Routing, Wavelength Assignment in Optical Networks Using an Efficient and Fair EDP Algorithm

Pallavi Manohar and V. Sridhar*

Applied Research Group, Satyam Computer Services Ltd.,
14 Langford Avenue, Lalbagh Road, Bangalore 560 025 INDIA.
{pallavi_mm, sridhar}@satyam.com

Abstract. Routing and wavelength assignment (RWA) problem in wavelength routed optical networks is typically solved using a combination of integer programming and graph coloring. Such techniques are complex and make extensive use of heuristics. RWA with maximum edge disjoint path (EDP) using simple bounded greedy algorithm is shown to be as good as previously known solution method. In this paper, we present shortest path first greedy algorithm for maximum EDP with construction of path conflict graph which gives fair and efficient solution to the RWA problem in optical networks.

Keywords: Optical Networks, Routing and Wavelength Assignment, Edge Disjoint Path Algorithms.

1 Introduction

Routing and wavelength assignment (RWA) for the lightpaths of the virtual topology in wavelength routed optical networks is typically solved in two parts – lightpath routing, to determine the route of the lightpaths on the physical topology, and wavelength assignment, to assign the wavelengths to each lightpath in the virtual topology such that wavelength constraints are satisfied for each physical link. The RWA problem for a given physical network and virtual topology is known to be NP-complete. Further, routing and wavelength assignment problems are each known to be NP-complete [3] and many heuristic solutions have been proposed [7,8]. See [3,10,11] for a survey of the various solutions to the RWA problem. Lightpath routing is done by first selecting the lightpaths and then assigning routes to them. The order in which the lightpaths are assigned can be selected by different heuristic schemes such as random, fixed, longest-first, or shortest-first. Routes can be found by techniques such as shortest path algorithm, weighted shortest path, or $K-$shortest path algorithms. The route is selected from the available candidates using different schemes such as random, first-fit, or probability. The RWA problem is also solved by formulating it as an

* *Member, IEEE*

M. Bubak et al. (Eds.): ICCS 2004, LNCS 3038, pp. 1178–1184, 2004.

integer linear program (ILP) optimization problem [9] with one of two objective functions – minimize the required number of wavelengths to establish the given set of lightpaths and maximize the number of established lightpaths with a fixed number of wavelengths. A hard version of the problem is when no wavelength conversion is used at the routing nodes of the physical network and this requires that the wavelength continuity constraint be satisfied, i.e., a lightpath should use the same wavelength on all the links in its path. Extensive and complex heuristics are used to solve the ILP. One solution considers a fractional relaxation with rounding techniques to obtain an integral solution and wavelength assignment with graph colouring method [1].

Solving the RWA problem using the mathematical programming is quite complex in view of the large number of variables and constraints, and complex problem size reduction heuristics. Further, it will not be known how close the solution is to the optimal. A simple and intuitive alternative method for RWA with performance comparable to that from mathematical programming techniques is given in [8]. In the next section we describe their method based on the simple greedy solution of a well known graph theory problem. Then, in Section 3 we present our solution for RWA using fair and efficient greedy solution for the same graph theory problem. We then give approximation bounds for the algorithm and conclude the paper in Section 5.

2 RWA Using Edge Disjoint Paths

In this section, we describe simple and intuitive solution to the RWA problem based on a well known graph theory problem [8]. Observe that the lightpaths that are assigned the same wavelength do not traverse through same physical link, i.e., these paths are *edge disjoint*. In graph theory, given a graph and a set of source-destination pairs and the requirement that a path be found for as many of the pairs as possible, finding the maximum set of edge disjoint paths is a well known problem and is formally defined as follows. Let $G = (V, E)$ be the graph of the physical network with no wavelength conversion, V the vertex set, and E the edge set. Connection request i is specified by a pair (s_i, t_i), $(s_i, t_i) \in V$. Let τ be the set of connections for whom edge disjoint paths need to be found, $\tau = \{(s_1, t_1), \ldots, (s_k, t_k)\}$. τ is said to be realizable in G if there exist mutually edge-disjoint paths $P_1, \ldots, P_k$ in G such that P_i has endpoints s_i and t_i. The *maximum edge disjoint paths problem* is to find a maximum size realizable subset of τ, given G and τ. Let the maximum size of a realizable subset of τ in G by $\alpha(\tau)$. It is easy to see that the maximum edge disjoint path (EDP) problem is a combinatorial optimization problem and is known to be NP-hard [5].

