Implicit and Explicit Higher Order Time Integration Schemes for Fluid-Structure Interaction Computations

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Abstract. In this paper higher order time integration schemes are applied to fluid-structure interaction (FSI) simulations. For a given accuracy, we investigate the efficiency of higher order time integration schemes compared to lower order methods. In the partitioned FSI simulations on a one-dimensional piston problem, a mixed implicit/explicit (IMEX) time integration scheme is employed: the implicit scheme is used to integrate the fluid and structural dynamics, whereas an explicit Runge-Kutta scheme integrates the coupling terms. The resulting IMEX scheme retains the order of the implicit and explicit schemes. In the IMEX scheme considered, the implicit scheme consists of an explicit first stage, singly diagonally implicit Runge-Kutta (ESDIRK) scheme, which is a multistage, L-stable scheme.

1 Introduction

For many engineering applications, fluid-structure interaction (FSI) phenomena are important for an efficient and safe design. Increased computational power has enabled the simulation of FSI, through coupling of existing flow and structure solvers. However, the simulation of long term dynamic behavior is still very time consuming. Therefore efficiency of the FSI solver is of the utmost importance.

It has already been demonstrated that for flow applications, higher order time integration schemes are computationally more efficient than popular lower order schemes, even for engineering levels of accuracy [2]. This drives the idea to use higher order time integration schemes for fluid-structure interaction simulations as well. So far we only found examples of fluid-structure interaction computations based on at most second order implicit time integration methods [3,5,11]. For the coupled fluid-structure simulations we envisage a partitioned scheme, meaning that an existing flow and structure solver can be used, each solving efficiently their own equations on a separate domain and coupling is obtained through boundary conditions. A partitioned strategy enables the re-use of all the effort put into the development and optimization of such codes, especially the

* Funded by the Netherlands Organisation for Scientific Research (NWO)

M. Bubak et al. (Eds.): ICCS 2004, LNCS 3039, pp. 604–611, 2004.

iterative solvers. In the partitioned scheme, the coupling terms between fluid and structure are not solved in a fully coupled system, but are given as a boundary condition. In order to obtain a stronger coupling, a predictor-corrector scheme is sometimes suggested, but we only found applications with first or second order time integration schemes. Also the use of sub-iterations at every time step is sometimes proposed, which comes, however, at an increased computational cost. An other possibility is to integrate one system with an explicit scheme, which can only be done efficiently in the absence of stiffness for that system.

In this paper we consider a mixed implicit/explicit (IMEX) time integration scheme based on higher order implicit and explicit Runge-Kutta methods. The separate fluid and structural dynamics are integrated by an unconditionally stable, implicit scheme and only the coupling component is integrated by the explicit scheme. The IMEX scheme is applied to a linear and a nonlinear, one-dimensional piston, which resembles a simple fluid-structure interaction problem. The order and accuracy of the resulting scheme are investigated and efficiency is compared to lower order methods.

2 Fluid-Structure Interaction Model Problem

The test case considered is the one-dimensional piston problem (Fig. 1), which is often used as an initial test case for fluid-structure interaction simulations [1, 10].

Fig. 1. The one-dimensional piston problem

The flow is modelled as a one-dimensional, isentropic, inviscid flow. Usually, the governing equation for the flow are written in the arbitrary Lagrangian-Eulerian (ALE) to cope with the moving and deforming mesh [4,6]. In this paper, however, we only consider the fluid on a non-moving mesh. The governing equation for the fluid yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_K w_\mathrm{f}\mathrm{d}x + \int_{\partial K} F(w_\mathrm{f})\cdot n\mathrm{d}s = 0, \tag{1}$$

wherein $w_\mathrm{f} = \begin{pmatrix}\rho\\ \rho u\end{pmatrix}$ the fluid state vector, ∂K is the boundary of K, $F(w_\mathrm{f}) = \begin{pmatrix}\rho u\\ \rho u^2+p\end{pmatrix}$ is the flux vector and n is the unit normal vector perpendicular to ∂K pointing outward. Coupling is achieved through an inflow/outflow boundary condition at the interface $u(x=0)=0$, $u(x=L)=\dot{q}$.

