A Study on Haptic Interaction and Simulation of Motion and Deformation of Elastic Object

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Abstract. An approach to haptic interaction and simulation of motion and deformation of elastic object is proposed. In our approach, the motion of an object is defined by changes of kinetic momentum and angular momentum after impulse force is applied to each degree of freedom, and the deformation of the object is defined by a set of temporal deformation patterns after impulse force is applied to each degree of freedom, which we call extended impulse response deformation model. The time complexity of computing interaction force is independent of the complexity of the model. This feature is advantageous for time-critical applications. Also, the time complexity of computing object deformation is linearly proportional to the complexity of the model. Through implementation of a prototype environment and evaluation of its performance, the feasibility of the proposed approach is demonstrated.

Keywords: Elastic objects, deformation model, dynamic motion, record and retrieval approach.

1 Introduction

Simulation of deformable object is one of the greatest interests in the field of Virtual Reality. Escpecially in the field of haptics, presentation of force in the moving and deforming interaction is a challenging topic. A difficulty of the problem is the amount of computation to simulate the behavior of deformable objects.

One way to avoid the problem of the computational complexity of such simulations is to employ the record and retrieval (or record reproduction) approach [7,4,5,2], where the behavior of an object under interaction is measured on a real object or pre-computed off line using a precise computation algorithm and then the data is used to generate a reaction of the virtual object in a real-time application. In a previous paper, we proposed an impulse response deformation model using this record and retrieval approach [6]. This model is a potential candidate because of the low computational complexity of calculation of interaction force, however, this approach cannot take into account the motion of the object.

In this paper, we propose an extending method for the impulse response deformation model to solve the problem of considering the object's motion. In our

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approach, advantages of the impulse response deformation model are taken over, moreover motion of the object is considered.

Our approach enables haptic interaction with dynamic deformable and movable object that is too complex to be solved by a common Finite Element Method (FEM). Also, an analysis on computational complexity suggests that our approach becomes more advantageous than FEM as the complexity of the model increases.

In the next section, the concept of our approach is discussed and an outline of the implementation is described. Experimental systems and results will be presented in Section 3.

2 Extended Impulse Response Deformation Model

In previous paper, we proposed the impulse response deformation model[6]. In our approach, the behavior of an object is defined by a set of temporal deformation patterns after impulse force is applied to each degree of freedom. Deformation resulting from interaction is obtained by computing convolution of the model and the history of interaction force.

The idea of the impulse response deformation model is based on the premise that the model is linear, which means that the influences caused by impulse forces on different degrees of freedom or at different times are independent of each other, and the resulting deformation is computed as the sum total of the influences. This assumption makes it possible to perform haptic interaction with dynamically deformable and complex object in real-time. However, part of the object must be fixed to a floor, a wall, etc., in order to avoid computational complexity of changing boundary conditions and nonlinerity resulting from rotary motion.

In this study, to solve these problems, the following a novel assumption is introduced.

2.1 Separating Impulse Response into Component of Deformation and Motion

Suppose an object is remainning stationary under gravity free conditions. When a unit impulse force is applied to some degree of freedom on the object, definite amounts of kinetic momentum and angular momentum are produced. Although component of the object's translation is proportional to the input force, component of the object's rotation has nonlinearity resulting from change of inertia matrix and centrifugal force. If the angular rate of the object is not large, effect of the nonlinearity is able to assume negligible small. At the same time, this assumption makes it possible to improve degrees of freedom of interaction dramatically.

In our apprpach, this assumption is introduced. In this assumption, when a unit impulse force is applied to some degree of freedom on the object, definite amounts of acceleration $\ddot{x}$ and angular acceleration $\ddot{\theta}$ are produced. Using this assumption, deformations after the impulse force is applied are separated into components of deformation and motion.

2.2 Deformation Model

In our approach, the following deformation model based on a idea of impulse response is used.

