Protective Measurements

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Protective measurement [1] is a method for measuring an expectation value of an observable on a single quantum system. The quantum state of the system can be protected by a potential, when the state is a nondegenerate energy eigenstate with a known gap to neighboring states, or via Zeno effect by frequent projection measurements.

Apart from protection, the procedure consists of a standard von Neumann measurement with weak coupling which is switched on and, after a long time, switched off, adiabatically. The interaction Hamiltonian for protective measurement of O is:

$$H_{int} = g(t)PO,\tag{1}$$

where P is a momentum conjugate to Q, the pointer variable of the measuring device. The interaction Hamiltonian is small as in *weak measurements* [2]. In both cases the initial state of the pointer is such that $\langle Q \rangle_{in} = 0$, $\langle P \rangle_{in} = 0$. In weak measurement, the weakness is due to small uncertainty in P which requires a large uncertainty of the pointer variable Q. Thus, although for the final wave function of the pointer, $\langle Q \rangle_{fin} = \langle \Psi | O | \Psi \rangle$, a single measurement does not allow obtaining significant information about $\langle \Psi | O | \Psi \rangle$. In protective measurement, the pointer is well localized at zero, which requires large uncertainty in P and the weakness is due to a small value of the coupling q(t). The coupling to the measurement device is weak, yet long enough so that we still have $\int g(t)dt = 1$. The result is again $\langle Q \rangle_{fin} = \langle \Psi | O | \Psi \rangle$, but this time, the pointer is well localized, so we can learn the value of the expectation value from a single experiment. This is so if during the measurement, the quantum state of the system remains close to $|\Psi\rangle$. Given the adiabatic switching of the measurement interaction, its small value, and the protection of the state, this is indeed the case.

One of the basic results of quantum mechanics is that when a measurement of a variable O with eigenvalues o_i is performed on a quantum system described by the state $|\Psi\rangle$, the probabilities p_i for obtaining outcome o_i satisfy:

$$\langle \Psi | O | \Psi \rangle = \sum p_i o_i. \tag{2}$$

This is why the expression $\langle \Psi | O | \Psi \rangle$ is called the expectation value of O. In protective measurements we obtain this value not as a statistical average, but as a reading of a measuring device coupled to a *single* system.

A sufficient number of protective measurements performed on a single system allow measuring its quantum wave function. This provides an argument against the claim that the quantum wave function has a physical meaning only for an ensemble of identical systems. Therefore, protective measurements have some merit even when the protection is achieved via frequent projection measurements on the state $|\Psi\rangle$ with no new information obtained during the whole procedure. If the protection of the state is via a known energy gap to any orthogonal state, then the protection measurement provides new information: we can find the whole wave function. Thus, protective measurement of the quantum wave function of an ion in a trap can yield the the trap's potential.

Numerous objections to the validity and meaning of protective measurements have been raised [4, 5, 6, 7, 8]. The validity of the result was questioned due to misunderstanding of what the protective measurement is [9, 10, 11]. The issue of meaning: "Is the wave function of a single particle an ontological entity?" [3] is open to various interpretations. Some will say 'yes' even before hearing about protective measurement, others say 'no' just because protective measurements are never 100% reliable. The protective measurement procedure is not a proof that we should adopt one interpretation instead of the other, but it is a good testbed which shows advantages and disadvantages of various interpretations. For example, the Bohmian interpretation does not provide a natural explanation of how a protective measurement can "draw" the whole wave function of an ion in a ground state of a trap, since the Bohmian position of the ion hardly changes during the measurement [12, 13].

The protective measurements method can be extended to pre- and postselected systems described by a two-state vector $\langle \Phi | | \Psi \rangle$ [14]. It requires separate different protections for the forward and backward evolving quantum states which are achieved by pre- and post-selection of quantum states of systems which provide the protection [15]. The outcome of such protective measurements is not the expectation value, but the *weak value*, $\frac{\langle \Phi | O | \Psi \rangle}{\langle \Phi | \Psi \rangle}$ [2]. A realistic setup for such protective measurement is a weak coupling to a variable of a decaying system which is post-selected not to decay [16].

Theoretical analysis of protective measurements leads to deeper understanding of quantum reality while its experimental realization (which seems feasible in a near future) might be useful for more effective gathering of information about quantum systems [17].

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