The structure is modelled as a simple harmonic oscillator with spring stiffness k, mass m and without any physical damping under an external pressure force due to a difference between the ambient pressure and the pressure in the flow at the interface, yielding

$$m\ddot{q} + kq = p(x = L) - p_0, \tag{2}$$

wherein $\ddot{q}$ denotes the structural acceleration $\frac{\mathrm{d}^2}{\mathrm{d}t^2}q$, $p(x = L)$ is the pressure in the flow at the interface and p_0 is the ambient pressure.

The flow is discretized using a cell-centered Finite Volume (FV) method on a uniform mesh of N cells. A standard second order central scheme and a first order upwind scheme [8] are used. In the fluid domain two ghost cells K_0 and K_{n+1} are introduced to cope with the boundary conditions. The structural dynamics are written as a system of two ordinary differential equations for which the state vector is denoted by w_s. The coupled, nonlinear system is written in semi-discrete form

$$\dot{w}_\mathrm{f} = F_\mathrm{f}(w_\mathrm{f}, w_\mathrm{s}), \qquad \dot{w}_\mathrm{s} = F_\mathrm{s}(w_\mathrm{s}, w_\mathrm{f}), \tag{3}$$

wherein F_f the flux for the fluid under boundary condition w_s and F_s the flux for the structure under boundary condition w_f. The system, linearized around an equilibrium state, yields

$$\dot{w}'_\mathrm{s} = A_\mathrm{s}w'_\mathrm{s} + A_\mathrm{sf}w'_\mathrm{f}, \tag{4}$$

$$\dot{w}'_\mathrm{f} = A_\mathrm{fs}w'_\mathrm{s} + A_\mathrm{f}w'_\mathrm{f}, \tag{5}$$

wherein w'_s and w'_f denote the perturbation in the structure and fluid states respectively and $A_\mathrm{f} = \frac{\partial F_\mathrm{f}}{\partial w_\mathrm{f}}$, $A_\mathrm{fs} = \frac{\partial F_\mathrm{f}}{\partial w_\mathrm{s}}$, $A_\mathrm{sf} = \frac{\partial F_\mathrm{s}}{\partial w_\mathrm{f}}$ and $A_\mathrm{s} = \frac{\partial F_\mathrm{s}}{\partial w_\mathrm{s}}$ all evaluated at the equilibrium. The matrix A_f is a $2N \times 2N$ band-matrix which contains the discretization of the fluid domain and A_s is a 2×2 matrix. The coupling matrices A_fs $(2N \times 2)$ and A_sf $(2 \times 2N)$ will generally only have a relatively small amount of non-zero entries, since the coupling only takes place at the boundary of the domain.

3 Time Integration by Mixed Implicit/Explicit Schemes

We consider any system of the form

$$\dot{w} = F(w, t), \tag{6}$$

which can be any (semi-discrete) system describing e.g. structural and/or fluid dynamics. Since future applications involve engineering problems a large range of eigenvalues will be introduced due to a wide range of scales in the flow (for example in boundary layers [9]), giving rise to the stiffness of the system. Stiffness can cause the time step to be limited by stability rather than accuracy considerations. Hence, we only consider L-stable, implicit time integration methods, which can cope with stiffness in a robust fashion and dissipate the high frequency modes. Some well-known unconditionally stable implicit methods include the first

and second order multi-step Backward Differentiation Formula (BDF) schemes [7] and the trapezoidal rule method. The third and higher order multi-step BDF methods, however, are only $L(\alpha)$-stable, making them impractical in engineering codes. Additionally the multi step methods are not self-starting. Less known and also less applied in engineering codes are implicit Runge-Kutta (IRK) methods [7]. The IRK methods can be designed with arbitrary high order and L-stability.

When a problem with easily separable stiff and nonstiff components is considered, a combination of implicit and explicit Runge-Kutta methods can be used. The implicit method is used to integrate the stiff component in a stable fashion and the nonstiff component of the system is integrated using the explicit scheme. These combined implicit/explicit (IMEX) schemes are already used for convection-diffusion-reaction problems in [9].

