3^{wsi} = \frac{59643a-27400bh-39762I_2^{wsi}}{109122b}$. Note the the third-period optimization problem for the retailer and supplier are concave because $\frac{\partial^2 \pi_{r3}^{wsi}}{\partial p_3^{wsi2}} = -2b < 0$ and $\frac{\partial^2 \pi_{r3}^{wsi}}{\partial p_3^{wsi2}} \frac{\partial^2 \pi_{r3}^{wsi}}{\partial I_3^{wsi2}} - \left(\frac{\partial^2 \pi_{r3}^{wsi}}{\partial I_3^{wsi} \partial p_3^{wsi}}\right)^2 = \frac{423}{289} > 0$; and $\frac{\partial^2 \pi_{m3}^{wsi}}{\partial w_3^{wsi2}} = -\frac{18187b}{6627} < 0$. The second-period profit function for the retailer is obtained as follows:

$$\pi_{r2}^{wsi} = p_2^{wsi}(a - bp_2^{wsi}) - (a - I_1^{wsi} + I_2^{wsi} - bp_2^{wsi})w_2^{wsi} - hI_2^{wsi} +$$
$$\frac{0.208932a^2-0.335467abh+1.17731b^2h^2+0.721424aI_2^{wsi}-0.776356bhI_2^{wsi}-0.240475I_2^{wsi2}}{b}$$

Corresponding optimal retail price and SI are $p_2^{wsi} = \frac{a+bw_2^{wsi}}{2b}$ and $I_2^{wsi} = \frac{3a}{2} - \frac{2b(5288037907h+2976902721w_2^{wsi})}{2863480311}$, respectively. Substituting optimal response, the profit function for the supplier in second-period is obtained as $\pi_{m2}^{wsi} = 2aw_2^{wsi} - I_1^{wsi}w_2^{wsi} + b(2.62087h^2 - 1.41726hw_2^{wsi} - 1.79158w_2^{wsi2})$, and corresponding wholesale price is $w_2^{wsi} = \frac{0.558166a-0.395535bh-0.279083I_1^{wsi}}{b}$. Note the the second-period optimization problem for the retailer and supplier are concave because $\frac{\partial^2 \pi_{r2}^{wsi}}{\partial p_2^{wsi2}} = -2b < 0$ and $\frac{\partial^2 \pi_{r2}^{wsi}}{\partial p_2^{wsi2}} \frac{\partial^2 \pi_{r2}^{wsi}}{\partial I_2^{wsi2}} - \left(\frac{\partial^2 \pi_{r2}^{wsi}}{\partial I_2^{wsi} \partial p_2^{wsi}}\right)^2 = 0.961899 > 0$; and $\frac{\partial^2 \pi_{m2}^{wsi}}{\partial w_2^{wsi2}} = -3.58316b < 0$, respectively. Finally, the first-period profit function for the retailer is obtained as follows:

$$\pi_{r1}^{wsi} = \frac{1}{b}[0.285445a^2 - bI_1^{wsi}(2.1416h + w_1^{wsi}) + ab(p_1^{wsi} - w_1^{wsi} - 0.716806h)$$
$$+0.714555aI_1^{wsi} - 0.178639I_1^{wsi2} + b^2(3.19862h^2 - p_1^{wsi}(p_1^{wsi} - w_1^{wsi}))]$$

Correspondingly optimal retail price and SI are $p_1^{wsi} = \frac{a+bw_1^{wsi}}{2b}$ and $I_1^{wsi} = 2a - 5.99421bh - 2.79895bw_1^{wsi}$. Substituting the optimal response for the retailer, the profit function for the supplier in first-period is obtained as follows:

$$\pi_{m1}^{wsi} = 2.5aw_1^{wsi} + b(5.54404h^2 - 2.41898hw_1^{wsi} - 2.20576{w_1^{wsi}}^2)$$

