Performance Evaluation of Partially Coherent MC/DS-CDMA System with MOC Sequence

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Abstract. This paper deal with the mutually orthogonal complementary (MOC) sequences to assigning a spreading sequences to each user and propose the partially coherent equal gain combined multicarrier direct-sequence code-division multiple access (MC/DS-CDMA) system. And, we analyze the reverse link capacity and BER performance in Rayleigh fading plus multiple-access interference (MAI) channel, and evaluate the effect of phase error on receiver and transmission activity in a cell. Each user is assumed to have a distinct set of spreading sequences, with a different spreading sequence for each carrier in each user's set. By selecting MOC sequences, MAI of asynchronous MC/DS-CDMA system can be eliminated when compared to systems employing a single spreading sequence to each carrier for a particular user in the phase noise channel, and either data rate or channel capacity can be increased.

1 Introduction

Recently, there has been great interest in applying multi-carrier techniques to obtain diversity effect in communications systems. One example is MC/DS-CDMA [1]-[3], in which each of the M_C carriers is multiplied by a spreading sequence unique to each user. This MC/DS-CDMA system has a number of desirable features, including narrow-band interference suppression and a lower required chip rate than that of a single-carrier system occupying the same total bandwidth. The lower required chip rate is a result of the fact that the entire bandwidth is divided equally among M_C frequency bands. In addition, it is easier to implement the parallel receiver architecture of a number of carriers than a larger order RAKE [4].

The MC/DS-CDMA system using MOC sequences does have some advantages when compared to the single DS-CDMA system. Main advantage is that the reduction in MAI reduces the effect of the near-far problem and the autocorrelation sidelobes. Therefore, the MC/DS-CDMA system can support more users and more information symbols for a fixed error probability constraint. And hence increase the data rate achievable by a single user. And the disadvantage of MC/DS-CDMA system using

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MOC is that the system is not as resistant to frequency-selective fading. However, even with this disadvantage, the system appears well suited to the fiber optical channels or Rician channels with a strong line-of-sight path.

In this paper, we introduce a MC/DS-CDMA that employs a set of spreading sequences for each user. Each user applies a different sequence to each sub-carriers. By selecting these sequences to be MOC sequences, MAI can be eliminated in the ideal phase-coherent channel, and either data rate or capacity can be increased. In the receiver, despreading is accomplished on a carrier-by-carrier basis. Hence, excluding MAI and noise, the output of the matched filter corresponding to a particular carrier channel is the autocorrelation function of the corresponding spreading sequence. The MAI part of the output of the matched filter is the summation of cross correlation functions between the intended user's and unintended users' spreading sequences. After adding the output of all matched filters, the autocorrelation sidelobes and MAI are zero by the defining property of MOC sequences. In this paper, the capacity and BER performance of MC/DS-CDMA system combined MOC sequences and EGC scheme for the reverse link (mobile-to-base) over Rayleigh fading channel is analyzed. The MOC sequences can provide high data rate and capacity for multi-users. The impacts of phase error of receiver and transmission activation are also considered.

2 Partially Coherent MC/DS-CDMA System with MOC Sequences

The relation of spectrum of the transmitted signal for single-carrier and multi-carrier system is given by

$$W_{MC} = M_C \cdot W_{SC} \tag{1}$$

where, M_C is the number of carriers, and we assume a strictly band-limited chip waveform with bandwidth W_{SC}. The symbol duration is $T = M_C \cdot T_C$, where the chip period T_C is M_C times larger than that of single-carrier systems. Similarly, $E = M_C \cdot E_C$ is the bit energy, where E_C is the chip energy. Without loss of generality, we assume user $k = 1$ is the intended user. The phase and delay of user $k = 1$ are assumed to be zero without loss of generality. Users $k = 2, \cdots, K$ are interfering users. They are assumed to be independent.

2.1 Transmitter

Let $d_h^{(k)}$ be the data stream for the k th user, and let $\left\{C_i^{(k)}, i = 1, 2, \cdots, M_C\right\}$ be one of the mutually orthogonal complementary sets of sequences of length L_S for the user k. The transmitted signal of user k in the i th branch is given by

$$S_i^{(k)} = \sum_{n=-\infty}^{\infty} d_h^{(k)} c_{n,i}^{(k)} \phi(t - nT_C - \tau^{(k)}) \tag{2}$$

$$k = 1, \cdots, K, \qquad i = 1, \cdots, M_C$$

where $h = \lfloor n/L_S \rfloor$, $\tau^{(k)}$ is the delay for user k, and the chip waveform $\phi(t)$ has Fourier transform $\Psi(f)$. We assume $\Psi(f)$ is strictly band-limited.

