Clustering Using WCC Models

Miguel Adán[1], Antonio Adán[2], and Andrés S. Vázquez[2]

[1] Departamento de Matemática Aplicada, UCLM
13071 Ciudad Real, Spain
{Miguel.Adan@uclm.es}

[2] Departamento de Ingeniería Eléctrica, Electrónica y Automática, UCLM
13071 Ciudad Real, Spain
{Antonio.Adan,Andres.Vazquez}@uclm.es

Abstract. In this paper we show how Weighted Cone-Curvature *(WCC)* Models are suitable to carry out clustering tasks. *CC* is a new feature extracted from mesh models that gives an extended geometrical surroundings knowledge for every node of the mesh. *WCC* concept reduces the dimensionality of the object model without loss of information. A similarity measure based on the *WCC* feature has been defined and implemented to compare 3D objects using their models. Thus a similarity matrix based on *WCC* corresponding to an object database is the input of a fuzzy c-means algorithm to carry out an optimal partition of it. This algorithm divides the object database into disjoints clusters, objects in the same cluster being somehow more similar than objects in different clusters. The method has been experimentally tested in our lab under real conditions and the main results are shown in this work.

1 Introduction

Clustering is a well known topic in the image processing field. Roughly speaking, the clustering's goal is to achieve the best partition over a set of objects stored in a database in terms of similarity. For partitioning we need to extract or define features of each object in such a way as to be well characterized. Depending on how that information was dealt with, several strategies of clustering can be found in the literature.

Methods based on perceptual/functional organizations aim to make hierarchical procedures where the step from one level to another must be controlled in some way. A perceptual organization mechanism is developed by Sengupta and Boyer in [1, 2] where surfaces belonging to the same object are identified. A graph is constructed, each surface corresponding to a node in this graph. Finally a partitioning scheme is carried out by comparing graphs corresponding to the objects. Selinger et al. argue that a single level of perceptual grouping is inadequate for recognition and use four levels of perceptual grouping [3]. In [4] a generic clustering scheme combining structural and functional approaches is presented. In this, a mapping of functionality to the primitive shape parts is the base for classifying objects.

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Similarity-base clustering is a simple technique that uses a similarity measure to guarantee if two objects are similar enough to put them in the same cluster. The similarity measure is usually defined through features of the object. In this sense, a good similarity measure is essential to carry out further clustering tasks. Lately, several works can be mentioned in this area. Yeung and Wang [5] introduce a similarity measure based on feature weight learning which is a reduction of the uncertainty existing in the clustering process. Thus clustering performance is improved. A stochastic clustering algorithm over silhouettes is accomplished in [6] where, in order to obtain a silhouettes database, a large number of views of each object is processed. Then, a dissimilarity matrix is obtained and a clustering algorithm is run over it. Cyr and Kimia [7] measure the similarity between two views of the 3D object by a metric that measures the distance between their corresponding 2D projected shapes. Ohbuchi et al. [8] present a version of the shape functions proposed by Osada [9] for 3D polygonal mesh models that allow them to make an efficient shape similarity search.

Lately we have developed a new strategy for 3D objects recognition using a flexible similarity measure based on spherical mesh models called Cone Curvature models [10, 11]. The difference between other strategies and ours is that we are able to compare two objects taking any part of information of the mesh model. In this sense, it can be said that our method is 'flexible' to experimental specifications. Consequently, an adaptable (or flexible) similarity measure contrary to previous fixed similarity measures is defined in our case. Secondly, we have carried out a reduction of the dimensionality of the object representation. So, unlike other techniques, no redundant information but a synthetic characterization is used. This reduction notoriously simplifies the volume of data handled in the model without loss of information and alleviates the computational cost of algorithms based on the model.

Now we present the applicability of such a method for clustering where our similarity matrix is the input of a fuzzy c-means algorithm. To show this we have structured the paper as follows. In Section 2, Weighted Cone Curvature model is briefly described defining Cone-Curvature and Weighted Cone Curvature as features of the object. Section 3 is devoted to defining a similarity measure and presenting the clustering algorithm. Clustering experimentation is dealt with throughout Section 4 presenting the results of a set of the tests.

