

First Competitive Ant Colony Scheme for the CARP

**Lacomme Philippe¹ Christian Prins²
Alain Tanguy¹**

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¹ Université Blaise Pascal
Laboratoire d'Informatique (LIMOS) UMR CNRS 6158,
Campus des Cézeaux, 63177 Aubiere Cedex
lacomme@sp.isima.fr, tanguy@sp.isima.fr

² Université de Technologie de Troyes,
STIT (équipe OSI) FRE CNRS 2732
12, Rue Marie Curie, BP 2060, F-10010 Troyes Cedex (France)
prins@utt.fr

FIRST COMPETITIVE ANT COLONY SCHEME FOR THE CARP	1
Keywords: Ant Colony, Capacitated Arc Routing	3
Mots-clés : colonies de fourmis, tournées sur arcs.....	3
1 Introduction	4
1.1 The Capacitated Arc Routing Problem	4
1.2 Ant Colony.....	4
2 Ant Colony Proposal	5
2.1 Notations.....	5
2.2 Ant Colony Framework	6
2.3 Generation of Initial Solutions	8
2.4 Solution Improvement	8
2.5 Local Search	9
3 Numerical Evaluation	10
3.1 Instances and Ant Colony Scheme Parameters	10
3.2 Numerical Experiments	11
4 Concluding Remarks and Future Research	17
References	17
Index.....	20

Abstract

This paper addresses the Capacitated Arc Routing Problem (CARP) using an Ant Colony Optimization scheme. Ant Colony schemes can compute solutions for medium scale instances of VRP. The proposed Ant Colony is dedicated to large-scale instances of CARP with more than 140 nodes and 190 arcs to service. The Ant Colony scheme is coupled with a local search procedure and provides high quality solutions. The benchmarks we carried out prove possible to obtain solutions as profitable as CARPET ones can be obtained using such scheme when a sufficient number of iterations is devoted to the ants. It competes with the Genetic Algorithm of Lacomme *et al.* regarding solution quality but it is more time consuming on large scale instances. The method has been intensively benchmarked on the well-known instances of Eglese, DeArmon and the last ones of Belenguer and Benavent. This research report is a step forward CARP resolution by Ant Colony proving ant schemes can compete with Taboo search methods and Genetic Algorithms

Keywords: *Ant Colony, Capacitated Arc Routing*

Résumé

Cet article concerne la résolution du Capacitated Arc Routing Problem avec un algorithme de type colonies de fourmis. Il a déjà été prouvé que de tels algorithmes permettaient de résoudre des problèmes de VRP de tailles modestes. L'algorithme proposé a pour ambition de résoudre des instances de CARP de très grandes tailles comportant plus de 140 nœuds et 190 arcs. La méthode proposée est hybridée avec une méthode de recherche locale qui accroît de manière significative la convergence vers de bonnes solutions. Nous montrons sur des exemples de la littérature que la méthode proposée permet d'obtenir des solutions comparables à celle de la méthode CARPET au prix d'un nombre d'itérations relativement élevé. La méthode permet même de concurrencer l'algorithme génétique de Lacomme *et al.* mais elle s'avère plus longue en terme de temps de calcul. Toutes les comparaisons ont été réalisées sur les instances de Eglese, DeArmon et sur les instances de Belenguer et Benavent. Ce rapport de recherche tente de prouver que les techniques d'optimisation à base de colonies de fourmis peuvent concurrencer les méthodes « tabou » ainsi que les algorithmes génétiques.

Mots-clés : *colonies de fourmis, tournées sur arcs*

1 Introduction

1.1 The Capacitated Arc Routing Problem

The *Capacitated Arc Routing Problem (CARP)* is defined in the literature on an undirected network $G = (V, E)$ with a set V of n nodes and a set E of m edges. A fleet of identical vehicles of capacity Q is based at a depot node s . A subset R of edges requires service by a vehicle. All edges can be traversed any number of times. Each edge i has a traversal cost $c_i > 0$ and a demand $q_i \geq 0$. The CARP consists on determining a set of vehicle trips with minimum total cost. Each trip starts and ends at the depot, each required edge is serviced by one single trip, and the total demand handled by any vehicle does not exceed Q . The cost of a trip is the sum of the costs of its serviced edges and of its intermediate connecting paths.

Golden B-L. and R.T.Wong [1], Benavent E., V. Campos and A. Corberan, E. Mota [2], Belenguer J.M. and E. Benavent [3] have investigated integer linear programming formulations and they have proposed lower bounds for the CARP. Since exact algorithms (like the branch-and-bound method of Hirabayashi *et al.* [4]) are limited to small instances (30 edges), larger instances must be tackled in practice by heuristic approaches. Powerful greedy heuristics include Path-Scanning [1], improved Construct-Strike [5], Augment-Insert [6], Augment-Merge [7] and Ulusoy's tour splitting method [8]. Concerning metaheuristics, Li [9] applied simulated annealing and taboo search to a road gritting problem, and Eglese [10] designed a simulated annealing approach for a multi-depot gritting problem with side constraints. The most efficient metaheuristics published so far are: a sophisticated taboo search method (CARPET) of Hertz *et al.* [11] and a hybrid genetic algorithm proposed by Lacomme *et al.* [12].

1.2 Ant Colony

Ant Colony schemes have been successfully applied to numerous combinatorial optimization problems including graph coloring [13], Quadratic Assignment Problem (QAP) [14], Traveling Salesman Problem (TSP) [15], Vehicle Routing Problem (VRP) [16, 17, 18], Vehicle Routing Problem with Time Window (VRPTW) [19]. A presentation of Ant Colony for routing is described in [20]. An important result in the field of ant algorithms was published in [21]: Gutjahr gave a

formal proof that, under certain conditions, a slightly limited version of the Ant System (called Graph-based Ant System) converges to the optimal solution of the given problem with a probability that can be made arbitrarily close to 1.

