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Series Editors

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Editors

Algebraic Geometry and Geometric Modeling

With 52 Figures, 27 in Color and 7 Tables, 1 in Color

Springer

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Preface

Algebraic Geometry and Geometric Modeling are two distinct domains of research, with few interactions up to now, though closely linked. On the one hand, Algebraic Geometry has developed an impressive theory targeting the understanding of geometric objects defined algebraically. On the other hand, Geometric Modeling is using every day, in practical and difficult problems, virtual shapes based on algebraic models. Could these two domains benefit from each other? Recent and interesting developments in this direction are about to convince us to answer yes. In this book, we have collected articles which reinforce, in some way, the natural bridge which exists between these two areas. The confrontation of the different points of view should result in a better analysis of the key problems and related methods to solve them. This was the aim of the workshop entitled *Algebraic Geometry and Geometric Modeling*, held from September 27 to September 29, 2004, at the University of Nice-Sophia Antipolis. This workshop was organized, in the context of the European project GAIA II (IST-2002-35512).

The articles of this book are grouped into three main families:

- Implicitization problems
- Classification problems
- Intersection problems

The first group of articles is about Implicitization problems, namely the conversion of a parametric representation of an algebraic variety into an implicit one. This problem is fundamental in Computer Aided Geometric Design (CAGD), where geometric objects are often given parametrically and some operations like testing if a point is in a variety, intersecting two varieties, determining the singular locus of a variety, is better understandood via implicit representations.

In his contribution, Goldman claims that the main contribution of Algebraic Geometry to Geometric Modeling is insight, not computation, and that we should not confuse concrete theoretical tools given by Algebraic Geometry with efficient computational methods needed by Geometric Modeling.

He defends this thesis using some examples, as solving polynomial systems, intersecting curves and surfaces, and implicitization problem.

Chardin presents in his paper a survey of the recent method of approximation complexes and its application to the implicitization problem of a parameterization given by a rational map from $\mathbb{P}^{n-1}$ to $\mathbb{P}^n$.

Shalaby, Thomassen, Wurm, Dokken and Jüttler compare four methods for approximate implicitization by piecewise polynomials. This comparison is performed by applying algorithms to academic surfaces and industrial ones provided by the CAGD vendor, Think3, a partner in the European project GAIA II.

Lazard in his paper shows that, using the Gb-RS software, Gröbner basis computations allow efficient computation in order to solve some problems like testing if a point is on a variety, computing the intersection of parameterized varieties, or studying the singularities of parametric varieties without implicitization.

Aigner, Szilágyi, Jüttler and Schicho study from a numerical point of view the computation of the implicit representation of a curve that approximates a given parametric curve in a domain of interest. They show that for any approximate parameterization of the given curve, the curve obtained by an approximate implicitization with a given precision is contained within a perturbation region.

The theme of the second string of papers in this volume is, loosely speaking, the theory of classification of algebraic surfaces. Traditionally in algebraic geometry, this theory has been developed mainly in the complex, projective setting. The important issue for CAGD applications, however, is an understanding of *real* algebraic curves and surfaces in affine real 3-space, or in bounded regions of this space. The understanding of the complex projective case helps understand the real case, for example one can obtain bounds, coming from the complex case, for the number of connected components and the number and type of singularities.

The paper by Dimca discusses how the presence of singularities affects the geometry and topology, or shape, of complex projective hypersurfaces. Relevant theory of deformations and singularities is surveyed, and examples are given in the curve and surface cases.

The topology of real algebraic surfaces is the topic of Kharlamov's contribution. Applying Smith theory and Hodge theory to real surfaces and higher dimensional varieties, he provides various prohibitions on the total Betti number, the Euler characteristic, and the signature. As an example, he discusses surfaces of low degree. The possible topological types of cubic and of (nonsingular) quartic surfaces are known, but already for quintics the knowledge is very limited.

Holzer and Labs illustrate the classification of real projective cubic surfaces. They show how to choose affine equations so that no singularity or real line lies in the plane at infinity. Using their new visualization tool SURFEX, the images of all types can be shown. A useful feature of SURFEX is that it allows visualizations of deformations of surfaces.

Most surfaces used in CAGD are rational, in fact given by a parameterization. In his paper, Krasauskas shows how to use toric surfaces to find Bezier patches of optimal degree on a given surface with boundary curves. This is done by studying all parameterizations of the given surface and applying the theory of Universal Rational Parameterization.

Elkadi, Galligo, and Lê also study parameterized surfaces, more precisely tensor surfaces of low bidegree. They work in the complex projective case, and consider equivalences of parameterizations and the corresponding moduli space. The number of moduli is determined and normal forms are provided; also the implicit equations and the nature of the singularities are studied.

The third group of papers deals with intersection problems in geometry. This is the corner-stone of many operations on shapes. Numerical difficulties have to be tackled, together with purely geometric questions such as computing the topology of the constructed objects. Behind these geometric problems, are hidden questions about the resolution of polynomial systems of equations and their simplification.

The paper by Fioravanti, Gonzalez-Vega and Necula deals with the intersection problems of rational surfaces of a special type, namely revolution and canal surfaces. These surfaces appear naturally in modeling problems, where the characterizations of shapes by distances are used.

The problem handled in the next paper by Galligo and Pavone is about computing the self-intersecting points of a parametric surface, that is the points where this parameterization is not injective. A subdivision approach is described and its practical efficiency illustrated on real CAGD models.

An important problem in geometric modeling concerns the assembling of objects by imposing some constraints on basic geometric primitives. The paper by Peter, Sitharam, Zhou, and Fan considers the 3D case and describes techniques to analyze it, from a combinatorial point of view.

The paper by Emiris and Tsigaridas is of another flavor. The problem that they consider is the decomposition of polynomials as products, approached from a convex geometry point of view. Namely, the decomposition of a polytope as the Minkowski sum of two other polytopes is analyzed in detail for polygons, in dimension two.

Finally the paper by Carlini handles the problem of reducing the number of variables in the presentation of a polynomial. Tools from effective algebraic geometry are used for this purpose, illustrating this interaction between algebra and geometry.

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