The IMEX schemes we consider in this paper consist of an explicit Runge-Kutta (ERK) and a stiffly-accurate explicit, singly diagonally implicit Runge-Kutta (ESDIRK) scheme, for which the solution at t^{n+1} can be made of arbitrary high order by cancellation of the lower order errors. The ESDIRK scheme is an L-stable, implicit scheme with an explicit first stage, which allows the implicit stages to be second order accurate. For every stage k we solve

$$w^{(k)} = w^n + \Delta t \sum_{i=1}^{k} a_{ki} F^{(i)}, \quad k = 1 \dots s, \tag{7}$$

wherein $F^{(i)} = F(w^{(i)})$ is the flux at stage i. After computing s stages, the solution at the next time level is found by

$$w^{n+1} = w^n + \Delta t \sum_{i=1}^{s} b_i F^{(i)}. \tag{8}$$

In this paper we consider third to fifth order IMEX methods. At least 37 different combinations have been found in the literature, but we only use the ones reported in [9], which are optimized for the Navier-Stokes equations. The third, fourth and fifth order schemes consist of a 4, 6 and 8-stage algorithm respectively. In order to distinguish between the implicit scheme and explicit schemes, we denote a_{ki} for the implicit and $\hat{a}_{ki}$ for the explicit schemes. Both schemes use the same b_i coefficients to obtain the higher order solution. An s-stage ESDIRK scheme needs to solve $s-1$ implicit systems within one time step compared to only one for the multi-step methods. So the question is whether the higher computational cost of the ESDIRK methods is compensated by their higher order accuracy.

4 Partitioning Algorithm

When the discretized fluid and structural equations are written as in (6), the monolithic or fully coupled solution is obtained by direct integration of (6) with any time integration scheme. For this academic problem, the monolithic solution

is still manageable, but for real-world applications a partitioned approach is more favorable. In the proposed partitioning algorithm, both the fluid and structure are integrated by the ESDIRK scheme. At every stage a Gauss-Seidel type of partitioning is applied. When the linear systems (4) and (5) are considered, the structure is advanced to stage k by

$$(I - a_{kk}\Delta t A_{\mathrm{s}})w_{\mathrm{s}}^{(k)} = w_{\mathrm{s}}^{n} + \Delta t \sum_{i=1}^{k-1} a_{ki} F_{\mathrm{s}}^{(i)} + \Delta t \sum_{j=1}^{k-1} \hat{a}_{kj} F_{\mathrm{sf}}^{(j)}, \tag{9}$$

showing that the coupling from fluid to structure is integrated by the ERK scheme. The coupling fluxes are treated in a consistent, explicit way in order to retain the higher order of the scheme. Due to the combined implicit/explicit nature of the scheme, we refer to it as IMEX. The same scheme is obtained when we define a flux predictor for $F_{\mathrm{sf}}^{(k)}$ as

$$F_{\mathrm{sf}}^{(*)} = \sum_{i=1}^{k-1} \frac{\hat{a}_{ki} - a_{ki}}{a_{kk}} F_{\mathrm{sf}}^{(i)}. \tag{10}$$

Next the fluid is integrated to stage k by

$$(I - a_{kk}\Delta t A_{\mathrm{f}})w_{\mathrm{f}}^{(k)} = w_{\mathrm{f}}^{n} + \Delta t \sum_{i=1}^{k-1} a_{ki} \left(F_{\mathrm{f}}^{(i)} + F_{\mathrm{fs}}^{(i)} \right) + a_{kk}\Delta t F_{\mathrm{fs}}^{(k)}, \tag{11}$$

wherein the coupling flux $F_{\mathrm{fs}}^{(k)}$ is already known from (9). After solving all s stages of the integration scheme, the state at the next time level is obtained

$$w_{\mathrm{s}}^{n+1} = w_{\mathrm{s}}^{n} + \Delta t \sum_{i=1}^{s} b_i \left(F_{\mathrm{s}}^{(i)} + F_{\mathrm{sf}}^{(i)} \right), \tag{12}$$

$$w_{\mathrm{f}}^{n+1} = w_{\mathrm{f}}^{n} + \Delta t \sum_{i=1}^{s} b_i \left(F_{\mathrm{f}}^{(i)} + F_{\mathrm{fs}}^{(i)} \right), \tag{13}$$

which completes one partitioned integration.