One Dimensional Model. In case of a one-dimensional continuous system, the relationship between the force $f(t)$ and displacement $u(t)$ is described using impulse response $r(s)$ by

$$u(t) = \int_0^\infty r(s) f(t-s) ds. \tag{1}$$

In case when $f(t)$ is a delta function at $t = 0$, it becomes identical to $r(t)$.

With the convenience for digital processing, we use a temporary discrete model. In the following explanation, we use s and t as indexes of discrete time, and the time for each value is indicated in a superscript with square brackets. We also confine the duration of the impulse response in a limited time T. Then the one-dimensional model is described as follows:

$$u^{[t]} = \sum_{s=0}^{T-1} r^{[s]} f^{[t-s]}. \tag{2}$$

In a precise sense, if we consider acceleration and angular acceleration of the object after the impulse force is applied, equation (2) becomes as follows:

$$u^{[t]} = \sum_{s=0}^{T-1} (r^{[s]} + \ddot{x}^{[s]} \cdot \Delta t^2 + l^{[s]} \ddot{\theta}^{[s]} \Delta t^2) f^{[t-s]}, \tag{3}$$

where $l^{[s]}$ is distance between the center of gravity of the object and point of application of the force, and Δt is time steps of the simulation. In usual haptic interaction, Δt becomes very small, therefore terms of $\ddot{x}^{[s]} \cdot \Delta t^2$ and $l^{[s]} \ddot{\theta}^{[s]} \Delta t^2$ are negligible. So, we used the approximate model described by equation (2).

In the interaction using a haptic interface device, or more precisely in the case when position-input/force-output device is used, the interface point (i.e. a representative point through which user interacts with object) of the device causes a displacement boundary condition on the deformation model. In other words, $u^{[t]}$ is given and $f^{[t]}$ is unknown. The unknown force is obtained by solving:

$$u^{[t]} = r^{[0]} f^{[t]} + \tilde{u}^{[t]}, \tag{4}$$

where $\tilde{u}^{[t]}$ is the current displacement derived from the past force and it is formulated by:

$$\tilde{u}^{[t]} = \sum_{s=1}^{T-1} r^{[s]} f^{[t-s]}. \tag{5}$$

Since all past force is known at the present time, the value of $\tilde{u}^{[t]}$ can be computed.

2.3 Multi Degrees of Freedom Model

Suppose we have a n degrees of freedom system. We describe the force and displacement on all degrees of freedom by vectors $F^{[t]}$ and $U^{[t]}$ $(n \times 1)$, respectively. Also, we describe the impulse response by a matrix $R^{[s]}$ $(n \times n)$, because the impulse response is defined for all combinations of n input impulses and n output displacements. Then their relationship is written by:

$$\begin{aligned} U^{[t]} &= \sum_{s=0}^{T-1} R^{[s]} F^{[t-s]} \\ &= R^{[0]} F^{[t]} + \tilde{U}^{[t]}, \end{aligned} \tag{6}$$

where

$$\tilde{U}^{[t]} = \sum_{s=1}^{T-1} R^{[s]} F^{[t-s]}. \tag{7}$$

In a usual deformation operation, the displacement boundary condition is given to some degrees of freedom, on which current displacement is given and the force is unknown. On other degrees of freedom, the current displacement is unknown and the force is not applied (or 0). To formulate this relationship, we introduce a description separated by the boundary conditions, as follows:

$$\begin{pmatrix} U_o^{[t]} \\ U_c^{[t]} \end{pmatrix} = \begin{pmatrix} R_{oo}^{[0]} & R_{oc}^{[0]} \\ R_{co}^{[0]} & R_{cc}^{[0]} \end{pmatrix} \begin{pmatrix} F_o^{[t]} \\ F_c^{[t]} \end{pmatrix} + \begin{pmatrix} \tilde{U}_o^{[t]} \\ \tilde{U}_c^{[t]} \end{pmatrix}, \tag{8}$$

where suffix c and o indicates the degrees of freedom on which the displacement boundary condition is applied and not applied, respectively. By solving the equation, the unknown values of $F_c^{[t]}$ and $U_o^{[t]}$ are obtained by

