Reconstruction of Surfaces from Cross Sections Using Skeleton Information

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Abstract. Surface reconstruction from parallel cross sections is an important problem in medical imaging and other object-modeling applications. Shape and topological differences between object contours in adjacent sections cause severe difficulties in the reconstruction process. A way to approach this problem is using the skeleton to create intermediate sections that represent the place where the ramifications occur. Several authors have proposed previously the use of some type of skeleton to face the problem, but in an intuitive way and without giving a basis that guarantees a complete and correct use. In this paper, the foundations of the use of the skeleton to reconstruct a surface from cross sections are expounded. Some results of an algorithm that is based on these foundations and has been recently proposed by the authors are shown that illustrate the excellent performance of the method in especially difficult cases not solved previously.

1 Introduction

The problem of reconstructing the surface of a solid object from a series of parallel planar cross sections (referred hereinafter simply as sections) has captured the attention in the Computer Graphics and Vision literature during the past three decades (see [1,6,9,10,11,13]). This important problem is found, for instance, in the processing of medical images that represent cross sections of the human body interior and are obtained through non-invasive methods like Computerized Tomography (CT) and Magnetic Resonance. Other applications are the non-destructive digitization of three-dimensional (3D) objects from their slices and the reconstruction of 3D models of the terrain from topographic elevation contours.

In general, the data consist of a series of sections that are separated to a constant distance. Each one of them is formed by a set of closed contours that define the boundary of the material of interest to be modeled. The problem resides in finding a set of planar closed convex polygons (usually triangles) that connect the vertices of the contours, so that the surface of a geometrically complete object (see definition in [3]) is built. As the sections are consecutive, the problem can be reduced to that of

A. Sanfeliu and J. Ruiz-Shulcloper (Eds.): CIARP 2003, LNCS 2905, pp. 180–187, 2003.

finding the set of polygons that join the contour vertices corresponding to each pair of adjacent sections.

Until the beginnings of the eighties, the methods proposed to solve the problem forced the connection of each vertex of a section with some vertices of the adjacent sections. However, as a certain distance separates the sections, the information of the places where the ramifications occur in the surface of interest is often missing. This causes differences in shape (Fig. 1a) and in the number of contours (Fig. 1d) between adjacent sections. In these cases, the restriction aforementioned does not only make impossible the treatment of the ramifications (Fig. 1d), but rather it causes a not very real tiling, even producing unavoidable interceptions between the triangles that are formed (Fig. 1b).

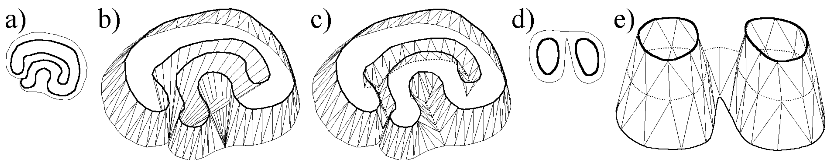

Fig. 1. Adjacent sections with shape differences. a), b) y c) are modified from [1]

A way to approach this problem is creating intermediate sections that represent the place where the ramifications occur (*dotted line* in Figs.1c, e). To this end, several authors [1,6,8,9,10] have proposed previously the use of a skeleton, but in an intuitive way and without giving a basis that guarantees a complete and correct use.

The previous related works that approach the problem using some type of skeleton are commented in the next section. The relationships between the concepts of image, skeleton and section are studied in Sections 3 and 4, and they constitute the foundations of the use of the skeleton to reconstruct a surface from cross sections. Finally, some results of an algorithm recently proposed by the authors which is based on these foundations [11] are shown that illustrate the excellent performance of the method.

2 Overview of Previous Related Work

Sloan and Hrechanyk [13] were the first ones to suggest the creation of artificial intermediate sections between adjacent sections in the cases where these sections are very different. Then the tiling between this artificial section and each one of the two that originated it could be made using any of the proposed methods. In this way, the model would better fit the reality, representing the place where the ramification occurs in the intermediate section.

However, it is not until the work of Levin [8] where the first method is proposed that builds a set of intermediate contours between contours of adjacent original sections in order to solve the ramification and tiling problems. The Levin's method is based on calculating the *distance field* for each point of each section. This value is the signed distance between the analyzed point and its nearest contour. In terms of distance fields, contours can be regarded as isocurves with an isovalue of zero. The value is positive or negative depending on whether the point is inside or outside the contour, respectively. The distance fields of the intermediate section are obtained by adding, for each point, the values of the corresponding distance fields in the original contiguous sections. The main limitation of this method is the very large number of triangles that the obtained surface presents.