In the beginning, no collective memory is used and ants use only heuristic information. Pheromone deposition is proportional to the fitness that can be defined for minimization objective as the inverse of the solution quality or solution cost. As stressed by Donati *et al.* [22] the fitness may be defined in different ways to reflect the optimization objectives: a combination of the travel distance, the travel time, the waiting times and so on. This pheromone trail guides ants in their future decision making paths with high pheromone concentration more attractive. Since pheromone is not permanent but evaporates over time, unused paths become less and less attractive while those frequently used attract more ants. During one iteration, the ants construct one solution based on heuristic scheme and on the pheromone trails. The pheromone trails are updated using the solutions fitness. Local search can also be applied to ant solutions increasing the convergence rate. The process is iterated until a lower bound is reached or a maximal number of iterations is carried out.

The remainder part of the paper is organized as follows. First, an Ant Colony framework with an elitist strategy dedicated to the CARP is proposed. Second, an intensive benchmark is provided to evaluate the method performances and behavior.

2 Ant Colony Proposal

2.1 Notations

f	number of ants
f_e	number of elitist ants
μ	notation used for one ant, its rank and the solution computed by the ant
I_{\max}	maximal number of iterations
n_s	number of iterations without improvement before pheromone erasing
w_{ij}	shortest path length from the required arc i to the required arc j
M_d	shortest path of maximal length between two required arcs: $M_d = \underset{i,j}{\text{Max}} w_{ij}$

p_{LS}	probability for one ant to experiment a local search procedure
p_p	probability to ignore pheromone trails for combining required arcs
k	maximal size of Ψ_i^μ and Ω_i^μ
α, β	relative influence of criteria (saving measure of moving to another required arc and pheromone attraction)
F^μ	the weight affected to the ant number μ
ρ	the trail persistence, $0 \leq \rho \leq 1$
s_{ij}	saving measure of moving from the required arc i to the required arc j : $s_{ij} = (M_d - w_{ij}) / M_d$
τ_{ij}	existing amount of pheromone from the required arc i to the required arc j
$\Delta\tau_{ij}^\mu$	deposit amount of pheromone laid by ant number μ when moving from required arc i to required arc j
L^μ	current value of the solution found by the ant number μ
t_μ	taboo list of ant μ
Ω_i^μ	set of required arcs not in t_μ and yielding the best savings
Ψ_i^μ	set of required arcs not in t_μ and yielding the best pheromone level
P_{ij}^μ	probability, for the ant μ , to combine the required arcs i and j

2.2 Ant Colony Framework

The network is stored as a directed internal graph using two opposite arcs per edge and one dummy loop on the depot. The nodes are dropped out and an arc index list is used. In any solution, each trip is stored as a sequence of required arcs with a total cost and a total load. Shortest paths between tasks are not stored. The cost of a trip is the collecting costs of its required arcs plus the traversal costs of its intermediate paths. The graph and solutions are represented according to the data structure described in [12] and with respect to the proposal of Lacomme *et al.*, the solutions are

giant tours with no trip delimiters sorted in increasing cost order. The giant tours are split into solutions regarding the vehicle constraints, using the Split procedure.

The ant system framework consists in the following steps (figure 1):

1. generation of solutions by powerful constructive heuristics dedicated to CARP;
2. generation of solutions by ants according to pheromone information;
3. application of a local search to the ant solutions with a fixed probability;
4. updating the pheromone information;
5. iterate steps 2 to 4 until the lower bound or some completion criteria are reached.

Generation of the initial set of f solutions

$I_c = 1$

Repeat

Pheromone trails deposit

For $\mu := 1$ to f **do**

Repeat

Select a required arc i to be serviced next

Add i in the current solution under construction

Update the taboo list t_μ of the ant μ

until ant μ has completed a tour

with probability P_{LS} , apply Local Search

Calculate the solution cost

If μ is not elitist **Then**

save the solution whatever the cost

Else

save solution only if a better tour is obtained by ant μ

EndIf

EndDo

Sort the ants in decreasing cost order

If for ant f no improvement occurs during n_s iterations **Then**

Erase the pheromone trails

EndIf

$I_c = I_c + 1$

Until (the lower bound is reached) **or** ($I_c = I_{\max}$)

Figure 1. Ant Colony algorithm

The population is divided in two sets: f_e elitist ants and $f - f_e$ non-elitist ants. All ants start at the anthill: depot node. Note that for VRP, several authors promote initial assignment of ants at each customer node at the beginning of iterations which implies that the number of ants is equal to the number of customers [22]. The elitist ants tend to favor the convergence of the algorithm and the non-elitist ones attempt to control the diversification process. Whatever the solution cost found by a non-elitist ant, it is stored and replaces the previous one. For the elitist ants, the solution is replaced only if it is more promising. To decrease the probability of being captive in a local minimum, the pheromone is erased when n_s iterations have been performed without improvement (figure 1).

2.3 Generation of Initial Solutions

Three well-known CARP heuristics have been used: Path-Scanning [7], Augment-Merge [1] and Ulusoy's tour splitting method [8]. The initial set of solutions is completed by random feasible solutions. Each solution is composed of one giant tour without the vehicle capacity constraint. This giant tour is a list of required arcs linked by shortest paths and the Split procedure [12] breaks this tour optimally into trips. Path Scanning algorithm builds one trip at a time. In constructing each trip, the sequence of arcs is extended by joining the arcs looking the most promising ones, until the vehicle capacity is reached. Possible criteria are minimization of the distance, maximization of productivity. Augment-Merge is composed of two phases. First, each required arc is serviced by a separate trip. Second, Augment considers the trips one by one, starting with the longest one and evaluates the concatenation of trips yielding the largest saving. Ulusoy's algorithm is composed of two steps. The first step relaxes capacity to build a giant tour S containing the required arcs. In a second step, S is optimally split into trips under the vehicle capacity constraint using Ulusoy's algorithm.