Here, we describe mutually orthogonal complementary sets of sequences. Each of the M_C sequences assigned to an individual user (one sequence per carrier) is distinct. So each user has a unique set of spreading sequences, and each of the spreading sequences in a user's set is different. Binary and multiphase complementary sets of sequences have been used in radar applications for reducing both range and doppler sidelobes, allowing the detection and resolution of objects that would otherwise be hidden in the range sidelobes of large nearby scatterers. One complementary set of sequences is a mate of the other, and they are MOC sets, because the sum of cross correlation functions is zero. However, to illustrate the basic ideas of the MC/DS-CDMA system and to simplify the analysis, we only consider binary MOC sets in the proposed system.

2.2 Receiver

The chip-matched filters i.e., ideal bandpass filters are used to separate the M_C multi-carrier frequency bands. Matched fillers matched to $c_i^{(k)}$, $i = 1, 2, \cdots, M_C$ are then used for despreading in the receiver. The M_C matched filler outputs are then summed and sampled. Note that it is natural to use a fast Fourier transform to perform this processing.

We assume the bit rate is the same for our system and the system described in [1]. We assume perfect symbol and chip synchronization for user $k = 1$, and we evaluate the performance of the first user. It is standard to assume $d_h^{(k)} c_{n,i}^{(k)}$, $k = 2, \cdots, K$ are independent sequences. We first look at the *i*th branch of the receiver for user $k = 1$.

From the figure 1, the chip-matched filter output is given by

$$Y_i(t) = S_i(t) + I_i(t) + N_i(t) \tag{3}$$

where $S_i(t)$ is the signal component, $I_i(t)$ is the MAI term, and $N_i(t)$ is the noise passing through bandpass and lowpass filters. After matched filtering and sampling, we have decision statistic $Z_i = S_{Z_i} + I_{Z_i} + N_{Z_i}$.

We use equal gain combine techniques at the receiver, i.e.,

$$Z = \sum_{i=0}^{M_C} Z_i \tag{4}$$

Denote $\bar{\alpha} = (\alpha_{1,i}, \cdots, \alpha_{1,M_c})$. Then

$$E\left[Z \mid \bar{\alpha}\right] = L_S \sqrt{E_C} \sum_{i=1}^{M_C} \alpha_{1,i}, \tag{5}$$

$$\text{var}\left[N_Z\right] = \sum_{i=1}^{M_C} \text{var}\left[N_{z_i}\right] = M_C L_S \eta_0 / 2, \tag{6}$$

and

$$\text{var}\left[I_Z\right] = \sum_{i=1}^{M_C} \text{var}\left[I_{Z_i}\right] = \sum_{i=1}^{M_C} \left[L_S R_{I,i}(0) + 2 \sum_{i=1}^{M_C - 1} R_{I,i}(lT_C) \sum_{j=1}^{M_C - 1} c_{j,i}^{(1)}, c_{j-l,i}^{(1)} \right] \tag{7}$$

where $R_{I_i}(\tau)$ is the autocorrelation function of interference.

By the correlation properties of complementary sequences at any user k, $\sum_{i=1}^{M_C} c_{n,i}^{(k)} c_{n+l,i}^{(k)} = M_C \delta(l)$, and when a sinc function with unit energy $(E_C = 1)$ is used as the chip waveform, the conditional SIR_{MOC-MC} can then be written as

$$SIR_{MOC-MC} = \frac{E^2\left[Z \mid \bar{\alpha}\right]}{\text{var}\left[I_Z\right] + \text{var}\left[N_Z\right]} = \frac{2 M_C^2 E_C \mu \cos^2\theta}{M_C L_S (K-1) E_C + M_C L_S \eta_0} \tag{8}$$

where $\mu = \sum_{i=1}^{M_C} \alpha_{1,i}^2$ is a chi-square random variable with $2M_C$ degrees of freedom.

As we stated it before, $C^{(k)}(\lambda)$ is an all-zero sequence. Therefore, the total $MAI = \sum_{k=2}^{K} MAI_k$ is zero no matter what data bits the other users are transmitting. In addition, the MAI is independent of the chip waveform. The receiver output signal for user 1 has a very narrow peak at $t = T$. At all other times, the output is zero. The error probability is

$$P_g(e) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{SIR_{MOC-MC}}}^{\infty} \exp\left(\frac{-x^2}{2} \right) dx \tag{9}$$

The bit-error probability remains the same as the number of interfering users increases, assuming perfect carrier synchronization. Of course, the number of interfering users cannot increase arbitrarily, as there must be MOC sequences to accomplish them.