For the nonlinear system (3), the separation of flux contributions to be used with the implicit and explicit schemes needs extra attention. In order to obtain separated flux contributions, a linearization of the flux is made around t_n

$$F_{\mathrm{f}}(w_{\mathrm{f}}^{(k)}, w_{\mathrm{s}}^{(k)}) = F_{\mathrm{f}}(w_{\mathrm{f}}^{n}, w_{\mathrm{s}}^{n}) + \Delta w_{\mathrm{f}}^{(k)} \frac{\partial F_{\mathrm{f}}}{\partial w_{\mathrm{f}}} + \Delta w_{\mathrm{s}}^{(k)} \frac{\partial F_{\mathrm{f}}}{\partial w_{\mathrm{s}}} + \text{h.o.t.}. \tag{14}$$

We define the flux at stage k as

$$F_{\mathrm{f}}^{(k)} = \bar{F}_{\mathrm{f}}^{(k)} + \Delta F_{\mathrm{fs}}^{(k)}, \tag{15}$$

wherein

$$\bar{F}_{\mathrm{f}}^{(k)} = F_{\mathrm{f}}(w_{\mathrm{f}}^{(k)}, w_{\mathrm{s}}^{n}) \quad \text{and} \quad \Delta F_{\mathrm{fs}}^{(k)} \approx \Delta w_{\mathrm{s}}^{(k)} \frac{\partial F_{\mathrm{f}}}{\partial w_{\mathrm{s}}}. \tag{16}$$

The contribution $\bar{F}_{\mathrm{f}}^{(k)}$ can be considered the flux in the flow under a constant boundary condition w_{s}^{n} and $\Delta F_{\mathrm{fs}}^{(k)}$ can be considered the change in flux at the interface caused by a change in structural state within the time step. The integration of the nonlinear system follows the IMEX partitioning algorithm as described in Eqs. (9–13), with $F_{\mathrm{f}} = \bar{F}_{\mathrm{f}}$, $F_{\mathrm{fs}} = \Delta F_{\mathrm{fs}}$ and similarly $F_{\mathrm{s}} = \bar{F}_{\mathrm{s}}$ and $F_{\mathrm{sf}} = \Delta F_{\mathrm{sf}}$.

5 Results and Discussion

The piston problem, for which an exact solution exists in the linear case, has only one single structural node with a mass $m = 2$ and a spring stiffness $k = 1.429$. With these settings the fluid has a strong influence on the structural motion without dominating the structural dynamics. In all computations 64 finite volume cells (FVC) are used to discretize the flow. For the linear simulations the standard central scheme without artificial dissipation is used. However for the nonlinear simulations stabilization is necessary and a first order upwind scheme is employed. The coupled simulations are performed with the IMEX scheme, using third to fifth order ESDIRK schemes for the implicit time integration and third to fifth order ERK schemes for the integration of the coupling terms. In the linear computations a Gaussian elimination is used to solve the implicit systems. In the nonlinear simulations, a Picard iteration is used for the monolithic BDF2 scheme and a Newton iteration is used in the flow solver. Since the iterative solvers have different efficiencies, it is not justified to determine the work by the total amount of CPU time. Therefore the work is defined as the number implicit stages that need to be solved during the simulation.

The linear system has a coupled radial frequency of $\omega_c = 1.01$ and a coupled period of $P = 6.19$. The computational efficiency of the higher order partitioned IMEX schemes is investigated by comparing them to the monolithic (or fully coupled) solution with the second order BDF time integration scheme. This way the efficiency of BDF2 is not diminished by partitioning. The energy error versus work is presented in Fig. 2. Since every time integration scheme has a fixed number of implicit stages to solve within one time step, we can obtain the order of the schemes by measuring the slope of the curves in the asymptotic range, as displayed in Fig. 2. The IMEX schemes show design order in the asymptotic range. In addition, the efficiencies of fourth and fifth order IMEX are much higher than monolithic BDF2. The partitioned third order IMEX performs equally with respect to monolithic BDF2. The effect of the consistent explicit coupling flux treatment can be seen by comparing the result for fifth order IMEX to the partitioned ESDIRK5 scheme, which uses the fluid state at the previous time level as a predictor for the implicit flux calculations. The scheme with the predictor does not show design order and needs about 2.5 times as much work to obtain the same accuracy as third order IMEX.