2.4 Solution Improvement

The pheromone update is done using the well-known formula:

$$\tau_{ij} \leftarrow \rho \cdot \tau_{ij} + \sum_{\mu=1}^n \Delta \tau_{ij}^{\mu} \quad \text{with} \quad \Delta \tau_{ij}^{\mu} = F^{\mu} / L^{\mu} \quad (1)$$

The contribution level of this global information depends on the quality of the solution. The basic weight $F^{\mu} = 1$ denotes that no ant is considered more promising than another one and the same

weight (value 1) is affected to each ant. To have a quantity of pheromone laid by ants depending on their rank μ it is possible to choose: $F^\mu = \mu$. The solution we promote is to consider some graph properties and specially the maximal distance between two required arcs. One can suppose the objective function is more “chaotic” when large distances occur in the network and it is possible to link F^μ and M_d according to the following formula:

$$F^\mu = \mu \times (M_d - 1) / (f - 1) + (f - M_d) / (f - 1) \quad (2)$$

With probability p_p , its next required arc is chosen taking into account the unproductive shortest path from its current position to another required arc. The following formulae are applied respectively with probability p_p and $1 - p_p$:

$$P_{ij}^\mu = \begin{cases} 1/k & \text{if } j \in \Omega_i^\mu \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$P_{ij}^\mu = \begin{cases} \frac{[s_{ij}]^\alpha [\tau_{ij}]^\beta}{\sum_{k \in \Psi_i^\mu} [s_{ik}]^\alpha [\tau_{ik}]^\beta} & \text{if } j \in \Psi_i^\mu \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

The first one is similar to the ranking strategy described, for instance, by Bullnheimer [23]. The second one uses pheromone trails.

2.5 Local Search

To improve the performances of metaheuristics it is a common practice to include a local search procedure. Previous published works on Ant Colony for the VRP prove that local search coupled with the Ant Colony method considerably improves the solutions quality (see for example [22]). The local search scheme dedicated to the CARP and proposed by Lacomme *et al.* [12] is used to improve solution with probability p_{LS} . It is an iterative improvement procedure based on three moves:

1. remove one required arc and reinsert it at another location;
2. remove two consecutive required arcs and reinsert them at another location;
3. two-opt moves.

The local search algorithm detects and performs the first feasible and improving move. This process is iterated until no such move is found. The split procedure [8] is applied to get the solution cost.

3 Numerical Evaluation

In this section, we present numerical experiments for the proposed Ant Colony scheme compared to the best methods for the CARP including CARPET and the Genetic Algorithm [12, 24]. The experiments were carried out on a Pentium III 800 MHz under Windows 2000. The scheme has been implemented using Delphi 6.

3.1 Instances and Ant Colony Scheme Parameters

The instance and parameters setting is summarized up in table 1. The following extra notations are introduced: τ the number of required arcs and n the number of nodes.

Table 1. Instances and parameters

$f = 60$	$k = 10$	$I_{\max} = 200$
$f_e = 10$	$P_{LS} = 50\%$	$n_s = 10$
$\rho = 0.90$	$p_p = 10\%$	$\alpha = \beta = 1$

The benchmark has been performed using the well-known instances of DeArmon, Eglese, Belenguer and Benavent using the parameters values presented in table 1. For each instance, the notations of table 2 are used. To evaluate the performances three experiments have been carried out with different seeds for the random generator. Results over three restarts are in column BACO (Best Ant Colony Optimization).

Table 2. Notations used in table of results

LB	Lower Bound
C	CARPET algorithm of Hertz
GW	Golden and Wang's algorithm
AM	Augment Merge heuristic
UL	Ulusoy
Ant	Ant Colony algorithm
Time	computation time for I iterations
BACO	best results over three experiments
Avg Time	average computational time for BACO
Dev	deviation regarding the LB, for a cost x : $Dev = \frac{x - LB}{LB}$
Av.Dev	average deviation regarding the LB
Nb hits	number of optima proved by the method
I	iteration at which the best GA value is found
GA	Genetic Algorithm

Grey boxes show solutions equal or better than CARPET and bold denote solutions equal of better than Genetic Algorithm ones. Note Lacomme *et al.* propose a GA with restarts denoted *Std MA* in [24] and results under various settings (*Best MA*). The GA column in tables refers to *Std GA* of [24].

3.2 Numerical Experiments

3.2.1 Results on DeArmon's Instances

The Ant Colony Optimization scheme (table 3) outperforms CARPET for the three experiments. Whatever the experiments, the deviation is about 0.34% that is better than CARPET deviation (0.50%). Let us note that whatever the experiment, for the Gdb8 instance the lower bound is not reached but the value 350 is better than CARPET solution (value 352) and equal to the solution of the Genetic Algorithm except for Gdb9, Gdb13 and Gdb23.

Ant System configuration over iterations.

The initial population is composed of random solutions and three heuristic solutions. Because no optimization has been applied on the initial population, the ant population is well spread from 360 (best heuristic solution) to 800 (worst random solution). The generation of the initial set of solutions provides a high diversification of solutions. Figure 2 gives a graphical representation of ants sorted by increasing cost after 20 iterations. The convergence of the algorithm produces stepwise modifications of the cost distribution of the ant population and the convergence is initialized.