2 WCC Models

2.1 Cone-Curvature Feature

Our solid representation model is defined on a mesh of h nodes from the tessellation of the unit sphere. Let T_I be this initial spherical mesh and T_M the mesh fitted to the object . For building T_M, T_I is deformed until it fits into the normalized surface of the object. In this process, mesh regularizing/smoothing tasks are also included. Then several geometric features are extracted from T_M and finally, mapped again into T_I.

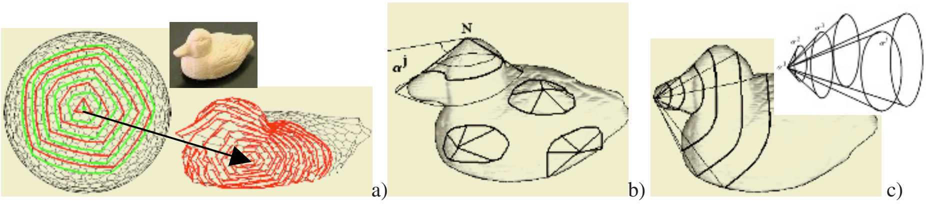

Fig. 1. a) MW drawn over T_I and a detail of WFs over T_M. b) Definition of CCs. b) Visualization of the CCs vector for a node N.

On the initial tessellation T_I, a topological structure called *Modeling Wave* (MW) [12] organizes the nodes of T_I in disjointed subsets following a new relationship. In this sense a three-neighbour relationship is just a kind of local topology. Each subset contains a group of nodes spatially disposed over the sphere as a closed quasi-circle, resulting in subsets that look like concentric rings on the sphere. Since this organization resembles the shape of a wave, this has been called *Modeling Wave (MW)*. Consequently each of the disjointed subsets is known as *Wave Front (WF)* and the first WF is called *Focus*. Of course, MW structure remains after the modeling process has finished. In other words, the WF structures remain in T_M (see Figure 1 a)).

From the previous definition it can be deduced that any node of T_I may be *Focus* and, therefore, it can generate its MW. Therefore *h* different MWs can be generated.
Although several kinds of features have been mapped into T_I in previous works [12] in this case Cone-Curvature (CC) is defined as a new and intuitive feature based on the MW structure taking into account the location of the WFs inside the model T_M. Its formal definition is as follows:

Let N be *Initial Focus* on T_M. We call *j*th Cone Curvature α^j of N, the angle of the cone with vertex N whose surface inscribes the *j*th *Wave Front* of the *Modeling Wave* associated to N.

The range of CC values is [-π/2, π/2], being the sign assigned taking into account the relative location of O, C^j, and N, where O is the origin of the coordinate system fixed to T_M and C^j is the barycentre of the *j*th WF. Negative values are for concave zones, values next to zero correspond to flat areas and positive values correspond to convex zones. Figure 1 b) illustrates this definition.

Note that a set of values $\{\alpha^1, \alpha^2, \alpha^3, \ldots \alpha^q\}$ gives an extended curvature information around N until the *q*th WF, where the word 'curvature' has a non-local meaning. So for each node N a set of *q* values could be used for exploring its surroundings (see Figure 1 c)).

On the other hand, it can be said that a vector $C=\{c^1, c^2, c^3 \ldots c^q\}$ where $c^j : T_I \rightarrow [-\pi/2, \pi/2] \quad j=1,\ldots q$, *q* being the number of *Wave Front* considered, is established for all the nodes of T_I. The whole Cone Curvature information is stored in a CC-matrix Θ, of *h*x*q* dimension. Note that Θ is invariant, unless row permutations, to changes in the pose of the object.

2.2 Weighted Cone Curvature

Now we show a study for the dimensionality reduction of the CCs by defining a new feature called Weighted Cone-Curvature (WCC). To do that a principal component analysis must be carried out over the CC vectors.