For the nonlinear problem an exact solution is not obtained. A “temporally exact solution” is obtained with the fifth order IMEX scheme and $\Delta t = 1/1024$. At $t = 0$ the flow is at rest and the piston has an initial displacement $q_0 = 0.5$.

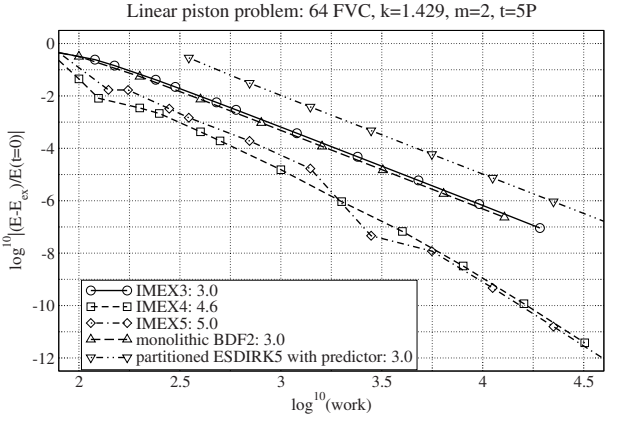

Fig. 2. Energy efficiency of IMEX schemes compared to monolithic BDF2 and partitioned ESDIRK5 without consistent predictor

The piston is released and the simulation is run until $t = 7$ which is a little over one period ($P \approx 6.34$). At $t = 7$ the L_2-norm of the error in the fluid density field is determined. For the structure the error for the displacement and velocity are computed. Simulations are performed with time steps ranging from $\Delta t = 1$ to $\Delta t = 1/512$. In Fig. 3 the L_2-norm of the error in the fluid density field is shown versus work for the third to fifth order IMEX schemes and the monolithic BDF scheme. From the results the order of the scheme are obtained by calculating

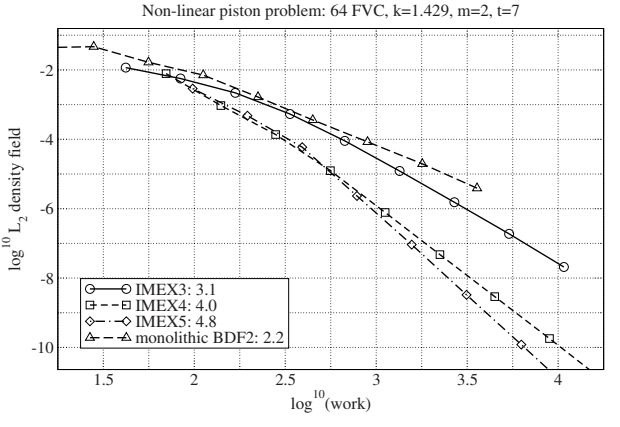

Fig. 3. Fluid density field efficiency for the third to fifth order IMEX schemes compared to monolithic BDF2

the slope of the graphs in the asymptotic range. In the asymptotic range the IMEX scheme have design order. For the larger time steps ($\Delta t = 1, 1/2$), the

order of the scheme is diminished, but the accuracy is still far better than the second order monolithic scheme with the same time step. When the error level is considered between -2 and -4, we find that the monolithic BDF2 scheme needs ≈1.3–2.5 times as much work compared to the fourth and fifth order IMEX schemes. For higher accuracy requirements, the efficiency of the higher order schemes increases.

6 Conclusions and Future Work

For the partitioned simulation of fluid-structure interaction third to fifth order IMEX schemes are used. Applied to a linear and nonlinear piston problem, the IMEX schemes are more efficient than the monolithic BDF2 scheme.

Up to this point only a simple one-dimensional problem has been considered. Future research focusses on the application of the IMEX scheme to more realistic, multidimensional problems.

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