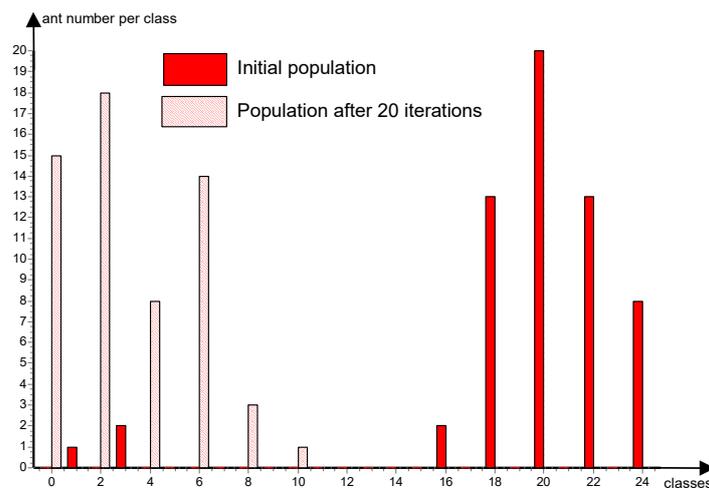


Figure 2. Population distribution after 20 iterations (Gdb9)

Table 3. DeArmon's instances

FILE	n	τ	LB	C	Exp. 1										Exp. 2				Exp. 3																
					Dev	GA	Dev	GW	AM	UL	Ant	Dev	Time	I	Ant	Dev	Time	I	Ant	Dev	Time	I	BACO	Dev											
gdb1	12	22	316	316	0.00	316	0.00	350	349	330	316	0.00	0.49	1	316	0.00	0.44	1	316	0.00	0.44	1	316	0.00											
gdb2	12	26	339	339	0.00	339	0.00	366	370	353	339	0.00	1.10	2	339	0.00	2.15	4	339	0.00	2.08	4	339	0.00											
gdb3	12	22	275	275	0.00	275	0.00	293	319	297	275	0.00	0.49	1	275	0.00	0.55	1	275	0.00	0.49	1	275	0.00											
gdb4	11	19	287	287	0.00	287	0.00	287	302	320	287	0.00	0.05	0	287	0.00	0.05	0	287	0.00	0.06	0	287	0.00											
gdb5	13	26	377	377	0.00	377	0.00	438	423	407	377	0.00	1.65	3	377	0.00	4.18	5	377	0.00	0.61	1	377	0.00											
gdb6	12	22	298	298	0.00	298	0.00	324	340	318	298	0.00	1.48	3	298	0.00	1.43	2	298	0.00	0.49	1	298	0.00											
gdb7	12	22	325	325	0.00	325	0.00	363	325	330	325	0.00	0.11	0	325	0.00	0.11	0	325	0.00	0.06	0	325	0.00											
gdb8	27	46	344	352	2.33	350	0.02	463	393	388	350	0.02	171.92	112	350	0.02	68.87	43	350	0.02	151.04	100	350	0.02											
gdb9	27	51	303	317	4.62	303	0.00	354	352	358	306	0.01	330.1	184	309	0.02	125.56	66	309	0.02	146.27	91	306	0.01											
gdb10	12	25	275	275	0.00	275	0.00	295	300	283	275	0.00	0.55	1	275	0.00	0.55	1	275	0.00	0.94	2	275	0.00											
gdb11	22	45	395	395	0.00	395	0.00	447	449	413	395	0.00	5.77	4	395	0.00	9.06	6	395	0.00	7.03	5	395	0.00											
gdb12	13	23	448	458	2.23	458	0.02	581	569	537	458	0.02	1.27	3	458	0.02	3.62	9	458	0.02	3.35	9	458	0.02											
gdb13	10	28	536	544	1.49	536	0.00	563	560	552	542	0.01	26.58	51	544	0.01	1.15	2	544	0.01	0.60	1	542	0.01											
gdb14	7	21	100	100	0.00	100	0.00	114	102	104	100	0.00	0.44	1	100	0.00	0.44	1	100	0.00	0.44	1	100	0.00											
gdb15	7	21	58	58	0.00	58	0.00	60	60	58	58	0.00	0.11	0	58	0.00	0.16	0	58	0.00	0.17	0	58	0.00											
gdb16	8	28	127	127	0.00	127	0.00	135	129	132	127	0.00	2.20	4	127	0.00	2.04	3	127	0.00	15.27	29	127	0.00											
gdb17	8	28	91	91	0.00	91	0.00	93	91	93	91	0.00	0.17	0	91	0.00	0.22	0	91	0.00	0.16	0	91	0.00											
gdb18	9	36	164	164	0.00	164	0.00	177	174	172	164	0.00	1.10	1	164	0.00	1.10	1	164	0.00	0.99	1	164	0.00											
gdb19	8	11	55	55	0.00	55	0.00	57	63	63	55	0.00	0.22	1	55	0.00	0.28	1	55	0.00	0.22	1	55	0.00											
gdb20	11	22	121	121	0.00	121	0.00	132	129	125	121	0.00	8.30	23	121	0.00	4.17	11	121	0.00	54.43	145	121	0.00											
gdb21	11	33	156	156	0.00	156	0.00	176	163	162	156	0.00	3.73	5	156	0.00	8.57	13	156	0.00	11.81	16	156	0.00											
gdb22	11	44	200	200	0.00	200	0.00	208	204	207	201	0.01	2.75	2	200	0.00	34.22	31	200	0.00	4.89	4	200	0.00											
gdb23	11	55	233	235	0.86	233	0.00	251	237	239	235	0.01	6.15	3	235	0.01	2.31	1	235	0.01	12.46	7	235	0.01											
Av.Dev (%) :					0.50						0.17						0.34						0.36						0.36						0.30
Nb hits :					18						21						17						18						18						18

