

Ernst Zermelo

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In Cooperation with Volker Peckhaus

Ernst Zermelo

An Approach to His Life and Work

With 42 Illustrations

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To the memory of
Gertrud Zermelo (1902–2003)

Preface

Ernst Zermelo is best-known for the explicit statement of the axiom of choice and his axiomatization of set theory. The axiom of choice led to a methodological enrichment of mathematics, the axiomatization was the starting point of post-Cantorian set theory. His achievements, however, did not unfold in an undisputed way. They became the object of serious criticism sparked, in particular, by the unconstructive character of the axiom of choice, making it one of the most debated principles in the history of mathematics. Zermelo defended his point of view with clear insights and discerning arguments, but also with polemical formulations and sometimes hurtful sharpness. The controversial attitude shining through here has become a dominating facet of his image. Further controversies such as those with Ludwig Boltzmann about the foundations of the kinetic theory of heat and with Kurt Gödel and Thoralf Skolem about the finitary character of mathematical reasoning support this view.

Even though these features represent essential constituents of Zermelo's research and character, they fall short of providing a conclusive description. Neither is Zermelo's major scientific work limited to set theory, nor his personality to controversial traits. His scientific interests included applied mathematics and purely technical questions. His dissertation, for example, promoted the Weierstraßian direction in the calculus of variations, he wrote the first paper in what is now called the theory of games, and created the pivotal method in the theory of rating systems. The complexity of his personality shows in his striving for truth and objectivity, and in the determination with which he stood up for his convictions. Well-educated in and open-minded about philosophy, the classics, and literature, he had the ability of encountering others in a stimulating way.

Due to serious illness, which hindered and finally ended his academic career, and due to growing isolation from the dominating directions in the foundations of mathematics, he became caught in a feeling of being denied due scientific recognition, and controversy seemed to gain the upper hand. Those close to him, however, enjoyed his other sides.

The present biography attempts to shed light on all facets of Zermelo's life and achievements. In doing so, quotations from various sources play a major role. Personal and scientific aspects are kept separate as far as coherence allows, in order to enable the reader to follow the one or the other of these threads. The discussion of Zermelo's scientific work does not require detailed knowledge of the field in question. Rather than aiming at an in-depth technical analysis of his papers, the presentation is intended to explore motivations, aims, acceptance, and influence. Selected proofs and information gleaned from drafts, unpublished notes, and letters add to the analysis.

The main text is followed by a *curriculum vitae* which summarizes the main events of Zermelo's life, now in a more schematical manner. It thus provides some kind of chronological index.

All facts presented are documented by appropriate sources. Whenever possible, English versions of German texts follow a published translation. In some special cases such as axioms, the original German version is given in the main text, as well. Original German versions which have not been published previously and whose wording may be of some importance or interest, are compiled in the appendix. In particular, the appendix contains all unpublished quotations from Zermelo himself, supplemented by samples of his literary activity.

There is no claim that the biography offers a complete picture. Rather, the description of Zermelo's life and work should be considered as an approach to a rich and multifaceted personality, which may invite the reader to take an even closer look.

When Zermelo's late wife Gertrud learnt about this project, she gave all possible support. The information and the documents which she provided with warm-hearted interest contributed in essential ways to the picture of Zermelo presented here. With deep gratitude, the biography is dedicated to her memory.

Freiburg, September 2006

Heinz-Dieter Ebbinghaus

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Editorial Remarks

Archives and *Nachlässe* which are frequently quoted are given in an abbreviated form as follows:

- ASD: Archiv der sozialen Demokratie, Bad Godesberg,
Nachlass Nelson;
- CAH: Center for American History, The University of Texas
at Austin, Dehn (Max) Papers, 3.2/86-4/Box 1;
- DMA: Deutsches Museum, Archiv;
- GSA: Geheimes Staatsarchiv Preußischer Kulturbesitz Berlin;
- SAZ: Staatsarchiv des Kantons Zürich;
- SUB: Niedersächsische Staats- und Universitätsbibliothek Göttingen;
- UAF: Universitätsarchiv Freiburg;
- UAG: Universitätsarchiv Göttingen;
- UAW: University Archives Wrocław;
- UBA: University of Buffalo Archives, Marvin Farber, Papers on
Philosophy and Phenomenology, 1920s–1980s, 22/5F/768.

The abbreviation “MLF” refers to the Abteilung für Mathematische Logik, Universität Freiburg. The corresponding documents are currently being incorporated into the Zermelo *Nachlass* held in the Universitätsarchiv Freiburg under signature C 129.

Photographs reprinted without reference to a source are contained in the Zermelo photo collection held in the Abteilung für Mathematische Logik at Freiburg University. This photo collection will also be integrated into the Zermelo *Nachlass*.

References of type OV n.xy, which follow an English translation of a German text, refer to the original German version given under the same reference in the Appendix (Chapter 7). The numbers $n = 1, 2, 3, 4$ correspond to Chapters 1 (Berlin), 2 (Göttingen), 3 (Zurich), 4 (Freiburg), respectively.

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