With the purpose of showing the degree of correlation existing between different curvature orders, correlation values for CCs from 2nd to 18th order are plotted in Figure 2 a) (notice that 1st order has no meaning). In this Figure, the correlation values are coded in grey levels (corresponding 0 to black and 1 to white). Each plotted value is computed as the average of the correlation values obtained for all the objects in the handled database (70 objects). It can be seen that, in general, there are very high correlation values for near orders of CC. On the contrary, small correlation values are found between lower orders of CC (2nd and 3rd) and all the others. It is also noticeable from the same Figure the high correlation existing between the upper orders of CC.

The meaning of h and q dimensions of the O CC-matrix could be explained as follows. The choice of a particular row N is equivalent to selecting the *Focus* of the MW whereas q provides CCs values corresponding to a specific depth; the higher order the more depth.

Our purpose is to obtain a single value for each row of the O matrix from the analysis of the principal components performed on all the rows, so that each row is reduced to a single representative value (see Principal Component Analysis or Karhunen-Loeve transform in [13]). Then, by means of the adequate linear combination, for each node N a single variable c_w will fuse the q values provided by its CCs. Therefore, every node will have just a variable c_w associated that is called Weighted Cone Curvature (WCC). Consequently, the O matrix is reduced to a vector C of $hx1$ dimension.

The variable Weighted Cone Curvature can be defined as a weighted combination of the different CCs given by the expression:

$$c_w = \sum_j v^j \cdot c^j \ . \tag{1}$$

The coefficients v^j of the linear combination are the coordinates of the eigenvector associated to the highest eigenvalue of the covariance matrix for the set of indexes j considered. These coefficients have been empirically determined by evaluating the principal component analysis over the models of our object database in our lab.

Note that different combinations of CC orders with respect to different criteria could be chosen. So, local, half, global, contiguous or discontiguous CCs could be chosen. That is why labeled our method as ‘flexible’. Because we are interested in using the CCs for partial views we have considered the CC of orders $j=4,5,\ldots9$. This means to consider only the CC information near to the *focus* but filtering also the first two orders (which are sensitive to noise).

The study of the principal components has been carried out over a database with 70 objects. Figure 2 b) shows a plot of the highest eigenvectors. Each line corresponds to the highest eigenvector obtained for a given object where the X axis represents the index and Y axis represents the eigenvector values. Very similar eigenvec-

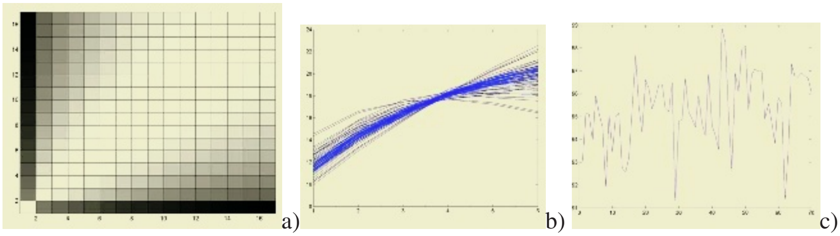

Fig. 2. a) Illustration of the correlation between different CC values from 2nd to 18th orders, in the object database. b) Plots of the highest eigenvectors. c) Percentage of the total variance that corresponds to c_w.

tors v^j can be appreciated for most of the objects showing higher dispersion for the greater indexes.

Finally, we consider the weighted array $\{v^j\}$ by computing the eigenvectors average obtained above. To evaluate the goodness of the dimensionality reduction an analysis of the percentage of the total variance that corresponds to the new variable is required. If a high percentage is achieved the new variable will satisfactorily explain the initial variables and the dimensionality reduction makes sense. In our study this percentage has been 95,34%. which means that the dimensionality reduction does not provoke any significant loss of information. Figure 2 c) shows, for all objects, the results of the percentage of the total variance that corresponds to the new variable c_w.

With these results it can be concluded that the dimensional reduction of the CC vector to a single variable c_w is very appropriate because it adequately summarizes the information provided by the entire vector. Each node of T_I will have a numeric value associated, called Weighted Cone Curvature, and the complete model can be characterized by the WCC vector C of h components.

3 Clustering Based on Similarity Matrix

In this section, a similarity measure for 3D shapes based-on WCC feature is firstly defined. After that the procedure of clustering is dealt with.