Given G and τ, let $\tau_1, \cdots \tau_i$ be partitions of τ such that partition is a solution to the maximum EDP problem on G, τ. Now the application to the RWA problem is follows. Given G and τ consider a solution of the maximum EDP problem that obtains a realizable set $\tau_1 = \alpha(G, \tau)$. Since the paths in τ_1 are edge disjoint they can be assigned the same wavelength, say λ_1. Now construct the set $\tau_1' = \tau - \tau_1$, i.e., τ_1' is the set of connections not contained in τ_1. An iteration of maximum

EDP on the G and τ_1' will give $\tau_2 = \alpha(G, \tau_1')$ and the paths in τ_2 can be assigned the same wavelength, say λ_2. This process can be repeated till τ_i' is empty for some i. The minimum such i will be denoted by $\chi(G, \tau)$. Finding $\chi(G, \tau)$ is the problem of minimizing the number of wavelengths in wavelength routed optical networks. It is easy to see that if the solution to the maximum EDP is optimum, then the optimum $\chi(G, \tau)$ can be obtained from above.

Simple greedy algorithm for maximum EDP is used in [8] and it is shown that it performs as good as that of more complex solution methods based on integer linear programming. They use bounded greedy algorithm (BGA) for the maximum EDP that was first described in [5]. Here, the order in which the lightpaths are selected is random. In this paper, we present improved greedy algorithm for maximum EDP. This uses shortest-first heuristics for the selection of lightpaths to be assigned and construction of path conflict graph [7] for assigning the routes from available candidate routes. In general, any network has the property that longer paths are likely to experience higher blocking than shorter ones. Consequently, the degree of fairness can be quantified by defining the unfairness factor as the ratio of the blocking probability on the longest path to that on the shortest path for a given RWA algorithm. Blocking of long lightpaths leaves more resources, such as wavelengths on the physical link in wavelength routed networks, available for short lightpaths. The problem of unfairness is more pronounced in networks without converters since finding long paths that satisfy the wavelength continuity constraint is more difficult than without this constraint. This motivates the use *shortest path first greedy* algorithm (SGA) [6] for RWA in wavelength routed optical networks. The first fit shortest path among the enumerated alternate shortest paths is selected in [8]. This may not give all possible disjoint paths in that partition. Basically this doesn't make use of the available alternate shortest paths which may lead to an inefficient solution. Further the algorithm can be improved with construction of *path conflict* graph from which a set of *desirable* routes for all (s_i, t_i) pairs in τ can be obtained. Thus, the route for each (s_i, t_i) pair is selected from this set. This efficiently packs disjoint paths for each partition and hence motivates its use for RWA especially in the case of reducing the blocking probability. In the following, we describe details of the solution of the RWA problem using the fair and efficient greedy algorithm to solve the maximum EDP.

3 RWA with Fair and Efficient Greedy EDP

For solving RWA using maximum EDP, in [8], a lightpath is selected randomly from traffic matrix and first fit shortest path is assigned to it. We improve this algorithm in two ways. First, by using SGA, that is, by selecting the lightpath from the traffic matrix which has minimum length shortest path. This implements fairness in routing for optical networks. The shorter paths are always assigned before the longer ones. Secondly, we make use of *alternate* shortest paths found for each (s_i, t_i) pair. Basically, we prune the set of routes further such that a route sharing minimum number of edges with routes for $(\tau - (s_i, t_i))$

is selected for each (s_i, t_i) pair. Then, from this *desirable* set, route is assigned. Note that it may not reduce the total number of wavelengths required in all the cases but it efficiently finds possible disjoint paths that are assigned the same wavelength. This improves reuse of the wavelength further. This *desirable* set of routes is obtained using path conflict graph as explained further in this section. We consider RWA for static case as selection of lightpath as well as route is performed offline while the algorithm in [8] can be applied to static and dynamic cases.

3.1 RWA with Shortest Path First Greedy EDP

In this section, we describe the details of the solution of the RWA problem using SGA to solve the maximum EDP. The inputs to the algorithm are the graph G of the physical network with no wavelength conversion and the lightpath set τ. The output of the algorithm is $\mathcal{P}$ which is the set of pairs $((s_i, t_i), P_i)$ where (s_i, t_i) is a routed lightpath and P_i is the physical path assigned to this lightpath. The algorithm, SGAforEDP(G, τ), operates as follows. (a) Select a lightpath (s_i, t_i) from the set τ, such that the shortest path P_i in G for this connection has minimum length among those of all requests in the set τ. If there are more than one candidates which can be selected, then choose one of them arbitrarily; (b) Find the shortest path P_i from *desired* set of routes (explained in Sect. 3.2) for this connection; (c) Add $((s_i, t_i), P_i)$ to the path set $\mathcal{P}$ and (s_i, t_i) to the set of routed lightpaths $\alpha(G, \tau)$ and delete the edges in G used by P_i; (d) Remove (s_i, t_i) from the set τ and repeat this while τ contains a request which can be routed in G. The algorithm is summarized in Fig. 1. Note that $\alpha(G, \tau)$ contains the lightpaths that get assigned the same wavelength.