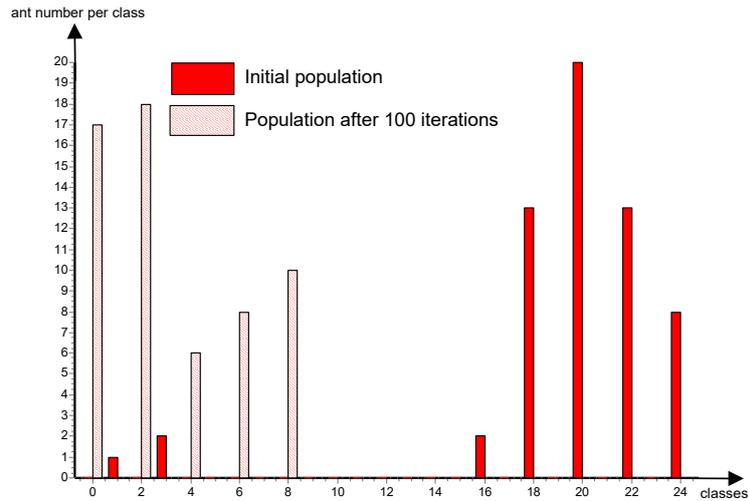


Figure 3. Population distribution after 100 iterations (Gdb9)

After 100 iterations, the distribution has changed (figure 3) and ants are concentrated around promising values.

Figure 4 gives the population evolution over iterations. To obtain a legible graphical representation, solutions are sorted in increasing cost order. After 50 iterations, there are 9 solutions better than Carpet one and a global trend appears to minimize the solution cost. Thanks to the non-elitist ants the minimization process continues over iterations (figure 4.c and 4.d) with an important diversity of the cost. After 184 iterations, 10 high quality solutions are identified and the population cost is spread over 320 and 450.

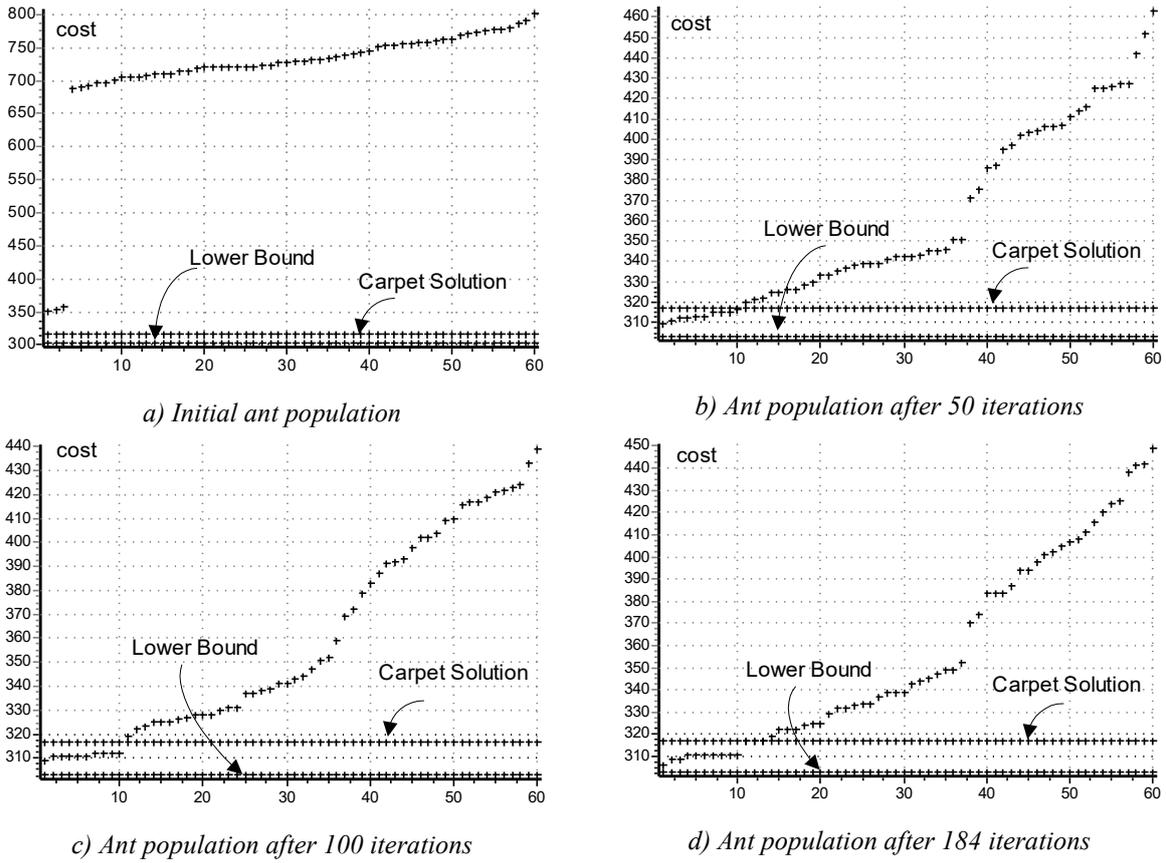


Figure 4. Ant population evolution for Gdb9 instance during the first experiment

Conclusion on DeArmon’s instances

The results prove the efficiency of the ants regarding both the taboo method CARPET and the Genetic Algorithm. For DeArmon’s instances (table 3), the ant optimization scheme ever provides solutions equal or better than CARPET solutions and 20 times it provides solutions equal or better than the Genetic Algorithm (table 4).