3 Evaluation of Partially Coherent MC/DS-CDMA with MOC Sequences

3.1 Capacity of MC/DS-CDMA with MOC Sequences in EG Combined Rayleigh Fading Channel

We consider two channels in this paper. First, we assume the system operates over an imperfect carrier-phase noise channel. Second, we analyze the system's performance in the Rayleigh slow fading channel. For user k, the impulse response of the i th frequency band is $\alpha_{k,i} e^{j\beta_{k,i}}$, where the $\alpha_{k,i}$ are independently, identically distributed (i.i.d.) Rayleigh random variables with unit second moment, so average received signal power is equal to transmitted signal power, and the $\beta_{k,i}$ are i.i.d. uniform random variables over $[0, 2\pi)$.

The channel capacity of Gaussian noise environment was an upper bound of the maximum transmission rate, and it can be expressed as

$$C = B \log_2 \left(1 + S/N\right) \tag{10}$$

where B is the channel bandwidth in Hertz and S/N is the signal to noise power ratio. This formulation is known as the Shannon-Hartley law. The S/N ratio of (10) for the fading channel is a random variable which should be replaced by SIR_{MOC-MC} of (8) for the reverse link. Therefore, the channel capacity is as follow

$$\overline{C}_{fading} = \int_0^\infty B \log_2 \left(1 + SIR_{MOC-MC}(\mu)\right) \cdot f(\mu) d\mu \tag{11}$$

where SIR_{MOC-MC} is expressed in (8) and $f(\mu)$ is the pdf of Rayleigh distribution and is given by [5], [6]

$$f(\mu) = \frac{\mu^{M_C} e^{-\mu/2} U(\mu)}{2^{M_C} (M_C - 1)!} \tag{12}$$

where $U(\mu)$ is a unit function.

3.2 BER of Partially Coherent MC/DS-CDMA System with MOC Sequences in MAI Channel

The base station receiver with partially coherent correlation type contains a random phase error. The random phase error assumed to be generated from PLL. The fading bandwidth is assumed to be much smaller than the loop bandwidth of PLL. At the output of the PLL, we have

$$\hat{\theta} = \theta + \Delta\theta \tag{13}$$

where the phase error $\Delta\theta$ have Tikhonov density function [7], [8]

$$f(\Delta\theta) = \frac{\exp\left(R\cos(\Delta\theta)\right)}{2\pi I_o(R)}, \qquad -\pi \le \Delta\theta \le +\pi \tag{14}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind and R is the loop SNR in PLL. The loop SNR R is proportional to system signal-to-noise ratio of E_b/N_0, i.e., $R = \rho * E_b/N_0$. When the loop SNR R is exceed 10 dB, $\cos(\hat{\theta})$ can be approximated by its expected value with respect to $\Delta\theta$. And this case not incur significant error. Therefore

$$\cos(\hat{\theta}) \approx E_{\Delta\theta}\left[\cos(\theta + \Delta\theta)\right] \tag{15}$$

where $E_{\Delta\theta}$ denotes the expectation with respect to the phase errors $\Delta\theta$.

The average error probability of partially coherent MC/DS-CDMA with MOC sequences in equal gain combined multipath Rayleigh fading channel is obtained as

$$P\left(e \mid k\right) = \int_0^{\infty}\int_{-\pi}^{\pi} P_g(e \mid k, \Delta\theta, \mu) f(\Delta\theta) f(\mu) d\theta d\mu \tag{16}$$

where $f\left(\Delta\theta\right)$ has the same form as Eq.(14).

Because of not every user in the cell is always transmitting simultaneously, the effect of transmission activity is include in the performance analysis. The probability that k out of K interferers are active can be described by a binomial distribution.

$$P(k) = \begin{pmatrix} K \\ k \end{pmatrix} a^k \left(1-a\right)^{K-k} \tag{17}$$

where a is the transmission activity factor. Therefore, the average error probability is

