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Wil Michiels · Emile Aarts · Jan Korst

Theoretical Aspects of Local Search

With 70 Figures and 3 Tables

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Authors

Wil Michiels
Jan Korst

Philips Research
High Tech Campus 34
5656 AE Eindhoven
The Netherlands
wil.michiels@philips.com
jan.korst@philips.com

Emile Aarts

Philips Research
High Tech Campus 34
5656 AE Eindhoven
The Netherlands
and
Eindhoven University of Technology
P.O. Box 513
5600 MB Eindhoven
The Netherlands
emile.aarts@philips.com

Series Editors

Prof. Dr. Wilfried Brauer

Institut für Informatik der TUM
Boltzmannstr. 3
85748 Garching, Germany
brauer@informatik.tu-muenchen.de

Prof. Dr. Juraj Hromkovič

ETH Zentrum
Department of Computer Science
Swiss Federal Institute of Technology
8092 Zürich, Switzerland
juraj.hromkovic@inf.ethz.ch

Prof. Dr. Grzegorz Rozenberg

Leiden Institute of Advanced
Computer Science
University of Leiden
Niels Bohrweg 1
2333 CA Leiden, The Netherlands
rozenber@liacs.nl

Prof. Dr. Arto Salomaa

Turku Centre of
Computer Science
Lemminkäisenkatu 14 A
20520 Turku, Finland
asalomaa@utu.fi

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Preface

In 1958 Croes published a twenty page article in Operations Research describing a new method to handle traveling salesman problems based on the iterative use of simple edge-exchange mechanisms. This article can be seen as the seminal paper on local search, reporting for the first time the success of simple neighborhood-based iteration methods when applied to combinatorial optimization problems.

Half a century of practice reveals that local search has become quite extensively used, and evidence exists that it has become one of the most frequently and widely applied heuristic search methods in engineering, operations research, and design. By October 2006 the well-known search engine Google returned over twenty million entries upon the query 'local search'. Applications of local search are abundantly available in real life, and they are by no means limited to small-scale or insignificant toy problems. On the contrary, fields are known where local search provides a competitive edge to business and product engineering. For instance, in chip design, the use of tools applying local-search-based techniques for determining area-effective silicon layouts has resulted in smaller chips than those that could be obtained with other placement techniques, resulting in cost savings of several million euros on a yearly basis as a result of increased production yield. The use of local search techniques in logistics has enabled the computation of cost-effective routes for the transportation of packets that need to be shipped over a complex network of roads and railways. This improves throughput and shortens delivery time, which in turn leads to substantial cost savings.

Over the years, a huge number of papers have appeared on local search. Closer examination indicates that the vast majority of these papers is of an experimental nature. This observation reflects very well the position that local search has been taking now for almost half a century: it is primarily seen as a practical tool for which not much can be said from a theoretical point of view. Although we largely agree with this classification, it would be short sighted to consider the theory of local search as being insignificant.

Apart from a number of specific isolated results, one can identify three main topics of theoretical work relating to local search. In the first place, performance guarantees have been studied regarding the quality of solutions obtained with local search. Secondly, one has started to investigate the time complexity of local search. On the one hand, this has led to problem-specific results on the number of iterations required to reach a local optimum. On the other hand, a general theory on the time complexity of local search has been developed. This theory is the local search counterpart of the theory of NP-completeness. It can be used to prove the

intractability of finding a local optimum for some local search problems. As a third topic of theoretical work, we mention the asymptotic convergence of local search in the case where a probabilistic iteration mechanism is applied. Simulated annealing is probably the best known example of such a probabilistic variant of local search.

The aim of this book is to bring together the theoretical results relating to local search, following the tripartition given above. We do not intend to completely cover all theoretical results presented in the literature. Instead, we use a small set of classical combinatorial optimization problems as running examples, particularly the traveling salesman problem and machine scheduling. The results we prove for these problems can be considered typical for the results that can be found in this area. Since we restrict ourselves to theoretical results, this book should not be considered as a handbook on how to apply local search in practice. For example, we do not give any specific implementation details of the various local search metaheuristics, such as simulated annealing and tabu search. However, we aim at giving a better understanding of the fundamental behavior of local search.

The organization of this book is as follows. The introductory chapter positions local search within the field of combinatorial optimization, and presents basic definitions and terminology. Chapter 2 presents basic neighborhood functions for a number of classical combinatorial optimization problems. Chapter 3 illustrates the use of indirect representations for several machine scheduling problems. Chapter 4 discusses properties of neighborhood functions. It considers degree and diameter of neighborhood graphs as well as dominance relations between different neighborhood functions. In Chapter 5, we give several examples of performance guarantees for local minima of given neighborhood functions and we show, for some specific examples, that local optimality implies global optimality. Chapter 6 addresses the computational complexity of finding a local optimum for a given combinatorial optimization problem and a given neighborhood function. It discusses the complexity class PLS, and gives examples of PLS-complete problems and related PLS-reductions. Most of the results presented up to Chapter 6 relate to iterative improvement. In Chapter 7 we discuss metaheuristics that potentially give higher-quality solutions. These metaheuristics either allow non-improving moves, such as simulated annealing and tabu search, or use multiple runs, such as iterated local search and genetic local search. Finally, Chapter 8 deals with the asymptotic convergence of simulated annealing using the theory of Markov chains.

The presentation of this book is at a graduate level. The book can serve as an introductory textbook on the theory of local search. It contains many theorems and corresponding proofs as well as bibliographical notes and clarifying exercises. This makes the book quite suitable for students in mathematics, computer science, and electrical and industrial engineering. We recommend reading the chapters in the order in which they are presented. However, most of the chapters are reasonably self-contained, and the reader might like to immediately jump to a chapter of his or her interest. Nevertheless, we recommend the reader to at least read Chapter 1 for basic terminology. Some of the sections are marked with an asterisk, indicating that they provide additional material that can possibly be skipped.

Eindhoven, October 2006 Wil Michiels, Emile Aarts, Jan Korst

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