Table 4. DeArmon’s instances

	BACO	CARPET	GA
better than		3	0
equal to		20	20
worse than		0	3

3.2.2 Results on Belenguer and Benavent’s Instances

BACO competes with CARPET (table 5) for almost all instances except for Val8C and Val10b, and provides better deviations than CARPET as regards the lower bound.

Table 5. Belenguer and Benavent's instances

FILE	n	τ	LB	C	Dev	GA	Dev	BACO	Dev	Av. Time
val1a	24	39	173	173	0.00	173	0.00	173	0.00	0.11
val1b	24	39	173	173	0.00	173	0.00	173	0.00	120.59
val1c	24	39	235	245	0.04	245	0.04	245	0.04	13.13
val2a	24	34	227	227	0.00	227	0.00	227	0.00	1.97
val2b	24	34	259	260	0.00	259	0.00	259	0.00	8.43
val2c	24	34	455	494	0.09	457	0.00	457	0.00	135.11
val3a	24	35	81	81	0.00	81	0.00	81	0.00	1.15
val3b	24	35	87	87	0.00	87	0.00	87	0.00	3.63
val3c	24	35	137	138	0.01	138	0.01	138	0.01	10.62
val4a	41	69	400	400	0.00	400	0.00	400	0.00	15.34
val4b	41	69	412	416	0.01	412	0.00	412	0.00	117.13
val4c	41	69	428	453	0.06	428	0.00	430	0.00	285.38
val4d	41	69	520	556	0.07	541	0.04	539	0.04	315.86
Val5a	34	65	423	423	0.00	423	0.00	423	0.00	49.53
Val5b	34	65	446	448	0.00	446	0.00	446	0.00	24.30
Val5c	34	65	469	476	0.01	474	0.01	474	0.01	200.31
val5d	34	65	571	607	0.06	581	0.02	597	0.05	193.82
val6a	31	50	223	223	0.00	223	0.00	223	0.00	3.77
val6b	31	50	231	241	0.04	233	0.01	233	0.01	78.39
val6c	31	50	311	329	0.06	317	0.02	317	0.02	91.57
val7a	40	66	279	279	0.00	279	0.00	279	0.00	11.17
val7b	40	66	283	283	0.00	283	0.00	283	0.00	6.59
val7c	40	66	333	343	0.03	334	0.00	334	0.00	569.27
val8a	30	63	386	386	0.00	386	0.00	386	0.00	15.38
val8b	30	63	395	401	0.02	395	0.00	395	0.00	259.49
val8c	30	63	517	533	0.03	527	0.02	534	0.03	358.06
val9a	50	92	323	323	0.00	323	0.00	323	0.00	969.03
val9b	50	92	326	329	0.01	326	0.00	326	0.00	1076.21
val9c	50	92	332	332	0.00	332	0.00	332	0.00	1368.47
val9d	50	92	382	409	0.07	391	0.02	404	0.06	633.98
val10a	50	97	428	428	0.00	428	0.00	428	0.00	341.81
val10b	50	97	436	436	0.00	436	0.00	437	0.00	683.42
val10c	50	97	446	451	0.01	446	0.00	448	0.00	515.79
val10d	50	97	524	544	0.04	530	0.01	538	0.03	916.10
					Av.Dev (%) :	1.96		0.61	0.90	
					Nb hits :	15		22	20	

CARPET has an average deviation of 1.90%. Over three experiments, the deviation of the ants is 1.11%, 1.05% and 1.04% and BACO has an average deviation of only 0.90%. For the 34 instances, BACO provides solutions equal or better than CARPET for 32 instances (table 6).

Table 6. Belenguer and Benavent's instances

	BACO	CARPET	GA
better than		17	0
equal to		15	27
worse than		2	7

The three experiments show that the Ant Colony scheme provides very low deviation as regards lower bound but its deviation is slightly higher than deviation of the Genetic Algorithm. However, BACO competes with the Genetic Algorithm for 27 instances and it is worse for only 7 instances. The average deviation of BACO (0.90%) is close to the Genetic Algorithm deviation (0.61%). The

computational time remains acceptable: 20 minutes are required for the last instances of the benchmark (Val9a - Val10d). This computation duration is two times larger than the execution time of the first instances. The first instances are solved in only 2 minutes of computation.

3.2.3 Results on Eglese's Instances

Eglese's instances (table 7) are more complex than the previous ones because non-required edges are spread in large-scale instances. For example, instances s1-A to s4-C have 140 required arcs and 190 nodes.