and corresponding wholesale price is $w_1^{wsi} = \frac{0.566697a-0.548333bh}{b}$. Note that the first-period optimization problem for the retailer and supplier are concave as $\frac{\partial^2\pi_{r1}^{wsi}}{\partial {p_1^{wsi}}^2} = -2b < 0$ and $\frac{\partial^2\pi_{r1}^{wsi}}{\partial {p_1^{wsi}}^2}\frac{\partial^2\pi_{r1}^{wsi}}{\partial {I_1^{wsi}}^2} - \left(\frac{\partial^2\pi_{r1}^{wsi}}{\partial I_1^{wsi}\partial p_1^{wsi}}\right)^2 = 0.714555 > 0$; and $\frac{\partial^2\pi_{m1}^{wsi}}{\partial {w_1^{wsi}}^2} = -4.41153b < 0$, respectively. By using back substitution, one can obtain the following optimal solutions:

$$w_1^{wsi} = \frac{0.783349a-0.274166bh}{b} \quad w_2^{wsi} = \frac{0.442669a+0.849025bh}{b} \quad w_3^{wsi} = \frac{0.335378a+1.73797bh}{b}$$

$$w_4^{wsi} = \frac{0.242614a+2.15087bh}{b} \quad w_5^{wsi} = \frac{0.161743a+2.10058bh}{b}$$

$$p_1^{wsi} = \frac{0.783349a-0.274166bh}{b} \quad p_2^{wsi} = \frac{0.721334a+0.424512bh}{b} \quad p_3^{wsi} = \frac{0.667689a+0.868984bh}{b}$$

$$p_4^{wsi} = \frac{0.621307a+1.07544bh}{b} \quad p_5^{wsi} = \frac{0.580871a+1.05029bh}{b}$$

$$I_1^{wsi} = 0.413846a - 4.45946bh \quad I_2^{wsi} = 0.579595a - 5.45874bh$$

$$I_3^{wsi} = 0.541729a - 4.28498bh \quad I_4^{wsi} = 0.338257a - 2.10058bh$$

$$\pi_{r5}^{wsi} = \frac{0.230379a^2-0.509631abh-3.30933b^2h^2}{b^2} \quad \pi_{m5}^{wsi} = \frac{2.20622(0.0769991a+bh)^2}{b^2}$$

$$\pi_{r4}^{wsi} = \frac{0.423153a^2-1.75473abh-4.75054b^2h^2}{b^2} \quad \pi_{m4}^{wsi} = \frac{0.0555915a^2+0.985682abh+4.59145b^2h^2}{b^2}$$

$$\pi_{r3}^{wsi} = \frac{0.546283a^2-3.20185abh-1.7504b^2h^2}{b^2} \quad \pi_{m3}^{wsi} = \frac{0.154342a^2+1.59964abh+5.12115b^2h^2}{b^2}$$

$$\pi_{r2}^{wsi} = \frac{0.550566a^2-3.71641abh+4.73697b^2h^2}{b^2} \quad \pi_{m2}^{wsi} = \frac{0.35107a^2+1.34668abh+3.91231b^2h^2}{b^2}$$

$$\pi_{r1}^{wsi} = \frac{0.362978a^2-1.25738abh+6.82633b^2h^2}{b^2} \quad \pi_{m1}^{wsi} = \frac{0.708371a^2-1.37083abh+6.20725b^2h^2}{b^2}.$$

2.2 Optimal decisions in Scenario BP

At the beginning of first period, the supplier determines a wholesale price (w_1^{bp}) and then the retailer procures $a - bp_1^{bp} + \sum_{t=1}^4 I_t^{bp}$ unit of products and sets the retail price (p_1^{bp}). In next four selling period, the supplier determines wholesale price (w_t^{bp}) and then the retailer procures ($q_t^{bp} = a - bp_t^{bp} - I_{t-1}^{bp}$)$(t = 2, \cdots, 5)$ units of product and sets retail price (p_t^{bp}) to satisfy market demand. The profit functions of the supplier and retailer for five consecutive selling periods are obtained as follows:

$$\pi_{r5}^{bp} = p_5^{bp}(a - bp_5^{bp}) - w_5^{bp}(a - bp_5^{bp} - I_4^{bp})$$

$$\pi_{m5}^{bp} = w_5^{bp}(a - bp_5^{bp} - I_4^{bp})$$

$$\pi_{r4}^{bp} = p_4^{bp}(a - bp_4^{bp}) - w_4^{bp}(a - bp_4^{bp} - I_3^{bp}) - hI_4^{bp} + \pi_{r5}^{bp}$$