Algorithm SGAforEDP(G,τ)
Begin
    $\alpha(G, \tau) = \phi$; $\mathcal{P}(G, \tau) = \phi$
    While (τ contains a request which can be routed in G) do
        If ((s_i, t_i) such that a shortest path P_i from
        s_i to t_i in G has minimum length among the
        requests in τ) then
            $\alpha(G, \tau) = \alpha(G, \tau) \bigcup \ (s_i, t_i)$
            $\mathcal{P}(G, \tau) = \mathcal{P}(G, \tau) \bigcup \ ((s_i, t_i), P_i)$
            $\tau = \tau - (s_i, t_i)$
            Delete edges in path P_i from G
        EndIf
    EndWhile
End

Fig. 1. Pseudo code for algorithm SGAforEDP

Each iteration of the SGAforEDP results in a set of lightpaths $\alpha(G, \tau)$ that can be assigned the same wavelength. And τ is the set of unassigned lightpaths. We run SGAforEDP with the physical topology graph G and the set of unassigned lightpaths to obtain the set of lightpaths that get assigned a distinct wavelength. This is repeated till all the lightpaths are assigned. The pseudo code is shown in Fig. 2.

```
Algorithm Greedy_EDP_RWA(G,τ)
Begin
    λ = 0
    While ( τ ≠ φ) do
        λ = λ + 1
        SGAforEDP(G,τ)
        Assign λ to all paths P_i ∈ 𝒫
    EndWhile
End
```

Fig. 2. Pseudo code for the algorithm Greedy_EDP_RWA

3.2 Obtain Desired Set of Routes

The shortest path for a selected lightpath in Fig. 1 is chosen from *desired* set of routes. This set is obtained after pruning enumerated alternate shortest paths using path conflict graph [7]. This is a graph having k partitions each for (s_i, t_i) pair. The path conflict graph $G_P = (V_P, E_P)$ has V_P vertices and each vertex represents one route. For an (s_i, t_i) pair there are one or more candidate routes that is alternate shortest paths which implies $|P_{s_i,t_i}| \geq 1$. Thus there are $\sum_{i=1}^{k} |P_{s_i,t_i}|$ vertices in V_P. Because of construction of G_P, the vertices in V_P can be partitioned into k disjoint sets such that $V_P = V_{s_1,t_1} \bigcup \ldots \bigcup V_{s_k,t_k}$. Each partition corresponds to one (s_i, t_i) pair. For any $u, v \in V_P, (u, v) \in E_P$ if and only if two routes corresponding to u and v share at least one edge in $G = (V, E)$. Now the question is how to choose the *best* vertex that is route from each partition to obtain *desired* set of routes. We use the heuristic given in [7] which is as follows. First remove all edges whose vertices are in the same partition, then choose a vertex with the smallest degree as the *best* vertex and remove all other vertices in the same partition. Continue this to obtain such a vertex from each partition. This gives us the *desired* set of routes.

3.3 Limited Wavelengths

The RWA can also be solved with objective of maximizing the number of lightpaths that are routed using a fixed number of wavelengths, say L. This is similar to the objective function of Sivarajan and Ramaswami [9]. The Greedy_EDP_RWA algorithm defined above can be used in this form of RWA as follows. Observe that there can only be a maximum of L partitions of the lightpath set τ with each partition being assigned the same wavelength. It is easy to see that we need to perform L iterations of the main loop of Greedy_EDP_RWA. The pseudo code for this algorithm called as Greedy_EDP_RWA_Limλ is shown in Fig. 3. The carried traffic τ_{carried} is the set of lightpaths for which route and wavelength is assigned whereas blocked traffic τ_{blocked} is the set of unassigned lightpaths which is blocked due to limited number of wavelengths available in the network. Use of *desired* set of routes makes more sense in this case as it efficiently packs disjoint routes for each wavelength. This increases carried traffic reducing the blocking probability further.

```
Algorithm Greedy_EDP_RWA_Limλ(G,τ)
Begin
    λ = 0
    τ_carried = φ; τ_blocked = φ
    For i = 1 to L
        λ = λ + 1
        SGAforEDP(G,τ)
        Assign λ to all paths P_i ∈ 𝒫
        τ_carried = τ_carried ⋃ α(G,τ)
    EndFor
    τ_blocked = τ
End
```