Table 7. Eglese's instances

FILE	n	τ	LB	C	Dev	GA	DEV	GW	AM	UL	BACO	Dev	Av Time
e1-A	77	98	3515	3625	0.03	3548	0.01	4115	4605	3952	3548	0.01	70.68
e1-B	77	98	4436	4532	0.02	4498	0.01	5228	5494	5054	4534	0.02	307.49
e1-C	77	98	5453	5663	0.04	5595	0.03	7240	6799	6166	5647	0.04	159.12
e2-A	77	98	4994	5233	0.05	5018	0.00	6458	6253	5716	5018	0.00	470.39
e2-B	77	98	6249	6422	0.03	6340	0.01	7964	7923	7080	6401	0.02	406.39
e2-C	77	98	8114	8603	0.06	8415	0.04	10313	10453	9338	8498	0.05	707.39
e3-A	77	98	5869	5907	0.01	5898	0.00	7454	7350	6723	5934	0.01	609.83
e3-B	77	98	7646	7921	0.04	7822	0.02	9900	9244	8713	7915	0.04	781.88
e3-C	77	98	10019	10805	0.08	10433	0.04	12672	12556	11641	10402	0.04	226.66
e4-A	77	98	6372	6489	0.02	6461	0.01	7527	7798	7231	6520	0.02	616.78
e4-B	77	98	8809	9216	0.05	9021	0.02	10946	10543	10223	9234	0.05	839.79
e4-C	77	98	11276	11824	0.05	11779	0.04	13828	13623	13165	11883	0.05	799.26
s1-A	140	190	4992	5149	0.03	5018	0.01	6382	6143	5636	5049	0.01	1010.53
s1-B	140	190	6201	6641	0.07	6435	0.04	8631	7992	7086	6541	0.05	2899.76
s1-C	140	190	8310	8687	0.05	8518	0.03	10259	10338	9572	8561	0.03	2388.90
s2-A	140	190	9780	10373	0.06	9995	0.02	12344	11672	11475	10368	0.06	4108.04
s2-B	140	190	12886	13495	0.05	13174	0.02	16386	15178	14845	13676	0.06	5377.59
s2-C	140	190	16221	17121	0.06	16795	0.04	20520	19673	19290	17115	0.06	3099.32
s3-A	140	190	10025	10541	0.05	10296	0.03	13041	11957	11956	10619	0.06	1392.07
s3-B	140	190	13554	14291	0.05	14053	0.04	17377	15891	15663	14264	0.05	6568.64
s3-C	140	190	16969	17789	0.05	17297	0.02	21071	19971	20064	17797	0.05	3160.04
s4-A	140	190	12027	13036	0.08	12442	0.03	15321	14741	13978	12868	0.07	8919.24
s4-B	140	190	15933	16924	0.06	16531	0.04	19860	19172	18612	17090	0.07	6360.03
s4-C	140	190	20179	21486	0.06	20832	0.03	25921	24175	23727	21314	0.06	4911.44
Av. Dev(%) :					4.74		2.47					4.11	
Nb hits :					0		0					0	

The Augment-Merge method, Golden and Wang's heuristic and Ulusoy's algorithm performances strongly decrease for these instances (see [12, 24] for details). On DeArmon's instances, the lower bound is reached by these heuristics for some instances. This remark remains true for Belenguer and Benavent's instances (Valla). For Eglese's instances, the deviation of the heuristics as regards the lower bound is high. CARPET provides an average deviation of 4.74%. BACO provides 15 solutions equal or better than CARPET ones and 9 solutions worse than CARPET ones (table 8). The Genetic Algorithm [12, 24] provides high quality solutions and the average deviation is low: 2.47%. The Ant Colony scheme provides an average deviation of 4.11% for BACO, which is two times worse than the Genetic Algorithm deviation.

Table 8. Eglese's instances

BACO	CARPET	GA
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better than	15	0
equal to	0	3
worse than	9	21

Significant enlargement of iterations improves solutions quality. For the small scale Eglese's instances, with 1000 iterations the Ant Colony scheme provides an average deviation of 2.94% which is close to the 2.15% of the Genetic Algorithm and better than the 3.84% of the CARPET algorithm (table 9).

Table 9. Resolution of the small scale Eglese's instances with 1000 iterations (one experiment)

FILE	n	τ	LB	C	Dev	GA	DEV	Ant	Dev	Time	I
e1-A	77	98	3515	3625	0.03	3548	0.01	3548	0.01	50.59	17
e1-B	77	98	4436	4532	0.02	4498	0.01	4514	0.02	622.86	214
e1-C	77	98	5453	5663	0.04	5595	0.03	5632	0.04	2204.31	757
e2-A	77	98	4994	5233	0.05	5018	0.00	5018	0.00	180.42	50
e2-B	77	98	6249	6422	0.03	6340	0.01	6406	0.02	2641.44	742
e2-C	77	98	8114	8603	0.06	8415	0.04	8479	0.05	3023.08	852
e3-A	77	98	5869	5907	0.01	5898	0.00	5902	0.01	4173.31	965
e3-B	77	98	7646	7921	0.04	7822	0.02	7853	0.04	3885.45	922
e3-C	77	98	10019	10805	0.08	10433	0.04	10401	0.04	3630.89	866
e4-A	77	98	6372	6489	0.02	6461	0.01	6547	0.02	4382.69	901
e4-B	77	98	8809	9216	0.05	9021	0.02	9214	0.05	1256.58	261
e4-C	77	98	11276	11824	0.05	11779	0.04	11883	0.05	4222.7	893
Av.Dev(%) :					3.84		2.15		2.94		
Nb hits:					0		0		0		

4 Concluding Remarks and Future Research

This paper presents a resolution scheme for the CARP based on Ant Colony. The Ant Colony scheme is competitive with the best methods previously published providing high quality solutions in rather short computational time. It outperforms the CARPET algorithm and competes with the Genetic Algorithm for small and medium scale instances. The computational time is acceptable but the Ant Colony scheme can not compete, for a computational point of view, with the powerful Genetic Algorithm. This work is a step forward for the CARP resolution based on Ant Colony and proves Ant Colony Scheme competes with Taboo Search and Genetic Algorithms. It strengthens the previous published attempt of Doerner *et al.* [25]. To the best of our knowledge the proposed ACO is the first one proposed for the CARP providing high quality results for large scale instances. However, the performance of the proposed algorithm does not reach state-of-the-art performance and further researches are required to increase the convergence rate and to reduce the computational times.