$$\pi_{m4}^{bp} = w_4^{bp}(a - bp_4^{bp} - I_3^{bp}) + \pi_{m5}^{bp}$$

$$\pi_{r3}^{bp} = p_3^{bp}(a - bp_3^{bp}) - w_3^{bp}(a - bp_3^{bp} - I_2^{bp}) - h(I_3^{bp} + I_4^{bp}) + \pi_{r4}^{bp}$$

$$\pi_{m3}^{bp} = w_3^{bp}(a - bp_3^{bp} - I_2^{bp}) + \pi_{m4}^{bp}$$

$$\pi_{r2}^{bp} = p_2^{bp}(a - bp_2^{bp}) - w_2^{bp}(a - bp_2^{bp} - I_1^{bp}) - h\sum_{t=2}^4 I_1^{bp} + \pi_{r3}^{bp}$$

$$\pi_{m2}^{bp} = w_2^{bp}(a - bp_2^{bp} - I_1^{bp}) + \pi_{m3}^{bp}$$

$$\pi_{r1}^{bp} = p_1^{bp}(a - bp_1^{bp}) - w_1^{bp}(a - bp_1^{bp} + \sum_{t=1}^4 I_1^{bp}) - \sum_{t=1}^4 I_1^{bp} + \pi_{r2}^{bp}$$

$$\pi_{m1}^{bp} = w_1^{bp}(a - bp_1^{bp} + \sum_{t=1}^4 I_1^{bp}) + \pi_{m2}^{bp}$$

The optimal solution for the retailer fifth-period optimization problem is obtained by solving $\frac{d\pi_{r5}^{bp}}{dp_5^{bp}} = 0$. On simplification, we have $p_5^{bp} = \frac{a+bw_5^{bp}}{2b}$. The optimal solution for the supplier fifth-period optimization problem is obtained by solving $\frac{\partial\pi_{m5}^{bp}}{\partial w_5^{bp}} = 0$. On simplification, one can obtain $w_5^{bp} = \frac{a-2I_4^{bp}}{2b}$. The profit function for the retailer and supplier in fifth-period are concave because $\frac{d^2\pi^{si}_{r5}}{d{p^{bp}_5}^2} = -2b < 0$ and $\frac{d^2\pi^{bp}_{m5}}{d{w^{bp}_5}^2} = -b < 0$, respectively. Similar to previous subsection, the profit function for the retailer in first-period is obtained as follows:

$$\pi^{bp}_{r1} = \frac{a^2-3{I^{bp}_1}^2+3a(I^{bp}_1+I^{bp}_2+I^{bp}_3+I^{bp}_4)-4bh(I^{bp}_2+2I^{bp}_3+3I^{bp}_4)-3({I^{bp}_2}^2+{I^{bp}_3}^2+{I^{bp}_4}^2)}{4b}$$
$$+(p^{bp}_1 - w^{bp}_1)(a - bp^{bp}_1) - (I^{bp}_1 + I^{bp}_2 + I^{bp}_3 + I^{bp}_4)w^{bp}_1 - h(I^{bp}_1 + I^{bp}_2 + I^{bp}_3 + I^{bp}_4)$$

Optimal solution for the retailer first-period optimization problem is obtained by solving $\frac{\partial\pi^{bp}_{r1}}{\partial p^{bp}_1} = 0$; $\frac{\partial\pi^{bp}_{r1}}{\partial I^{bp}_1} = 0$; $\frac{\partial\pi^{bp}_{r1}}{\partial I^{bp}_2} = 0$; $\frac{\partial\pi^{bp}_{r1}}{\partial I^{bp}_3} = 0$ and $\frac{\partial\pi^{bp}_{r1}}{\partial I^{bp}_4} = 0$, simultaneously. After solving, following solution is obtained: $p^{bp}_1 = \frac{a+bw^{bp}_1}{2b}$; $I^{bp}_1 = \frac{3a-4b(h+w^{bp}_1)}{6}$; $I^{bp}_2 = \frac{3a-4b(2h+w^{bp}_1)}{6}$; $I^{bp}_3 = \frac{3a-4b(3h+w^{bp}_1)}{6}$; $I^{bp}_4 = \frac{3a-4b(4h+w^{bp}_1)}{6}$.