$$F_c^{[t]} = (R_{cc}^{[0]})^{-1} (U_c^{[t]} - \tilde{U}_c^{[t]}), \tag{9}$$

$$U_o^{[t]} = R_{co}^{[0]} F_c^{[t]} + \tilde{U}_o^{[t]}. \tag{10}$$

In the experiments discussed below, the proposed approach is applied to an object model consisting of triangular patches, and interaction with one or two interface points or one plane is investigated. In point interaction, since each node has three degrees of freedom, one or two interface points interacting with one or two nodes cause displacement boundary condition on those nodes, respectively (i.e. $n_c = 3$ or 6). In the case when the interface points are interacting with triangle patches, rather than with nodes, we approximately determine the interaction force that is distributed to the vertex nodes of the triangle patches. We employed an interpolation approach that is similar to those used by Hirota et al[**?**], where interpolated interaction force is computed by repeatedly using the formula of one- and two-node interaction.

2.4 Motion Model

In our approach, motion of the object is described by Euler method, as follows:

$$\dot{x}^{[t+1]} = \dot{x}^{[t]} + \ddot{x}^{[t]} \cdot \Delta t \tag{11}$$

$$x^{[t+1]} = x^{[t]} + \dot{x}^{[t+1]} \cdot \Delta t \tag{12}$$

$$\dot{\theta}^{[t+1]} = \dot{\theta}^{[t]} + \ddot{\theta}^{[t]} \cdot \Delta t \tag{13}$$

$$\theta^{[t+1]} = \theta^{[t]} + \dot{\theta}^{[t+1]} \cdot \Delta t. \tag{14}$$

2.5 Computational Complexity

The computation of solving the equation of the deformation model becomes easy as the degree of freedom of displacement boundary condition (n_c) is small through time. Since only n_c factors of $F^{[t]}$ have non-zero values for any given t, the values of $\tilde{U}_c$ and $\tilde{U}_o$ are computed by $O(n_c^2 \cdot T)$ and $O(n \cdot n_c \cdot T)$, respectively. $R_{cc}^{[0]}$ becomes a matrix of $n_c \times n_c$, hence the Equation 9 is solved by $O({n_c}^3)$ even if it is solved by Gauss elimination method. Most memory resource is consumed to store the impulse response matrix $R^{[s]}$, which requires $O(n^2 \cdot T)$, and some more area is needed for storing past forces, $O(n_c \cdot T)$.

The computation of solving the equation of the motion model requires $O(N)$, where N is the number of the objects. This is negligible small, under the condition that the number of the objects is small.

3 Experiment

3.1 Pre-computation of Deformation Model

Pre-computation is a process where the measurement of an impulse response is performed by simulation. A unit force is applied for a unit time on each degree of freedom and displacements of all degrees of freedom are recorded. By repeating this process on all degrees of freedom, the impulse response matrix $R^{[s]}$ is obtained. The dynamic behavior of the object was simulated using a FEM model consisting of tetrahedral elements.

We employed a *cube* model, as shown in Figure 1. The complexity of the model is listed in Table 1. Also, Young's modulus $E = 2000\text{N/m}^2$, Poisson's ratio $\nu = 0.49$, density $\rho = 110\text{kg/m}^3$. The length of a side of the model is 10cm. All through the experiments described in this paper, the sampling frequency of the impulse response was set to 500Hz and the duration of response 1s, hence we obtained 500 samples for each element of an impulse response matrix (or

Table 1. Complexity of model

free nodes (n)	866
triangle patches	1728
entire nodes	2197
tetrahedral elements	8640
pre-computation time per d.o.f. (s)	92.8
data size per d.o.f. (MB)	9.9

$T = 500$). In the pre-computation process, however, to improve accuracy in relatively quick deformation followed immediately after the affection of force, the time step for the simulation was set to 1ms, and then averaged them for every 2ms. The computation time to obtain impulse response matrix for each model is also listed in the table, where the computation was performed by a PC (CPU:Itanium2 1.4GHz×4).

