Diagnosability of Fuzzy Discrete Event Systems

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***Abstract*— In order to more effectively cope with the real-world problems of vagueness, *fuzzy discrete event systems* (FDESs) were proposed recently, and the supervisory control theory of FDESs was developed. In view of the importance of failure diagnosis, in this paper, we present an approach of the failure diagnosis in the framework of FDESs. More specifically: (1) We formalize the definition of diagnosability for FDESs, in which the observable set and failure set of events are *fuzzy*, that is, each event has certain degree to be observable and unobservable, and, also, each event may possess different possibility of failure occurring. (2) Through the construction of observability-based diagnosers of FDESs, we investigate its some basic properties. In particular, we present a necessary and sufficient condition for diagnosability of FDESs. (3) Some examples serving to illuminate the applications of the diagnosability of FDESs are described. To conclude, some related issues are raised for further consideration.**

***Index Terms*— Discrete event systems, failure detection, fault diagnosis, fuzzy finite automata.**

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I. Introduction

A *discrete event system* (DES) is a dynamical system whose state space is discrete and whose states can only change as a result of asynchronous occurrence of instantaneous events over time. Up to now, DESs have been successfully applied to many engineering fields [4]. In most of engineering applications, the states of a DES are crisp. However, this is not the case in many other applications in complex systems such as biomedical systems and economic systems. For example, it is vague when a man's condition of the body is said to be "good". Moreover, it is imprecise to say at what point exactly a man has changed from state "good" to state "poor". Therefore, Lin and Ying [18,19] initiated significantly the study of *fuzzy discrete event systems* (FDESs) by combining fuzzy set theory with crisp DESs. Notably, FDESs have been applied to biomedical control for HIV/AIDS treatment planning [20,21]. And R. Huq *et al* have presented a novel intelligent sensory information processing using FDESs for robotic control recently [10, 11].

As Lin and Ying [19] pointed out, a comprehensive theory of FDESs still needs to be set up, including many important concepts, methods and theorems, such as controllability, observability, and optimal control. These issues have been partially investigated in [2, 3, 28]. Qiu [28] established the supervisory control theory of FDESs, and found a method of checking the existence of supervisors for FDESs; and independently, Cao and Ying [2, 3] significantly developed FDESs. As a continuation, this paper is to deal with the failure diagnosis for FDESs.

It is well known that the issues of diagnosability for DESs are of practical and theoretical importance, and have received extensive attention in recent years [5-9,12,13,15-17,23-27,29-39]. However, the observability and the failure set of events in the literature are usually *crisp*. Motivated by the fuzziness of observability for some events in real-life situation, in this paper, the observable set and failure set of events are *fuzzy*. That is, each event has certain degree to be observable and unobservable, and, also, each event may possess different possibility of failure occurring. We formalize the definition of diagnosability for FDESs using the fuzzy observable set and the fuzzy failure set of events.

Generally speaking, a fuzzy language generated by a fuzzy finite automaton is said to be diagnosable if, based on the degree of observability and the possibility of failure occurring on events, the occurrence of failures can be always detected within a finite delay according to the observed information of the traces. Through the construction of observability-based diagnosers of FDESs, we investigate some basic properties concerning the diagnosers. In particular, we present a necessary and sufficient condition for diagnosability of FDESs, that is, a fuzzy language is F_i-diagnosable if and only if there are no F_i-indeterminate cycles in the diagnoser with respect to each event. Our results may better deal with the problems of fuzziness, impreciseness and subjectivity in the failure diagnosis, and, generalize the important consequences in classical DESs introduced by Sampath *et al* in their seminal works [31, 32]. In order to illustrate the applications of the diagnosability of FDESs, some examples are provided to illuminate the results derived.

This paper is organized as follows. Section II recalls some preliminaries and notations concerning FDESs. In Section III, an approach to defining diagnosability for FDESs is presented. In Section IV, we construct the observability-based diagnosers of FDESs, and some main properties of the diagnosers are investigated. In particular, we present a necessary and sufficient condition for diagnosability of FDESs. Finally, some examples are provided to illustrate the condition of diagnosability for FDESs in Section V. To conclude, in Section VI, we summarize the main results of the paper and address some related issues.

II. Preliminaries

In this section, we briefly recall some preliminaries regarding fuzzy finite automata. For a detailed introduction, we may refer to [18, 19, 28].