We compute the following Hessian matrix to check concavity:

$$H^{bp} = \begin{pmatrix} \frac{\partial^2\pi^{bp}_{r1}}{\partial {p^{bp}_1}^2} & \frac{\partial^2\pi_{r1bp}}{\partial p^{bp}_1\partial I^{bp}_1} & \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_4} \\ \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_1} & \frac{\partial^2\pi^{bp}_{r1}}{\partial {I^{bp}_1}^2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_4} \\ \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial {I^{bp}_2}^2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_2\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_2\partial I^{bp}_4} \\ \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_2\partial I^{bp}_3} & \frac{\partial^2\pi^{bp}_{r1}}{\partial {I^{bp}_3}^2} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_3\partial I^{bp}_4} \\ \frac{\partial^2\pi^{bp}_{r1}}{\partial p^{bp}_1\partial I^{bp}_4} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_1\partial I^{bp}_4} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_2\partial I^{bp}_4} & \frac{\partial^2\pi^{bp}_{r1}}{\partial I^{bp}_3\partial I^{bp}_4} & \frac{\partial^2\pi^{bp}_{r1}}{\partial {I^{bp}_4}^2} \end{pmatrix} == \begin{pmatrix} -2b & 0 & 0 & 0 & 0 \\ 0 & -\frac{3}{2b} & 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{2b} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2b} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2b} \end{pmatrix}$$

The values of principal minors are $\Delta_1 = -2b < 0$; $\Delta_2 = 3 > 0$; $\Delta_3 = -\frac{9}{2b} < 0$; $\Delta_4 = \frac{27}{4b^2} > 0$ and $\Delta_5 = -\frac{81}{8b^3} < 0$, i.e. profit function for the retailer is concave. Substituting the optimal response for the retailer, the profit function for the supplier in first-period is obtained as $\pi^{bp}_{m1} = \frac{45aw^{bp}_1+b(120h^2-40hw^{bp}_1-41{w^{bp}_1}^2)}{18}$ and the corresponding wholesale price is $w^{bp}_1 = \frac{5(9a-8bh)}{82b}$. By using back substitution, one can obtain the following optimal solutions:

$$w^{bp}_2 = \frac{15a+14bh}{41b} \quad w^{bp}_3 = \frac{45a+124bh}{123b} \quad w^{bp}_4 = \frac{45ab+206bh}{123b} \quad w^{bp}_5 = \frac{3(5a+32bh)}{41b}$$
$$p^{bp}_1 = \frac{127a-40bh}{164b} \quad p^{bp}_2 = \frac{7(4a+bh)}{41b} \quad p^{bp}_3 = \frac{2(42a+31bh)}{123b} \quad p^{bp}_4 = \frac{84a+103bh}{123} \quad p^{bp}_5 = \frac{4(7a+12bh)}{41b}$$
$$I^{bp}_1 = \frac{11a-28bh}{82b} \quad I^{bp}_2 = \frac{33a-248bh}{246} \quad I^{bp}_3 = \frac{33a-412bh}{246} \quad I^{bp}_4 = \frac{11a-192bh}{82}$$
$$\pi^{bp}_{r5} = \frac{503a^2-4320abh-13824b^2h^2}{3362b} \quad \pi^{bp}_{m5} = \frac{2(6a+29bh)^2}{1089b}$$
$$\pi^{bp}_{r4} = \frac{3018a^2-23583abh-39074b^2h^2}{10086b} \quad \pi^{bp}_{m4} = \frac{5(405a^2+4446abh+12538b^2h^2)}{15129b}$$
$$\pi^{bp}_{r3} = \frac{4527a^2-31869abh-6254b^2h^2}{10086b} \quad \pi^{bp}_{m3} = \frac{6075a^2+55620abh+140756b^2h^2}{30258b}$$
$$\pi^{bp}_{r2} = \frac{3018a^2-18909abh+21770b^2h^2}{5043b} \quad \pi^{bp}_{m2} = \frac{10(405a^2+2970abh+7126b^2h^2)}{15129b}$$
$$\pi^{bp}_{r1} = \frac{5(5727a^2-15648abh+114976b^2h^2)}{80688b} \quad \pi^{bp}_{m1} = \frac{5(405a^2-720abh+4256b^2h^2)}{2952}$$