Figure 2 is an example of impulse response in the case when a downward impulse force by a point is applied to a node on the top surface. Elastic wave caused by the impulse force is propagating from the point where the impulse is applied, and in this example, it is taking approximately 16ms before entire nodes in the model start causing displacement. Also, Figure 3 is an example of impulse response in the case when downward impulse forces by a plane are applied to all nodes on the top surface.

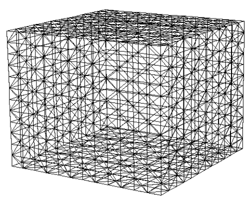

Fig. 1. Experimental model

3.2 Experimental System

The experimental system consists of two PCs and two PHANToM devices[3], as shown in Figure 4. All of the computation related to the model is performed by PC1 (CPU:Itanium2 1.4GHz×4, memory:16GB, OS:Linux); the computation of force and deformation are executed as separate processes using a thread mechanism.

In the *force* process, the following tasks are performed: the information of interface points of the devices is received from the *haptic update* process on PC2 through an Ethernet; collision between the interface points and the object surface is detected and the displacement cause by those points is determined; the impulse

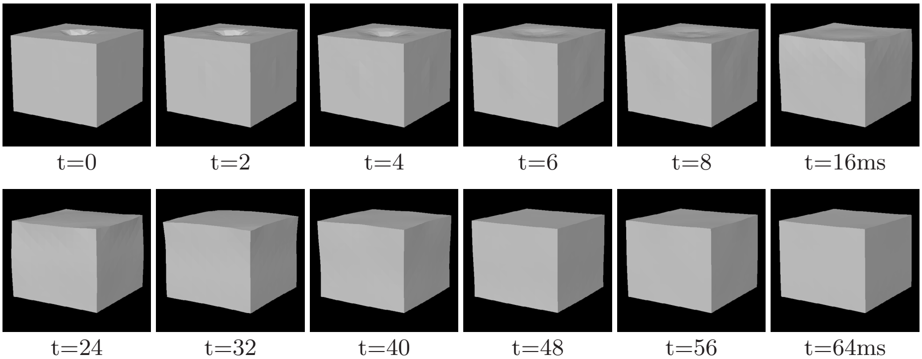

Fig. 2. Examples of impulse response (in the case when a downward impulse force by a point is applied to a node on the top surface)

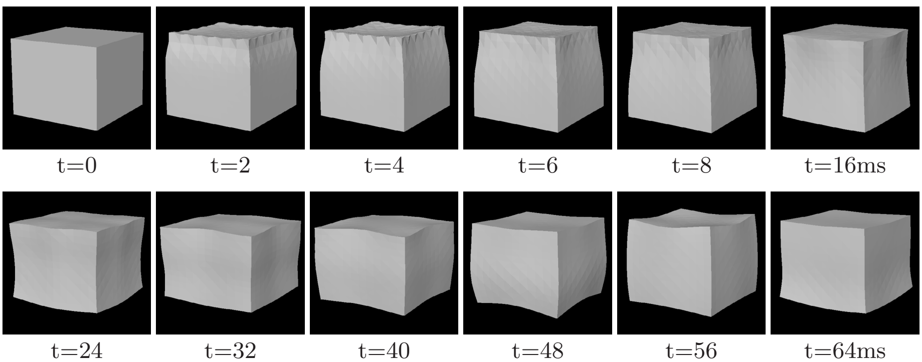

Fig. 3. Examples of impulse response (in the case when downward impulse forces by a plane are applied to all nodes on the top surface)

force (or force boundary condition at each time) and feedback force is computed and output to the haptic update process; the history of impulse forces is updated. The *force* process repeats this computation every 2ms. The *deformation* process computes the deformation of the entire model using the history of impulse forces, which is passed from the *force* process through a shared memory. The process runs asynchronously from the *force* process except for minimum exclusive access control while the *force* process is writing to the shared memory.