The polygonal form of skeleton called *medial axis* is used for the first time in Meyers' doctoral thesis [9]. However, it is not inserted between adjacent sections, but used to obtain information about the relationships of vicinity among the regions where ramifications occur. The form of skeleton used was called *shaved medial axis* (SMA) (Fig. 1d) and the possible types of connections among its loops helped to classify the ramifications. The method does not work correctly in the cases of ramifications from many-to-many contours. In addition, the projections on a same plane of the contours related with the ramification can intercept each other.

Bajaj *et al*. [1] detected the parts of contours with very different shape and applied a method similar to the one used by Geiger [4] to tile them. This method requires the *edge Voronoi diagram* (EVD), but due to the difficulty in implementing a numerically stable algorithm, Bajaj *et al*. proposed to find a rough medial axis using an iterative decomposition of the polygon, in which cutting edges are added until all polygons are convex. The authors did not specify how to implement this decomposition.

Oliva *et al*. [10] used a new type of skeleton called *angular bisector network* (ABN) that was calculated as an approximation to the EVD. Each segment of the analyzed contours was associated with a cell of the ABN. The cells that guarantee a certain level of proximity can be triangulated in a straightforward way. Otherwise, an intermediate contour is inserted that consists of the common border between cells corresponding to contour segments in different sections. This procedure can be repeated recursively until all the cells are triangulated.

In a recent work, Klein *et al*. [6] presented an algorithm that combines the approach proposed by Levin and the recursive triangulation proposed by Oliva *et al*. Instead of the complex calculation of the ABN, Klein *et al*. computed discrete distance fields to define intermediate contours and the needed correspondences. The main advantage of the method proposed is the use of the *z-buffer* of standard graphics hardware to obtain the medial axis that separates the projections of the analyzed contours and the proximity correspondences between the vertices of the medial axis and those of the analyzed contours. All the examples shown by Klein *et al*. were artificial and none of them included holes.

3 Relationship between Image and Skeleton

3.1 Definitions

An *image* can be defined as a function f: N x N → G where f(x, y) is the illumination of a pixel with spatial coordinates (x, y) belonging to the set N of natural numbers and G is the set of positive integer numbers representing their illumination. In this work, *binary images* will be used, where the two values represent the background and the object (*white* and *black pixels* in Fig. 2a, respectively).

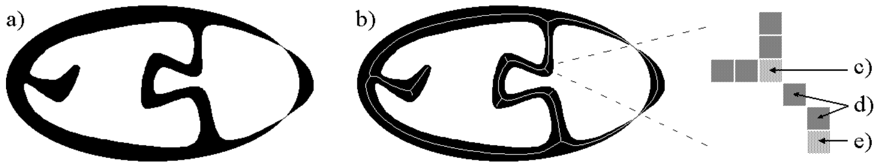

Fig. 2. Binary image (a), skeleton over it drawn in white (b) and pixel connectivity (c-e)

The skeleton or medial axis can be defined, in a general way, as a set of connected lines or curves that are equidistant with respect to the borders or limits of a figure [12]. If the figure is represented by a binary image, the skeleton is its narrowest representation (Fig. 2b). A specific definition is: the *skeleton* $E(I)$ of an image I is a set of points p located inside the boundary of I such that there exists, for each one of them, at least two points belonging to the boundary that are separated at a minimum distance from it (and, therefore, p is halfway).

The process for which the skeleton of an image is obtained is denominated *skeletonization*. Most of the skeletonization algorithms erode the borders of the binary image repeatedly until narrow lines or simple pixels remain. This erosion process is also known as *thinning*. Taking into account the comparative analysis of twenty thinning algorithms carried out in [7], we selected, among the algorithms that preserve connectivity, the Suzuki-Abe algorithm [14] to be used in this work, due to its high speed, simplicity and demonstrated success, even in recent works [5].

3.2 Connectivity of Skeleton Vertices

By definition of skeleton, the connectivity of each one of its points p_i is determined by the number of pixels that belong to the boundary of I and are at a minimum distance from p_i. This means that the connectivity of the $E(I)$ pixels is determined by the shape of the original image boundary. When the shape is similar along a certain trajectory what is obtained is just a path for this trajectory. For example, the skeleton of an image that represents a hand-written letter is an approximation of the way that the pencil tip goes through when drawing it.

