

# Cognitive Technologies

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# Modeling Decisions

Information Fusion  
and Aggregation Operators

With 55 Figures and 35 Tables

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To our wives, Mònica and Atsuko

and to our children,  
Martí, Aina, Meritxell  
Masaaki, Yoshifumi, Naohiro

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## Preface

*Pedra, paper i tisores  
un, dos, tres  
Jan Ken Pon  
A-i ko de sho<sup>1</sup>*

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World RPS Society [437]

Information fusion is a broad area that studies methods to combine data or information supplied by multiple sources. Aggregation operators are some of the functions that can be used for combining data.

This book is intended for those interested in methods for aggregating information and, specially, for those who need to embed such methods in applications. It constitutes an introduction to the field. The main focus is on functions that deal with numerical information although other kinds of functions (specially ones for ordinal scales) are considered as well. It is aimed at senior undergraduate and beginning graduate students of computer science, engineering, and mathematics.

This is an introductory book in the field of aggregation operators, focused on practical applications; we have tried, on the one hand, to limit the operators and results to a set of manageable size and, on the other hand, to include some descriptions and examples of such operators at work.

We have also included a few computational issues. It has to be said that although for most operators no implementation details are given, their implementation is usually straightforward. Most of the operators and methods appearing in the book have been implemented by the authors (in Java).

Due to our objective, results with a mainly mathematical interest are not included in the text. For example, only aggregation operators that combine a finite number of inputs have been studied in detail. Some definitions and results that can be useful for further study but are not relevant for real appli-

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<sup>1</sup> Rock, Paper, Scissors

cations have been included in separate figures. This is the case for definitions of fuzzy integrals of continuous functions.

## Organization

The book contains an introductory chapter, two chapters presenting some other introductory topics, and the main chapters.

The Introduction describes information integration at large, and locates aggregation operators in this setting.

Chapter 2 describes some of the tools that are needed later in the book. In particular, it focuses on measurement theory, probability and statistics, and fuzzy sets.

Chapter 3 gives an introduction to functional equations. Some well-known equations are reviewed, and a few notes on how to solve them are given.

Chapter 4 is devoted to the synthesis of judgements. It mainly reviews aggregation operators related to separability and quasi-arithmetic means, first without weights and then with them. At this point, the Bajraktarević's mean is defined. A few operators for ordinal scales are also presented.

Chapter 5 gives an overview of fuzzy measures. The most well-known families are studied: belief and plausibility and  $\perp$ -decomposable and distorted probabilities. Such fuzzy measures are later used in conjunction with fuzzy integrals.

Chapter 6 describes aggregation operators that can be expressed as particular cases of fuzzy integrals. Such operators include weighted means, OWA operators and weighted minimum and maximum. Fuzzy integrals, such as Choquet, Sugeno, t-conorm, and twofold integrals, are also defined and compared.

Chapter 7 is devoted to a few indices to evaluate aggregation operators and their parameters. This section includes descriptions for the Shapley and Banzhaf indices, interactions, average values and orness.

We finish, in Chapter 8, by considering the process of parameter determination for some particular operators, for example, for learning weights for the weighted mean and fuzzy measures for Choquet integrals. Two cases are considered, parameter determination with the help of an expert and parameter determination from examples.

To ease the reading, references have been grouped in bibliographical sections (Bibliographical Notes, at the end of each chapter). The full listing of the references is given at the end of the book. Examples have been given to illustrate the operators, and figures and tables have been included for the same purpose. In some cases, figures have been added to include some definitions or properties that have less interest for practical application (e.g., definitions of some fuzzy integrals in continuous domains). The book finishes with an Appendix where the main properties and some aggregation operators are listed. The lists are not exhaustive.

## How to Use This Book

The book does not assume specific previous knowledge of aggregation operators, and Chapters 2 and 3 give some preliminaries to make it self-contained. Although the chapters have been written to avoid dependences as much as possible, there are some dependences between chapters. The most important relationships are enumerated here. Chapter 4 uses functional equations reviewed in Chapter 3, and Chapter 6 defines fuzzy integrals that use the fuzzy measures described in Chapter 5. Evaluation methods (Chapter 7) are based on the particular operators and the particular parameters explained in previous chapters (e.g., Shapley value for a fuzzy measure). The problem of parameter determination for a given operator (Chapter 8) naturally needs the operator under consideration (described in previous chapters). Nevertheless, to prevent the reader from going back and forth, there are minor repetitions in the text.

The following equation is the most repeated one:

$$\min_i a_i \leq \mathbb{C}(a_1, \dots, a_N) \leq \max_i a_i$$

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The actual content of the book has been shaped by our own research and joint work in the last few years. The series of conferences we initiated on Modeling Decisions for Artificial Intelligence (2004-present) [412, 415, 416], has also, through its participants, influenced this work. Special thanks go to Professors J. Dujmović and R. Mesiar.

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Ultimately, the authors are fully responsible for all errors and omissions in this book.