$$P(e) = \sum_{k=1}^{K} P(e \mid k) P(k) \tag{18}$$

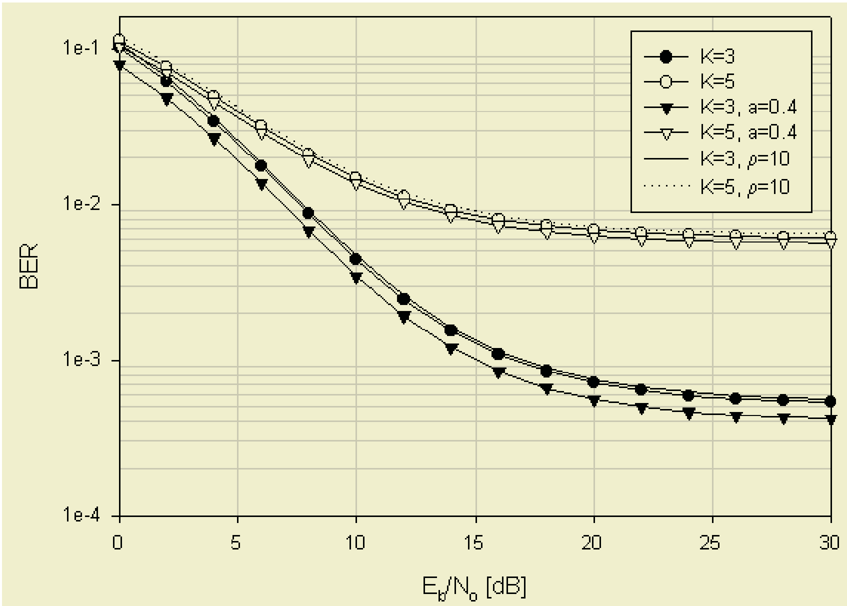

Fig. 1. Average error probability of MC/DS-CDMA using MOC sequences over the Rayleigh fading channel ($M_C = 8,\ L_S = 8$).

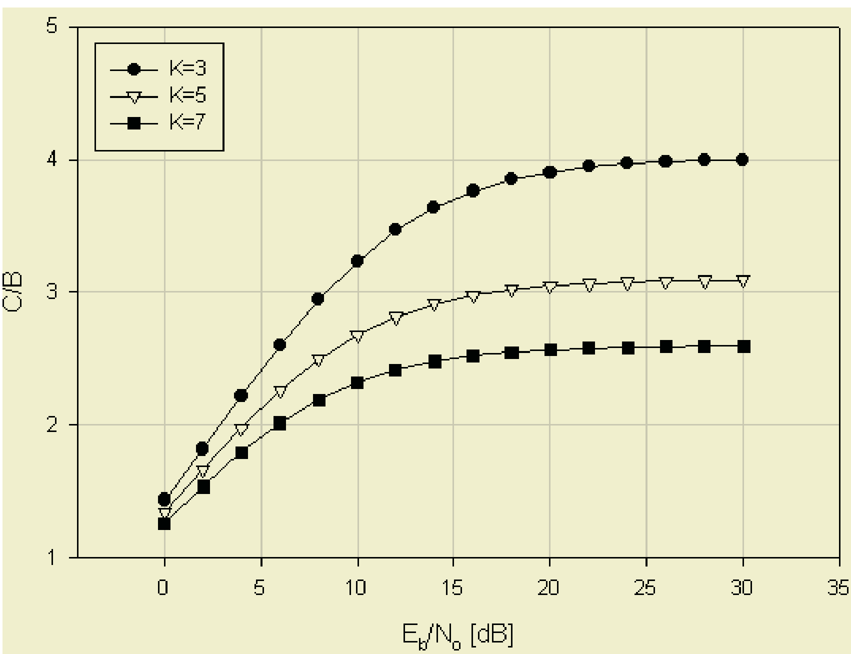

Fig. 2. Channel capacity of MC/DS-CDMA using MOC sequences according to the multiple-access user ($M_C = 8,\ L_S = 8$).

Figure 1 is numerically computed by using (18). This figure shows the BER versus the average received E_b/N_0 in the presence of Rayleigh fading, multiple-access interference, and phase noise channel. From the figure 1, we know that the effects of transmission activity and phase error in multiple-access interference channel. In fact, for a second-order PLL, R is proportional to the square of the received signal amplitude, which implies that R is proportional to E_b/N_0. Thus, a more realistic assignment for R would be $R = \rho * E_b/N_0$.

Figure 2 is obtained from (11) for Rayleigh fading and multiple-access interference channel. It is illustrated that average channel capacity of MC/DS-CDMA system for reverse link will decrease with an increase in the number of multiple-access users. And, the average channel capacity of MC/DS-CDMA system are obviously improved when the E_b/N_0 is increased.

4 Conclusion

The MC/DS-CDMA system using MOC sequences and EG combining is suited to phase noise and fading channels. In such channel, the effects of phase error and transmission activity on the system performance of a CDMA system is examined and quantified for reverse links. The closed form of SIR_{MOC-MC} derived in this paper can enable one to see the interrelationship of key system parameters, such as the number of sequence length, carriers, and multiple-access users. From the analytical results, it can be seen that the capacity and BER performance of a MC/DS-CDMA system using MOC sequences and EG combining in fading channel are degraded with an increase in the number of simultaneous users. Analytical results also show that the maximum transmission rate of a CDMA system decreases with an increase in the number of multiple-access users and transmission activity in a cell.

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