In the setting of FDESs, a fuzzy state is represented as a vector $[a_1, a_2, \cdots, a_n]$, which stands for the possibility distributions over crisp states, that is, $a_i \in [0,1]$ represents the possibility that the system is in the ith crisp state, ($i = 1, 2, \cdots, n$). Similarly, a fuzzy event is denoted by a matrices $\sigma = [a_{ij}]_{n\times n}$, and $a_{ij} \in [0,1]$ means the possibility for the system to transfer from the ith crisp state to the jth crisp state when event σ occurs, and n is the number of all possible crisp states. Hence, a fuzzy finite automaton is defined as follows.

Definition 1 [28]: A *fuzzy finite automaton* is a fuzzy system

$$G = (Q, \Sigma, \delta, q_0),$$

where Q is the set of some state vectors (fuzzy states) over crisp state set; q_0 is the initial fuzzy state; Σ is the set of matrices (fuzzy events); $\delta : Q \times \Sigma \to Q$ is a transition function which is defined by $\delta(q, \sigma) = q \odot \sigma$ for $q \in Q$ and $\sigma \in \Sigma$, where $\odot$ denotes the *max-min* operation in fuzzy set theory [14].

Remark 1: The transition function δ can be naturally extended to $Q \times \Sigma^*$ in the following manner:

$$\delta(q, \epsilon) = q, \quad \delta(q, s\sigma) = \delta(\delta(q, s), \sigma),$$

where Σ^* is the Kleene closure of Σ, ϵ denotes the empty string, $q \in Q$, $\sigma \in \Sigma$ and $s \in \Sigma^*$. Moreover,

δ can be regarded as a partial transition function in practice. In biomedical engineering [20], for example, although many treatments (fuzzy events) are available for a patient, but in fact, only one or a few treatments are adopted by doctors according to the patient's conditions (fuzzy states). We can see Example 2 later for details.

The fuzzy languages generated by G is denoted by $\mathcal{L}_G$ or $\mathcal{L}$ for simplicity [28], which is a function from Σ^* to $[0,1]$. Let $s \in \Sigma^*$. The postlanguage of $\mathcal{L}$ after s is the set of continuations of s in all physically possible traces, i.e.,

$$\mathcal{L}/s = \{t \in \Sigma^* : (\exists q \in Q)[\delta(q_0, st) = q \wedge \mathcal{L}(st) > 0]\}.$$

From [18, 19, 28], we know that each fuzzy event is associated with a degree of controllability, so, the uncontrollable set $\widetilde{\Sigma}_{uc}$ and controllable set $\widetilde{\Sigma}_c$ are two fuzzy subsets of Σ, and satisfy: for any $\sigma \in \widetilde{\Sigma}$,

$$\widetilde{\Sigma}_{uc}(\sigma) + \widetilde{\Sigma}_c(\sigma) = 1.$$

Analogously, we think that each fuzzy event is associated with a degree of observability. For instance, for some treatments (fuzzy events) in biomedical systems modelled by a fuzzy finite automaton, some effects are observable (headache disappears, for example), but some are unobservable (for instance, some potential side effects of treatment). Therefore, the unobservable set $\widetilde{\Sigma}_{uo}$ and observable set $\widetilde{\Sigma}_o$ are two fuzzy subsets of $\widetilde{\Sigma}$, too, and satisfy: for any $\sigma \in \widetilde{\Sigma}$,

$$\widetilde{\Sigma}_{uo}(\sigma) + \widetilde{\Sigma}_o(\sigma) = 1. \tag{1}$$

Furthermore, we define $\widetilde{\Sigma}_o(\epsilon) = 0$, and

$$\widetilde{\Sigma}_o(s) = \min\{\widetilde{\Sigma}_o(\sigma_i) : i = 1, 2, \ldots, m\} \tag{2}$$

for $s = \sigma_1\sigma_2 \ldots \sigma_m \in \Sigma^*$.