2.3 Benchmark model

In Scenario BM, the retailer does not maintain SI or procure products in bulk. The profit functions for the retailer and supplier in each selling period are $\pi^{bm}_r = (p^{bm} - w^{bm})(a - bp^{bm})$ and $\pi^{bm}_m = w^{bm}(a - bp^{bm})$, respectively. One may obtain the optimal response function of the retailer by solving first order condition of optimization as $p(w^{bm}) = \frac{a+bw^{bm}}{2}$. Substituting optimal response, the supplier's profit function is obtained as follows, $\pi_m = \frac{w^{bm}(a-bw^{bm})}{2}$ and the corresponding optimal wholesale price is $w^{bm} = \frac{a}{2b}$. Based on the optimal decisions, the closed form profit functions can be obtained as, $\pi_r^{bm} = \frac{a^2}{16b}$ and $\pi_m^{bm} = \frac{a^2}{8b}$. Note that in absence of additional inventory, wholesale and retail prices remain uniform in each period.

3 Managerial Implications

Proposition 1. In procurement scenario BP,
(i) the retailer and supplier sets maximum retail and wholesale price in first selling period, respectively.
(ii) the retail and wholesale prices increases from the second selling period.
(iii) the amount of products distributed by the retailer decreases as the selling period progress.
proof. The retail and wholesale prices, and SI in Scenario BP satisfy the following relations:

$$p_1^{bp} - p_2^{bp} = \frac{15a-68bh}{164b} > 0 \text{ and } p_2^{bp} - p_3^{bp} = p_3^{bp} - p_4^{bp} = p_4^{bp} - p_5^{bp} = -\frac{h}{3} < 0$$
$$w_1^{bp} - w_2^{bp} = \frac{15a-68bh}{82b} > 0 \text{ and } w_2^{bp} - w_2^{bp} = w_3^{bp} - w_4^{bp} = w_4^{bp} - w_5^{bp} = -\frac{2h}{3} < 0$$
$$I_1^{bp} - I_2^{bp} = I_2^{bp} - I_3^{bp} = \frac{2bh}{3} > 0$$

The above inequalities ensures proof.
Proposition 2. In procurement scenario WSI,
(i) the retailer and supplier sets maximum retail and wholesale price in first selling period, respectively.
(ii) the retail and wholesale prices decreases from the second selling period.
proof. The retail and wholesale prices, and SI in Scenario WSI:

$$p_1^{wsi} - p_2^{wsi} = \frac{0.0620142a-0.6986787bh}{b} > 0, \ p_2^{wsi} - p_3^{wsi} = \frac{0.053645a-0.444472bh}{b} > 0,$$
$$p_3^{wsi} - p_4^{wsi} = \frac{0.046382a-0.206451bh}{b} > 0, \ p_4^{wsi} - p_5^{wsi} = \frac{0.04044a+0.02515h}{b} > 0$$
$$w_1^{wsi} - w_2^{wsi} = \frac{0.12402847a-1.397357bh}{b} > 0, \ w_2^{wsi} - w_3^{wsi} = \frac{0.1072902a-0.888943bh}{b} > 0$$
$$w_3^{wsi} - w_4^{wsi} = \frac{0.092764a-0.41290bh}{b} > 0, \ w_4^{wsi} - w_5^{wsi} = \frac{0.080871a+0.05029bh}{b} > 0$$

The above inequalities ensures proof.
Proposition 3.
(i) The retailer decision to maintain SI always outperforms the single period procurement decision if $h \in \left[\frac{0.0591229a}{b}, \frac{0.125072a}{b}\right]$
(ii) Supply chain member receives higher profits in procurement scenarios under BP compared to BM.
proof. The following relations ensure that the average profits of the supplier always greater compere to the profit earns by the supplier in Scenario BM:

$$\pi_{m1}^{wsi}/5 - \pi_m^{bm} = \frac{0.016674272a^2-0.27416633ahb+1.2414495b^2h^2}{b} > 0$$
$$\pi_{m1}^{bp}/5 - \pi_m^{bm} = \frac{9a^2-180ahb+1064b^2h^2}{738b} = \frac{9(a-10bh)^2+164b^2h^2}{738b} > 0$$