Thus, each pixel of $E(I)$ can be classified according to its connectivity. A *terminal pixel* has connectivity one and is caused by the presence of a local maximum in the I boundary. A terminal pixel appears at the end of a segment where a protuberance or local convex shape occurs in I (Fig. 2e). An *intermediate pixel* has connectivity two and is obtained when the I boundary presents a similar shape in both sides of the line that goes approximately through the pixel and its two adjacent pixels (Fig. 2d). Lastly, a *branch pixel* has connectivity greater than two and is caused by a ramification involving two or more trajectories. A branch pixel appears in the place where the shape of I ramifies (Fig. 2c). Terminal and branch pixels will be called *extreme pixels*.

Analyzing the existent connections among the different skeleton pixels, a skeleton can be considered as a set of extreme pixels that are connected to each other through zero or more intermediate pixels. A group of intermediate pixels that connect a pair of extreme pixels will be called a *rail* of the skeleton. Note that a rail is equidistant from two portions of the image boundary that present a similar shape along it.

4 Relationship between Skeleton and Section

The problem of surface reconstruction from sections consists of obtaining a surface that connects the vertices of the contours that belong to each pair of adjacent sections. A supposition that covers most of the real cases is that the projection of this surface, on an intermediate plane parallel to the original sections, should be in the region that separates the material of interest of each pair of adjacent sections. The strange cases not covered by this supposition have not been dealt with by any of the consulted authors. Nevertheless, some results on these strange cases are included in Section 5.

4.1 Construction of the Skeleton from Adjacent Sections

When projecting the regions occupied by the material of interest of the adjacent sections on a parallel plane (Fig. 3b), the region not common to both regions is the one that separates the material of interest of these sections (Fig. 3c).

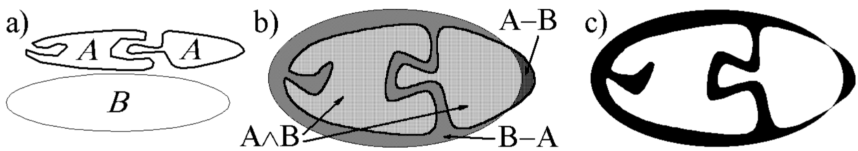

Fig. 3. Region that separates the material of interest in adjacent sections

This action can be expressed using logical operations on binary images. Each section is represented as a binary image where the region that occupies the material of interest has been determined. The image I that separates the material of interest of two adjacent sections is the result of the binary operation XOR ($\vee$ exclusive) on the corresponding two images. In this way, the only pixels in I that are drawn are those that belong to the region of the material of interest in only one of the analyzed sections. After this operation, it is necessary to include the pixels that form the boundary of each one of the contours involved in order to be able to include in the skeleton the pixels where the contours of the adjacent sections intercept (if any). To obtain the skeleton it would remain to apply some skeletonization technique (commented in 3.1) to the image I, whose result would be similar to the one shown in Fig. 2b.

4.2 Significance of the Skeleton

As has been discussed in 3.2 the connectivity of the $E(I)$ pixels is determined by the shape of the boundary that the image I represents. However, as has been defined in 4.1, the boundary of I is formed by the contours of the contiguous sections. Therefore, the shape of the analyzed contours determines the connectivity of the pixels of the resulting skeleton $E(I)$. The analysis that follows is very similar to the one discussed in 3.2, but taking into account that the shape of the image I is conditioned locally by the separation of the near contours in the adjacent sections.

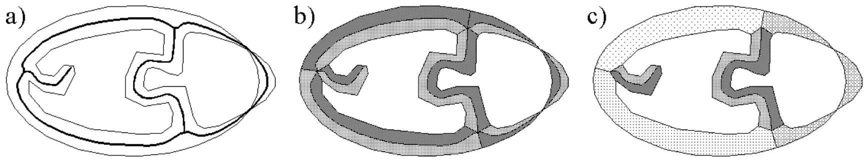

Fig. 4. Formation and fusion of ribbons

As has been explained in 3.2, a rail separates equidistantly two portions of the image boundary that present a similar shape along it. By construction of *E*(*I*), these portions of the *I* boundary correspond to the projections of portions of the contours that belong to the analyzed adjacent sections. It is deduced then that a rail of *E(I),* built according to 4.1, is halfway the projections of two nearby contour portions (PC_{Ca}, PC_{Cb}) with similar shape located in the contours C_a and C_b, respectively. If PC_{Ca} and PC_{Cb} belong to the same section, then the rail will represent the place where the necessary ramification occurs, so that these portions will be connected to each other at an intermediate height of the analyzed sections.