Keeping the WCC concept in mind, a distance d between two models T^i and T^j is defined as:

$$d(T^i, T^j) = \left|C^i - C^j\right| = \sqrt{\sum_{k=1}^{h} (C^i(k) - C^j(k))^2} \tag{2}$$

where C^i and C^j are sorted distributions of WCC vectors for both models.

Once the distance d has been defined and considering a model database where D is the maximum distance, a binary relationship through a *similarity function s* is established as follows.

$$s : X \times X \rightarrow [0,1] \qquad s(T^i, T^j) = s_{ij} = 1 - d(T^i, T^j) / D \tag{3}$$

X being the model database. Thus we can define a Similarity Matrix $S = (s_{ij})$ which stores the whole similarity information for a database.

Roughly speaking, clustering procedures yield a data description in terms of clusters or groups of data points that possess strong similarities [14]. In our case, we have an object database H that we want to divide in groups of objects. We may consider H as a set of n unlabelled samples $\mathbf{x_1}$, $\mathbf{x_2}$, …$\mathbf{x_n}$ where $\mathbf{x_i}$ is a feature vector of the ith object. For us, every object is characterized by the vector of similarity values with respect to the rest of the objects so $\mathbf{x_i}=(s_{i1,}....s_{in})$. Note that $\mathbf{x_i}$ corresponds to ith row into the *Similarity Matrix S*. Therefore the clustering procedure would be to divide H into p disjoint subsets H_1, H_2,…H_p, samples in the same cluster being somehow more similar than samples in different clusters.

Once we fix the clustering and similarity measurement problems, we are obliged to evaluate the partitioning. Then, the problem is one of finding the partition that extremizes a criterion function. For that we have used the fuzzy c-means function, which is a variation of sum-of-squared-error criterion. Although complete information about this can be found in [15], we will make a brief reference to this criterion function.

For a given integer $p>1$ and real $m>1$, the criterion function to be minimized is defined as follows:

$$J = \sum_{i=1}^{p} \sum_{k=1}^{n} (u_{ik})^m \left| x_k - m_i \right| \tag{4}$$

where $\mathbf{m_i}$ is the mean vector in H_i, $|\mathbf{x}|$ denotes the Euclidean norm of $\mathbf{x}$ and u_{ik} is the ith membership function on the kth sample $\mathbf{x_k}$ to the cluster H_i. The vectors $\mathbf{m_1}$, …,$\mathbf{m_p}$ can be interpreted as prototypes of cluster (called *cluster centers*). So, high memberships occur for samples close to the corresponding cluster centers. The number m is called the *exponent weight* and is used to control the contribution of the samples to J depending on their memberships values. Once chosen p and m, an iterative procedure recomputes $\mathbf{m_i}$ and u_{ik} until a local minimum of J is achieved.

4 Experimental Tests

Taking our similarity measure as the basis of the clustering procedure, we have evaluated it by carrying out several tests for different sets of objects. This experimentation has been accomplished with 41 free form objects sensed in the lab where different people participated in their election. Curiously, there were different opinions for establishing the clusters a priori. Obviously the choosing of natural groups was not completely clear for us. Finally we grouped the objects in sub-sets that we have labeled as: *cubes (1-6)*, *prisms (7-9)*, *round shapes (10-13)*, *cars (14-17)*, *polyhedral (18-23)*, *free shapes (24-29)*, *cone shapes (30-36)* and *cylinders (37-41)* (Figure 3). The goal of our experimentation is to know what the performance of our method in real environments is and what concordance exists between the clusters computed with the clusters established a priori.

In order to graphically illustrate the Similarity Matrix S we have presented it as a two dimension grey map where the grey level goes from 0, plotted as white, to 1, plotted as black. This election, that seems illogical, has been adopted in order to achieve better visualization. Note that reflexive (black diagonal) and symmetrical properties can be clearly seen on it. Looking at the grey levels for the ith row we can check the feature vector (called $\mathbf{x_i}$ in section 3) which is the similarity vector. Making a visual analysis of S we can sometimes appreciate dark grouping zones for a set of rows (or columns indistinctly). These groups might be thought of as candidate clusters.