Fig. 3. Pseudo code for the algorithm Greedy_EDP_RWA_Limλ

4 Approximation Bounds

The maximum EDP is NP-hard and is also hard to approximate and good heuristics are not well known [5]. The best known approximation guarantee for arbitrary graphs is $\mathcal{O}(\max \sqrt{m}, \mathtt{diam}(G))$, where m is the number of edges in set E and $\mathtt{diam}(G)$ is the diameter of the graph G [5]. Srinivasan and Baveja [2] improved the approximation algorithms based on LP relaxation. Guruswami *et al* [4] have shown that simple greedy algorithms and randomized rounding algorithms can yield approximations similar to those given in [2]. Greedy algorithms have been extensively studied in combinatorial optimization due to their elegance and simplicity. The approximability of the maximum edge disjoint paths problem (EDP) in directed graphs is seemingly settled by the $\Omega(m^{1/2-\epsilon})$-hardness result of Guruswami *et al* [4] and the $\mathcal{O}(\sqrt{m})$ approximation achievable for the greedy algorithm [5]. Let OPT denote the integral optimum value for the optimal solution for a given instance. Kolliopoulos and Stein [6] show similar bounds for the shortest first greedy algorithm obtaining $\max(OPT/\sqrt{m_0}\ ,\ OPT^2/m_0\ ,\ OPT/d_0)$ paths where m_0 and d_0 are the number of edges and average length of paths respectively, in some optimum solution. The underlying idea in SGA is that if one keeps routing commodities along sufficiently short paths the final number of commodities routed is lowerbounded with respect to the optimum. It is clear that the approximation ratio of SGA with construction path conflict graph is at least as good as that of BGA. As $E_0 \subseteq E$ is the set of edges used by the paths in an optimal solution and $m_0 = |E_0|$, SGA rather provides the better approximation bound than that of simple BGA. The approximation existentially improves when $|E_0| = o(|E|)$, i.e., when the optimal solution uses a small portion of the edges of the graph.

5 Conclusion

In this paper, we have presented shortest first greedy edge disjoint path based solution for the routing of lightpaths of the virtual topology of an optical network and the assignment of wavelengths to them. This method is appealing in its simplicity and employs fairness aspect providing better approximations than that of RWA with simple greedy for maximum EDP. Further, this algorithm can also be used to solve the RWA in which the total number of available wavelengths is upper bounded and the carried traffic can be improved with the construction of path conflict graph.

References

1. Banerjee D. and B. Mukherjee, "A Practical Approach for Routing and Wavelength Assignment in Large Wavelength Routed Optical Networks," *IEEE Journal on Selected Areas in Communications*, vol 14, no 5, pp. 903-908, June 1996.
2. Baveja A. and A. Srinivasan, "Approximation Algorithms for Disjoint Paths and Related Routing and Packing Problems," *Mathematics of Operations Research*, vol 25, pp. 255-280, May 2000.
3. Choi J., N. Golmie, F. Lapeyrere, F. Mouveaux, and D. Su, "A Functional Classification of Routing and Wavelength Assignment Schemes in DWDM Networks : Static Case," *Proceedings of VII International Conference on Optical Communication and Networks*, Jan 2000.
4. Guruswami V., S. Khanna, R. Rajaraman, B. Shepherd, M. Yannakakis, "Near-Optimal Hardness Results and Approximation Algorithms for Edge-Disjoint Paths and Related Problems," *Proc. of ACM Symp. on Theory Comput.*, pp. 19-28, 1999.
5. Kleinberg J., "Approximation Algorithms for Disjoint Paths Problems," *PhD Thesis*, MIT, Cambridge, MA, May 1996.
6. Kolliopoulos S., C. Stein, "Approximating Disjoint-Path Problems Using Greedy Algorithms and Packing Integer Programs," *Lecture Notes in Computer Science*, vol 1412, 1998.
7. Li G. and R. Simha, "The Partition Coloring Problem and its Application to Wavelength Routing and Assignment," *First Workshop on Optical Networks*, Dallas, TX, 2000.
8. Manohar P., D. Manjunath and R. K. Shevgaonkar, "Routing and Wavelength Assignment in Optical Networks from Edge Disjoint Path Algorithms," *IEEE Communications Letters*, vol 6, pp. 211-213, May 2002,.
9. Ramaswami R., K. Sivarajan, "Routing and Wavelength Assignment in All-Optical Networks," *IEEE/ACM Transactions on Networking*, vol 3, no 5, pp. 489-500, October 1995.
10. Rouskas G., "Routing and Wavelength Assignment in Optical WDM Networks," *Wiley Encyclopedia of Telecommunications*, 2001.
11. Zang H., J. Jue, and B. Mukherjee, "A Review of Routing and Wavelength Assignment Approaches for Wavelength-Routed Optical WDM Networks," *SPIE/Baltzer Optical Networks Magazine*, vol. 1, no. 1, Jan. 2000.