Similarly, the difference of average profits obtain under different scenarios with profits obtain in Scenario BM are

$$\pi_{r1}^{wsi}/5 - \pi_r^{bm} = \frac{0.0100956a^2-0.251475abh+1.36527b^2h^2}{b} \text{ if } h \in \left[\frac{0.0591229a}{b}, \frac{0.125072a}{b}\right]$$
$$\pi_{r1}^{bp}/5 - \pi_r^{bm} = \frac{171a^2-3912abh+28744b^2h^2}{20172b} > 0$$

The above inequalities ensures proof.

The graphical representation of the profit functions of the retailer and supplier are shown in Figures 1a and 1b.

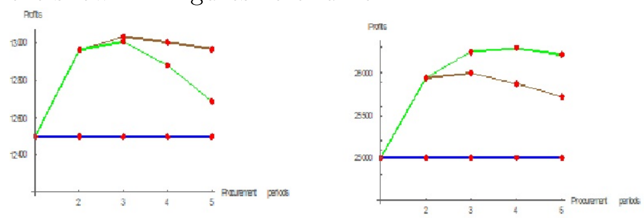

Fig 1a. Average profits of the retailer Fig 1b. Average profits of the manufacturer
a=200, b=0.2, and h=50 Scenario BP (green), WSI (brown), and BM (blue)

Figures 1a and 1b demonstrate the profits of the supply chain members if the retailer makes procurement planning for five consecutive cycle. It is found that Scenario BM is always outperformed by both scenarios BP and SI. It is found that the profit functions of the retailer does not demonstrate a cumulatively pattern. Due to additional procurement in the first selling period, the profit functions demonstrate that nature. However, one can not conclude with regards to the optimality of the procurement planning of the retailer.

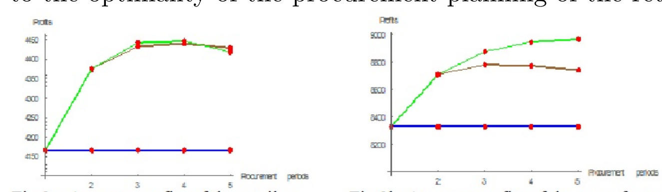

Fig 2a. Average profits of the retailer Fig 2b. Average profits of the manufacturer
a=200, b=0.6, and h=10 Scenario BP (green), WSI (Brown), and BM (blue)

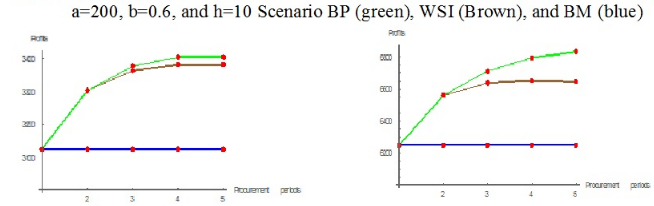

Fig 3a. Average profits of the retailer Fig 3b. Average profits of the manufacturer
a=200, b=0.8, and h=5 Scenario BP (green), WSI (Brown), and BM (blue)

Price elasticity and product holding cost are two extremely important factors affecting procurement decision and overall profitability. Price-elasticity is a critical factor ([10], [11]) influencing the demand. Therefore, more analytical investigations are required to obtain concrete conclusion.

3 Conclusion

The pricing and procurement decisions in a supplier-retailer five-period supply chain is explored in this study. Under price sensitive demand, impact of three procurement decisions are analyzed and corresponding Stackelberg equilibriums are compared. The comparison among equilibrium outcomes in perspective of profits of each supply chain members demonstrate how the procurement decision is influencing the overall preference of the supply chain members. In contrast to

Anand et al. [1], it is found that the build-up SI is not always profitable for the retailer, and manufacturer also. Price-elasticity and holding cost of the retailer are critical factors effecting procurement decision.
The present analysis can be extended to include several important features. For the analytical tractability, we consider five consecutive selling period. In future, one can extend the generalized version of the proposed model. One can also consider the effect of product deterioration or imperfect quality item.

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