Computation of collision was carried out by using the God-object Method[8]. This algorithm is advantageous for our purpose in that it is capable of providing the position of the collision (or ideal interface point) and that the displacement on the point can be computed as the relative position of the interface point from the ideal interface point.

The program on PC1 was developed using Intel Compiler and Performance Libraries [1]; the process of computing the current deformation caused by the past forces (i.e. $\tilde{\boldsymbol{U}}_c^{[t]}$) was parallelized using OpenMP Compiler allotting three CPUs. Also, the computation of *deformation* process was optimized using Math Kernel Library. PC2 (CPU:Pentium3 500MHz×2, OS:Windows) is considered as the controller for two PHANToM devices; the *haptic update* process serves just for converting the interface from GHOST API to the TCP/IP connection interface. GHOST library was used for basic interface with the device; the GHOST main loop runs at 1kHz, and in the main loop, the latest value of force that has been received from the force process is passed to the device and the position of interface point is updated.

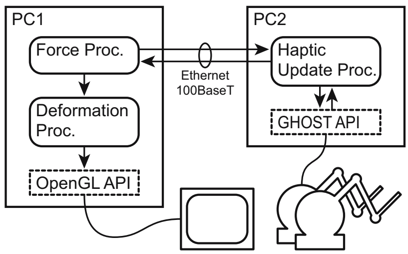

Fig. 4. Experimental system

3.3 Experimental Results

Some examples of interaction are shown in Figure 5, where time-series images of dynamic deformation are listed. Since it is difficult to store every frame of screen image in real-time, these images were generated off line based on the log of the sequence of the interaction force.

Figure 5(a) shows the case where the *cube* model is colliding and bouncing on a floor after free falling. Motion and deformation resulting from collision and inertia force is represented and relatively fast motion of the restoring process is presented.

An example of interaction cooperatively using two interface points is shown by Figure 5(b), where the user is holding both the left and the right side of *cube* model by both the right and the left interface points, and the user is shaking the model at 2Hz from side to side under gravity free conditions. Displacement on the left and right interface point causes motion and deformation of the *cube* model and consequently changes the forces on the left and right interface points.

The computation on interaction force required at most approximately 0.5ms, which is sufficiently fast for usual force feedback application. The entire *force* process including collision detection and communication was completed within each cycle-time of 2ms.

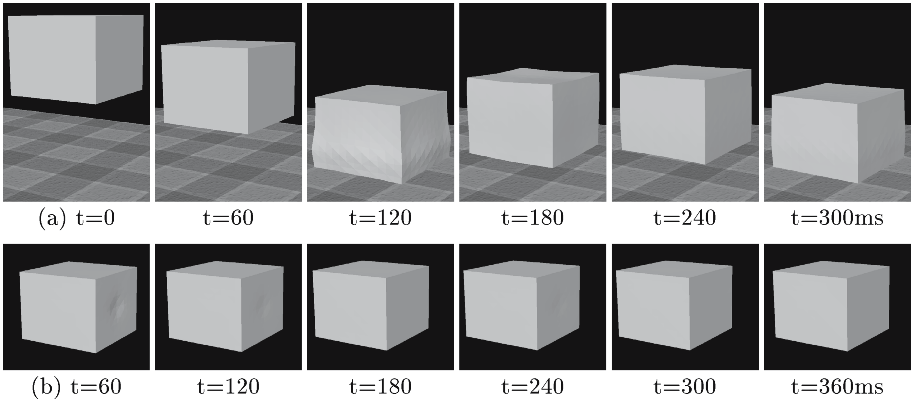

Fig. 5. Examples of dynamic motion and deformation

4 Conclusion

We presented the concept of an extended impulse response deformation model. The model provides an approach to simulating motion and deformation of elastic objects and computes the interaction force at haptic rate, which was barely attained by other methods such as FEM. We implemented a prototype interaction system and demonstrated the feasibility and performance of the proposed approach.

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