Fig. 3. Objects.

When a set H of n objects (samples) $\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}$ is considered, each object $\mathbf{x_i}$ is inside a n-dimensional space ($\mathbf{x_i}$ has n components) and therefore the resulting cluster is in a n-dimensional space. In order to graphically show the clusters we have included two-dimensional projections of that space (corresponding to first and last dimensions).

Table 1 summarizes the results obtained for twelve tests. For each test, a set of object H belonging to different groups are chosen. Then the clustering algorithm is run and clusters $H_1, \ldots, H_p$ are computed. The number of cluster p is set taking into account the mean values of the membership functions of the objects in their respective clusters (denoted as u in the table). In the first column the name of the groups set a priori appears and last rows shows the cases where an object is put in an unexpected cluster.

There are cases where clustering results coincide quite well with the natural groups (test nº: 1, 3, 7, 10). For example in Test nº1, the three clusters correspond to *cubes,* *round shapes* and *polyhedral*. This grouping can be seen clearly in the *Similarity Matrix* (Figure 4 above) because three disjointed dark zones appear. On the right, four groups are also evident in the *Similarity Matrix* corresponding to Test nº3.

On the other hand, when the number of cluster is forced to be less than the number of initial groups, cases exist where two groups are put in the same cluster (test nº: 2, 4, 5, 6, 8, 9, 11 and 12). Nevertheless, a coherent clustering is made in such cases. Test nº 2 is a good example confirming that. In this case *H* is formed by *cars*, *polyhedral*, *cones* and *cylinders*. When the clustering procedure is executed for *p=3*, *cones* and *cylinders* are put in the same cluster. Figure 4 below shows the nearness of such groups. Test nº 4 is a more complex case where this coherence can also be seen.

Table 1. Results for tests.

Test	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
p	3	3	4	4	3	2	3	3	3	3	3	5
u	0.93	0.92	0.72	0.81	0.90	0.86	0.88	0.88	0.82	0.85	0.84	0.76
n	16	20	18	37	37	18	19	19	23	11	22	41
Cubes	H_1			H1	H1	H1	H1				H1	H1
Prisms			H1	H1	H1			H1		H1	H1	H1
Round shapes	H2	H1		H2	H2			H2		H2	H2	H2
Cars		H2	H2						H1	H3	H3	H3
Polyhedrals	H3			H3	H3		H2	H3	H1			H3
Free shapes			H3	H3	H3			H3	H2			H4
Cones		H3		H4	H1	H2	H3		H3			H5
Cilinders		H3	H4	H4	H1	H2					H1	H5
Unexpected cases	$19 \to H_1$		$24 \to H_2$	$37 \to H_1$ $19 \to H_1$	$19 \to H_1$	$37 \to H_1$	$19 \to H_1$	$19 \to H_1$	$24 \to H_1$			$19 \to H_1$ $37 \to H_1$ $24 \to H_3$ $17 \to H_1$

