Baoding Liu

Uncertainty Theory

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Editor-in-chief
Prof. Janusz Kacprzyk
Systems Research Institute
Polish Academy of Sciences
ul. Newelska 6
01-447 Warsaw
Poland
E-mail: kacprzyk@ibspan.waw.pl

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Dr. Baoding Liu
Uncertainty Theory Laboratory
Department of Mathematical Sciences
Tsinghua University
Beijing 100084
China
liu@tsinghua.edu.cn

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Preface

It was generally believed that the study of probability theory was started by Pascal and Fermat in 1654 when they succeeded in deriving the exact probabilities for certain gambling problem. Great progress was achieved when Von Mises initialized the concept of sample space, and filled the gape between probability theory and measure theory in 1931. An axiomatic foundation of probability theory was given by Kolmogoroff in his *Foundations of Probability Theory* of 1933. Since then, probability theory has been developed steadily and has been widely applied in science and engineering. Probability theory will be introduced in Chapter 2.

Fuzzy set was initiated by Zadeh via membership function in 1965, and was well developed and applied in a wide variety of real problems. In order to measure a fuzzy event, Zadeh proposed the concept of possibility measure in 1978. Although possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu and Liu gave the concept of credibility measure in 2002. Credibility theory is a branch of mathematics that studies the behavior of fuzzy phenomena. An axiomatic foundation of credibility theory was given by Liu in his *Uncertainty Theory* of 2004. Chapter 3 will provide the credibility theory.

Sometimes, fuzziness and randomness simultaneously appear in a system. In order to describe this phenomena, a fuzzy random variable was presented by Kwakernaak as a random element taking "fuzzy variable" values in 1978, and a random fuzzy variable was proposed by Liu as a fuzzy element taking "random variable" values in 2002. More generally, a hybrid variable was introduced by Liu as a measurable function from a chance space to the set of real numbers in 2006. Fuzzy random variable and random fuzzy variable are instances of hybrid variable. Chance theory is a hybrid of probability theory and credibility theory. Chapter 4 will introduce the chance theory.

Uncertainty theory is a branch of mathematics based on normality, monotonicity, self-duality, and countable subadditivity axioms. It provides the commonness of probability theory, credibility theory and chance theory. Chapter 5 will present the uncertainty theory.

For this new edition the entire text has been totally rewritten, and the chapters on chance theory and uncertainty theory are completely new. The book is suitable for mathematicians, researchers, engineers, designers, and students in the field of mathematics, information science, operations research, industrial engineering, computer science, artificial intelligence, and manage-

ment science. The readers will learn the axiomatic approach of uncertainty theory, and find this work a stimulating and useful reference.

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Baoding Liu
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http://orsc.edu.cn/liu

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