In this way, we arrive to the basic structure of the reconstruction called *ribbon*, which is composed by a rail *L* and a contour portion *PC*, such that they are close to each other and keep some shape similarity. The proximity relationship implies that there is no other rail or contour portion between *L* and *PC*. We can take advantage of these ribbon properties to reconstruct the surface that the ribbon forms using some simple and quick algorithm [2]. Notice that, due to the skeleton construction, each rail has two associated ribbons, one for each one of its sides.

During the reconstruction process, the endpoints of each contour portion should be included so that the union of the contour portions associated with each rail of the skeleton yields the original contours (Fig. 4b). In this way, the reconstruction of the surface between two adjacent sections is reduced to the union of the reconstruction of all the ribbons that form it.

The large number of generated triangles is one of the drawbacks of some of the consulted methods that use the skeleton in the surface reconstruction process. In order to reduce the number of triangles, the ribbons that share a common rail and connect contour portions located in different sections can be fused together (Fig. 4c).

5 Results and Discussion

We have recently proposed a surface reconstruction method which implements all the ideas presented in the two previous sections [11]. Fig. 5 displays some results on synthetic and real examples that are discussed below. See [11] for more details.

The example shown along this work (Fig. 5a) contains two adjacent sections that not only have a different number of contours, but also a very marked difference in their shapes. The result on this example is shown in Figs. 5a and 5b. Another example refers to the existence of holes in some of the sections, and the corresponding result is shown in Figs. 5c and 5d. Finally, Figs. 5e-5i display the results on an example that has not been solved by any method of the consulted literature. It is a surface portion that twists abruptly among the sections, causing its projection to be found inside the region belonging to the material of interest in both sections.

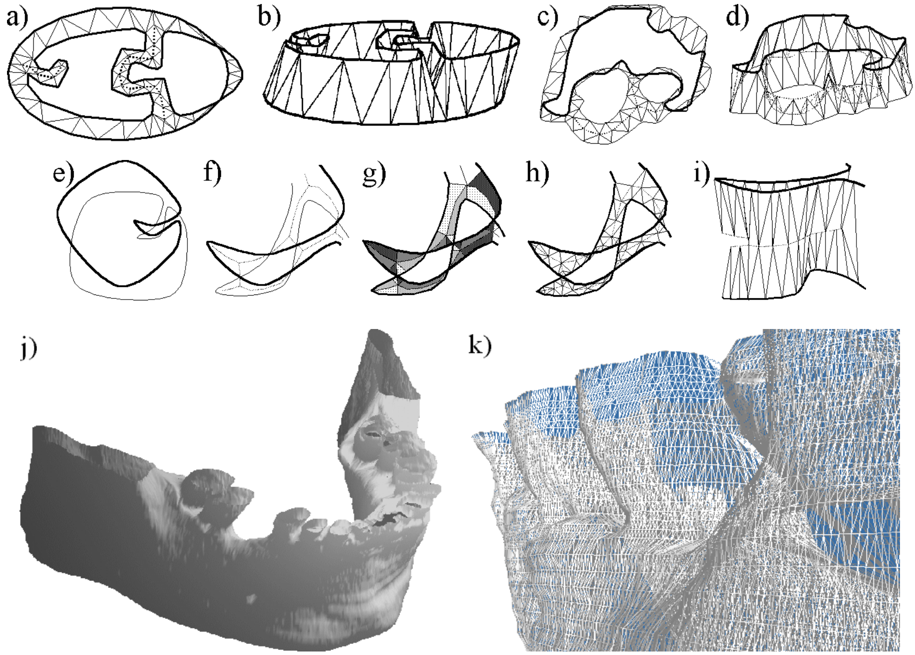

Fig. 5. Tiling result (*a, c, h*) and 3D view (*b, d, i, j, k*) of some reconstruction examples

In all synthetic cases it can be observed that the reconstructed surface is closed and does not intercept itself. The place where the ramifications occur is inserted in an intermediate height of the original sections and shown in the figures as dotted lines.