References

1. Golden B-L. and R.T. Wong. *Capacitated arc routing problems*. Networks, vol. 11, pp. 305-315, 1981.

2. Benavent E., V. Campos, A. Corberan and E. Mota. *The capacitated arc routing problem. lower bounds*. Networks, vol. 22, pp. 669-690, 1992.
3. Belenguer J.M. and E. Benavent. *A cutting plane algorithm for the capacitated arc routing problem*. Research Report, Dept. of Statistics and OR, Univ. of Valencia (Spain), 1997.
4. Hirabayashi R., Y. Saruwatari and N. Nishida. *Tour construction algorithm for the capacitated arc routing problem*. Asia-Pacific Journal of Oper. Res., vol. 9, pp. 155-175, 1992.
5. Pearn W.L. *Approximate solutions for the capacitated arc routing problem*. Computers and Operations Research, vol. 16, no. 6, pp. 589-600, 1989.
6. Pearn W.L. *Augment-insert algorithms for the capacitated arc routing problem*. Computers and Operations Research, vol. 18, no. 2, pp. 189-198, 1991.
7. Golden B.L., J.S DeArmon and E.K. Baker. *Computational experiments with algorithms for a class of routing problems*. Computers and Operation Research, vol. 10, no. 1, pp. 47-59, 1983.
8. Ulusoy G. *The Fleet Size and Mixed Problem for Capacitated Arc Routing*. European Journal of Operational Research, vol. 22, pp. 329-337, 1985.
9. Li L.Y.O. *Vehicle routing for winter gritting*. Ph.D. Dissertation, Department of OR and OM, Lancaster University, 1992.
10. Eglese R.W. *Routing winter gritting vehicles*. DAM, vol. 48, no. 3, pp. 231-244, 1994.
11. Hertz A., G. Laporte and M. Mittaz. *A Tabu Search Heuristic for the Capacitated Arc Routing Problem*. Operations Research, vol. 48, no. 1, pp. 129-135, 2000.
12. Lacomme P., C. Prins and W. Ramdane-Cherif. *Competitive genetic algorithms for the Capacitated Arc Routing Problem and its extensions*. Lecture Notes in Computer Science, E.J.W. Boers et al. (Eds.), LNCS 2037, pp. 473-483, Springer-Verlag, 2001.
13. Costa D. and A. Hertz. *Ants can colour graphs*. Journal of the Operational Research Society, vol. 48, no. 3, pp. 295-305, 1997.
14. Stutzle T. and M. Dorigo. *Algorithms for the Quadratic Assignment Problem*. In D. Corne, M. Dorigo and F. Glover (Eds.), New Ideas in Optimization, Mc Graw-Hill, 1999.
15. Dorigo M. and L.M. Gambardella. *Ant Colony System. A cooperative learning approach to the Travelling Salesman Problem*. IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 53-66, 1997.
16. Bullnheimer B., R.F. Hartl and C. Strauss. *Applying the ant systems to the vehicle routing problem*. In S. Voss, S. Martellon, I.H. Osman and C. Roucairol (Eds.), Meta-Heuristics. Advances and Trends in Local Search Paradigms for Optimization, Kluwer, Boston, 1999.

17. Bullnheimer B., R.F. Hartl and C. Strauss. *An improved ant system algorithm for the vehicle routing problem*. Annals of Operations Research, vol. 89, pp. 319-328. 1999.
18. Reimann M., M. Stummer and K. Doerner. *A Savings based Ant System for the Vehicle Routing Problem*. in Proceedings of the Genetic and Evolutionary Computation Conference (GECCO), Morgan Kaufmann, New-York, pp 1317-1325, 2002.
19. Gambardella L.M., E. Taillard and G. Agazzi. *Macs-VRPTW. A multiple Ant Colony System for Vehicle Routing Problems with Time Windows*. In D. Corne, M. Dorigo and F. Glover (Eds.), *New Ideas in Optimization*, Mc Graw-Hill, London, 1999.
20. Bundgaard M., T. C. Damgaard, F. Decara, and J. W. Winther. *Ant Routing System—a routing algorithm based on ant algorithms applied to a simulated network*. Report of Spring, University of Copenhagen, 2002.
21. Gutjahr W.J. *A graph-based Ant System and its convergence*. Future Generation Computing Systems, vol. 16, pp. 873-888. 2000.
22. Donati A.V., L.M. Gambardella, A.E. Rizzoli, N. Casagrande and R. Montemanni. *Time Dependent Vehicle Routing Problem with an Ant Colony System*. Technical Report IDSIA-02-03, Istituto Dalle Molle di Studi sull'Intelligenza Artificiale, November 2002.
23. Bullnheimer B., R.F. Hartl and C. Strauss. *A New Rank Based Version of the Ant System. a Computational Study*. Working Paper number 1, Working Papers Series, 1997.
24. Lacomme P., C. Prins, W. Ramdane-Chérif. *Competitive Memetic Algorithms for Arc Routing Problems*. Research Report, LOSI-01, University of Technology of Troyes 2001.
25. Doerner K., R.F. Hartl, V. Maniezzo, M. Reimann. *An Ant System Metaheuristic for the Capacitated Arc Routing Problem*, MIC'2003, The Fifth Metaheuristic International Conference, Kyoto, Japan, August 25-28, 2003.

Index

Table 1. Instances and parameters.....	10
Table 2. Notations used in table of results	10
Table 3. DeArmon's instances	12
Table 4. DeArmon's instances	14
Table 5. Belenguer and Benavent's instances.....	15
Table 6. Belenguer and Benavent's instances	15
Table 7. Eglese's instances	16
Table 8. Eglese's instances.....	16
Table 9. Resolution of the small scale Eglese's instances with 1000 iterations (one experiment)....	17
Figure 1. Ant Colony algorithm	7
Figure 2. Population distribution after 20 iterations (Gdb9).....	11
Figure 3. Population distribution after 100 iterations (Gdb9).....	13
Figure 4. Ant population evolution for Gdb9 instance during the first experiment.....	14