Sabadell (Catalonia) and Kunitachi (Japan),  
April 23rd, 2007 (St. Jordi's Day)

Vicenç Torra  
Yasuo Narukawa



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# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Fusion and Integration	5
1.2	An Architecture for Information Integration	7
1.3	Information Fusion Methods	9
1.3.1	Function Construction	10
1.4	Goals of Information Fusion	11
1.5	Bibliographical Notes	14
<b>2</b>	<b>Basic Notions</b>	19
2.1	Measurement Theory	19
2.1.1	Measurement	21
2.1.2	Representation and Uniqueness Theorems	22
2.1.3	Uniqueness Theorems and Scale Type	24
2.2	Probability and Statistics	25
2.2.1	Random Variables	28
2.2.2	Expectation and Moments	29
2.2.3	Independence	30
2.2.4	Parametric Models and Nonparametric Methods	33
2.2.5	Regression	34
2.2.6	Robust Statistics	39
2.2.7	M- and L-Estimators	45
2.2.8	Robust Regression	46
2.3	Fuzzy Sets	49
2.3.1	Operations on Fuzzy Sets	53
2.3.2	Implications	58
2.3.3	Fuzzy Relations	59
2.3.4	Truth Degrees	60
2.3.5	Fuzzy Inference Systems	62
2.4	Bibliographical Notes	64

<b>3</b>	<b>Introduction to Functional Equations</b>	67
3.1	Basic Functional Equations	69
3.2	Using Functional Equations for Information Fusion	74
3.3	Solving Functional Equations	77
3.4	Bibliographical Notes	79
<b>4</b>	<b>Synthesis of Judgements</b>	81
4.1	Associativity	81
4.1.1	Uninorms and Nullnorms	84
4.2	Separability: the Quasi-arithmetic Means	90
4.3	Aggregation and Measurement Scales	93
4.3.1	Ordinal Scales	97
4.3.2	Different Data in Different Scales	99
4.4	Weighted Means	101
4.4.1	Bajraktarević's Means	102
4.5	Bibliographical Notes	103
<b>5</b>	<b>Fuzzy Measures</b>	111
5.1	Definitions, Interpretations, and Properties	112
5.1.1	Interpretations	115
5.1.2	Properties	118
5.2	Belief and Plausibility Measures	120
5.2.1	Belief Measures from Unconstrained Ones	122
5.2.2	Possibility and Necessity Measures	124
5.3	$\perp$ -Decomposable Fuzzy Measures	126
5.3.1	Sugeno $\lambda$ -measures	127
5.3.2	Hierarchically $\perp$ -Decomposable Fuzzy Measures	129
5.4	Distorted Probabilities	134
5.4.1	$m$ -Dimensional Distorted Probabilities	137
5.4.2	Properties	139
5.5	Bibliographical Notes	142
<b>6</b>	<b>From the Weighted Mean to Fuzzy Integrals</b>	147
6.1	Weighted Means, OWA, and WOWA Operators	147
6.1.1	Properties	148
6.1.2	Interpretation of Weighting Vectors in WM and OWA	150
6.1.3	The WOWA Operator	154
6.1.4	OWA and WOWA Operators and Fuzzy Quantifiers	159
6.2	Choquet Integral	163
6.2.1	Construction of Choquet Integral	165
6.2.2	Properties	167
6.3	Weighted Minimum and Weighted Maximum	171
6.3.1	Properties of Weighted Minimum and Maximum	175
6.3.2	Dealing with Symbolic Domains	175
6.4	Sugeno Integrals	175

6.4.1	Properties .....	180
6.5	Fuzzy Integrals .....	182
6.5.1	The Fuzzy $t$ -Conorm Integral .....	182
6.5.2	Twofold Integral .....	187
6.6	Hierarchical Models for Aggregation .....	189
6.7	Bibliographical Notes .....	191
<b>7</b>	<b>Indices and Evaluation Methods .....</b>	<b>197</b>
7.1	Indices of Power: Shapley and Banzhaf Power Indices .....	198
7.1.1	Shapley Value .....	199
7.1.2	Characterization of the Shapley Value .....	200
7.1.3	Banzhaf Value .....	201
7.1.4	Properties .....	203
7.2	Interaction .....	203
7.3	Dispersion .....	204
7.3.1	Entropy for Fuzzy Measures .....	205
7.4	Average Values .....	207
7.5	Orness or the Degree of Disjunction .....	207
7.5.1	Orness for Fuzzy Quantifiers .....	209
7.5.2	Pointwise Orness: Orness Distribution Function .....	212
7.5.3	Interpretation .....	213
7.6	Bibliographical Notes .....	214
<b>8</b>	<b>Selection of the Model .....</b>	<b>219</b>
8.1	Analytic Hierarchy Process .....	220
8.2	OWA Weights from Orness .....	223
8.2.1	Orness and Dispersion .....	224
8.3	Extracting Parameters from Examples: Expected Outcome ..	226
8.3.1	Weighted Mean .....	229
8.3.2	OWA Operators .....	235
8.3.3	The WOWA Operator .....	236
8.3.4	Choquet Integral .....	237
8.4	Extracting Parameters from Examples: Preferences or Partial Orders .....	242
8.5	Analysis .....	243
8.6	Bibliographical Notes .....	244
<b>A</b>	<b>Properties .....</b>	<b>249</b>
<b>B</b>	<b>Some Aggregation Operators .....</b>	<b>251</b>
	<b>References .....</b>	<b>253</b>