The results of the proposed algorithm are also shown in the reconstruction of a human jaw from real TC images (Figs. 5j, 5k), where the correct reconstruction of ramifications is observed in the base of the teeth.

6 Conclusions

In this paper, the foundations of the use of the skeleton to reconstruct a surface from cross sections have been explained and illustrated.

After a review of the previously reported works that have used some type of skeleton to solve the surface reconstruction problem, it was concluded that all of them made an intuitive use of the skeleton and there was a lack of a basis that guaranteed a complete and correct use of the skeleton information.

We have argued that there exists a close relationship between the contours of two adjacent sections and the rails of the skeleton built from the region that separates the material of interest in both sections. By skeleton construction, each rail separates equidistantly the projections of two nearby contour portions with similar shape. This property can be used to reconstruct in an easy way the region, denominated ribbon, which is between the rail and one of the related contour portions. If both contour portions belong to the same section, then the rail can be used to represent the place where the ramification occurs in an intermediate height to the original sections. Otherwise, there is no ramification and, to reduce the number of triangles in the resulting surface, the rail may be discarded by fusing the adjacent ribbons that share it. The final surface reconstruction reduces to the union of the reconstruction of all the formed ribbons.

Some results of a surface reconstruction algorithm recently proposed by the authors have been shown. This algorithm [11] is based on the ideas presented here to give solution to the investigation problem. The examples displayed here have included difficult cases, even one not solved by the consulted literature. In all cases, a topologically correct surface is obtained. Moreover, all the cases are treated in a unified way, independently of whether the number of contours in the adjacent sections is the same or not, or whether the shapes of the involved contours are similar or not. Hence, a great deal of generality is achieved in the proposed solution.

References

1. Bajaj, C.L., Coyle, E.J., Lin, K.N.: Arbitrary topology shape reconstruction from planar cross sections. Graphical Models and Image Processing. 58 (1996) 524–543
2. Christiansen, H.N., Sederberg, T.W.: Conversion of contour line definitions into polygonal element mosaics, Computer Graphics. 13 (1978) 187–192
3. Foley, J.D., Van Dam, A., Feinier, S.K., Hughes, J.F.: Computer Graphics: Principles and Practice. Second Edition in C. Addison Wesley (1996)
4. Geiger, B.: Tree-dimensional modeling of human organs and its applications to diagnosis and surgical planning. Technical report, 2105, INRIA, France (1993)
5. Kégl, B., Krzyzak, A.: Piecewise linear skeletonization using principal curves. IEEE Transactions on Pattern Analysis and Machine Intelligence. 24(1) (2002) 59–74
6. Klein, R., Schilling, A., Straßen, W.: Reconstruction and simplification of surface from contours. Graphical Models. 62(6) (2000) 429–443
7. Lee, S.W., Lam, L., Suen, C.: A systematic evaluation of skeletonization algorithms. Int. J. of Pattern Recognition and Artificial Intelligence. 7(5) (1993) 1203–1225
8. Levin, D.: Multidimensional reconstruction by set-valued approximation. IMA J. Numerical Analysis. (6) (1986) 173–184
9. Meyers, D.: Reconstruction of surfaces from planar contours. Doctoral dissertation. University of Washington (1994)
10. Oliva, J.M., Perrin, M., Coquillart, S.: 3D reconstruction of complex polyhedral shapes from contours using a simplified generalized Voronoi diagram. Comp. Graphics Forum, 15(3) (1996) C397–C408
11. Pina, J., Alquézar, R.: Results of a new method for surface reconstruction from tomographic images. In Proc., V Cong. Soc. Cub. de Bioingeniería, T0090, Havana, Cuba (2003)
12. Ritter, G.X., Wilson, J.N.: Handbook of Computer Vision Algorithms in Image Algebra. Second Edition. CRC Press (2001)
13. Sloan, K.R., Hrechanyk, L.M.: Surface reconstruction from sparse data. In Proc., IEEE Conf. on Patt. Recog. and Image Processing, Dallas (1981) 45–48
14. Suzuki, S., Abe, K.: Sequential thinning of binary pictures using distance transformation. In Proceedings of the 8th Int. Conf. on Patt. Recog. (1986) 289–292