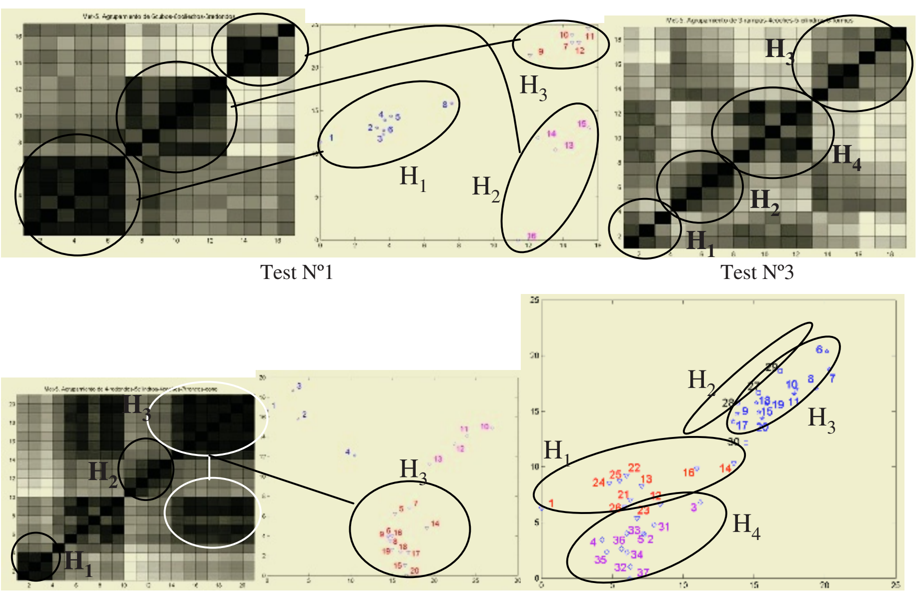

Fig. 4. Clustering results.

In conclusion, it can be said that the method has worked in a real environment which implies working with noise and error sources. None of the experiments has given unexpected clustering; on the contrary the clustering algorithm has given results according to the human grouping established by us. Nevertheless, we have started several initiatives to improve the method concerning to increase the database and deal with the problem of the validity of the clusters.

References

1. Sengupta K., Boyer K.L.: Creating Random Structural Descriptions of CAD Models and Determining Object Classes. Proc. IEEE Workshop on CAD-based Vision. Pp 38-45. 1994.
2. Sengupta K. and Boyer K.L.: Modelbase Partitioning Using Property Matrix Spectra. Computer Vision and Image Understanding Vol 70, No 2, pp 177-196, 1998.
3. Selinger A. and Nelson R.: A perceptual Gruping Hierarchy for Appearance-Based 3D Object Recognition. Computer Vision and Image Understanding, Vol76, No1 pp 83-92, 1999.
4. Froimovich G., Rivlin E., Shimshoni I.: Object Classification by Functional Parts. Fisrt Symposium on 3D Data, Proccesing, Viasualization and Trnasmision. Padova, pp 648-655. 2002.
5. Yeung D.S., Wang X. Z.: Improving Performance of Similarity-Based Clustering by Feature Weight Learning. IEEE Trans. On Pattern Analysis and Machine Intelligence. Vol 24, No 4, pp 556-561.
6. Gdalyahu Y. and Weinshall D.: Flexible Syntactic Matching of Curves and Its Application to Automatic Hierarchical Classification of Silhouettes. IEEE Trans. On Pattern Analysis and Machine Intelligence. Vol 21 No 12, pp 1312-1328. 1999.
7. Cyr C. Kimia M, B. B. : 3D Object recognition using shape similarity-based aspect graph. Int. Conference on Computer Vision. Vancouver. pp 254-261. 2001
8. Ohbuchi R., Minamitani T., Takei T.: Shape-Similarity search of 3D Models by Using Enhanced Shape Functions. Proc of the Theory and Practice of Computer Graphics (TPCG'03). 2003.
9. Osada R., Funkhouser T., Chazelle D., Dobkin D. Matching 3D Models with Shapes Distributions. Proc. Int. Conf. On Shape Modeling and Applications, pp 154-166. Genova. 2001.
10. Adán M., Adán A., Solids Characterization Using Modeling Wave Structures. Lecture Notes in Computer Science, LNCS 2652, pp 1-10. 2003.
11. Adán M., Adán A., Cerrada C., Merchán P., Salamanca S.: Weighted Cone-Curvature: Applications to 3D Similarity Shapes. The Fourth International Conference on 3-D Digital Imaging and Modeling (3D DIM). Banff. Pp 458-465. 2003.
12. Adán A., Cerrada C., and Feliú V.: Modeling Wave Set: Definition and Application of a new Topological Organization for 3D Object Modeling. Computer Vision and Image Understanding, 79, pp. 281-307. 2000
13. Jolliffe, I. T. , *Principle Components Analysis*, Springer-Verlag, New York, 1986.
14. Duda R. O. and Hart P. E.: Pattern Classification and Scene Análisis. Jonh Wiley & Sons . 1973.
15. Bezdek J. C and Pal S. K.: Fuzzy Models For Pattern Recognition. IEEE Press. 1992.