We define the maximal observable set Σ_{mo}, which is composed of the events that have the greatest degree of observability among Σ, i.e.,

$$\Sigma_{mo} = \{\sigma \in \Sigma : (\forall a \in \Sigma)[\widetilde{\Sigma}_o(\sigma) \geq \widetilde{\Sigma}_o(a)]\}. \tag{3}$$

Let $\mathcal{L}_G(q)$ is the set of all traces that originate from fuzzy state q. Denote

$$\begin{aligned}\mathcal{L}_1(q,\sigma) &= \{a \in \Sigma \cap \mathcal{L}_G(q) : \\ &\quad (a \in \Sigma_{mo}) \vee [\widetilde{\Sigma}_o(a) > \widetilde{\Sigma}_o(\sigma)]\},\end{aligned} \tag{4}$$

$$\begin{aligned}\mathcal{L}_2(q,\sigma) &= \{ua \in \mathcal{L}_G(q) : (\| u \| \geq 1) \\ &\quad \wedge [\widetilde{\Sigma}_o(\sigma) \geq \widetilde{M}_o(u)] \wedge [a \in \mathcal{L}_1(q,\sigma)]\},\end{aligned} \tag{5}$$

where $\| u \|$ denotes the length of string u, and $\widetilde{M}_o(u) = \max\{\widetilde{\Sigma}_o(\sigma) : \sigma \in u\}$. Intuitively, $\mathcal{L}_1(q,\sigma)$ collects all of single fuzzy event whose degree of observability is either the greatest among Σ or greater than $\widetilde{\Sigma}_o(\sigma)$. And $\mathcal{L}_2(q,\sigma)$ consists of the strings ua containing at least two fuzzy events, in which the degree of observability for any event of u is less than or equal to that of σ and $a \in \mathcal{L}_1(q,\sigma)$. We denote

$$\mathcal{L}(q,\sigma) = \mathcal{L}_1(q,\sigma) \cup \mathcal{L}_2(q,\sigma), \tag{6}$$

$$\mathcal{L}_a(q,\sigma) = \{s \in \mathcal{L}(q,\sigma) : s_f = a\}, \tag{7}$$

where $\mathcal{L}_a(q,\sigma)$ represents those strings in $\mathcal{L}(q,\sigma)$ that end with event a.

III. Approaches to Defining Diagnosability for FDESs

In this section, we will give a definition of the diagnosability for FDESs using the fuzzy observable set $\widetilde{\Sigma}_o$ and the fuzzy failure set $\widetilde{\Sigma}_f$.

As mentioned above, in biomedical systems modelled by a fuzzy finite automaton, some effects are observable, but some are unobservable, even some effects are undesired failures (for example, some potential side effects). Therefore, in the setting of FDESs, the failure set of events, as a subset of the unobservable set $\widetilde{\Sigma}_{uo}$, is also regarded as a fuzzy subset of Σ. We denote it as $\widetilde{\Sigma}_f$, and, for each fuzzy event $\sigma \in \Sigma$, $\widetilde{\Sigma}_f(\sigma)$ represents the possibility of the failure occurring on σ. Since diagnosis is generally based on the unobservable failures [31,32,36], without loss of generality, we can assume that $\widetilde{\Sigma}_f \widetilde{\subseteq} \widetilde{\Sigma}_{uo}$, that is, $\widetilde{\Sigma}_f(\sigma) \leq \widetilde{\Sigma}_{uo}(\sigma)$ for any $\sigma \in \Sigma$, which means that failures are always unobservable.

Usually, the failure set $\widetilde{\Sigma}_f$ is partitioned into a set of failure types $f_1, f_2, \ldots, f_m$, i.e.,

$$\widetilde{\Sigma}_f = \widetilde{\Sigma}_{f_1} \widetilde{\cup} \widetilde{\Sigma}_{f_2} \widetilde{\cup} \ldots \widetilde{\cup} \widetilde{\Sigma}_{f_m} \tag{8}$$

where $\widetilde{\cup}$ is Zadeh fuzzy OR operator [14], that is,

$$\widetilde{\Sigma}_f(\sigma) = \max\left\{\widetilde{\Sigma}_{f_i}(\sigma) : i = 1, 2, \ldots, m\right\}$$

for any $\sigma \in \Sigma^*$. Let s_f denote the final fuzzy event of $s \in \Sigma^*$. We define

$$\begin{aligned} \Psi_\sigma(\widetilde{\Sigma}_{f_i}) \ &= \{s \in \Sigma^* : (\exists q \in Q)[\delta(q_0, s) = q] \\ &\wedge[\mathcal{L}(s) > 0] \wedge [\widetilde{\Sigma}_{f_i}(s_f) \geq \widetilde{\Sigma}_{f_i}(\sigma)]\}. \end{aligned} \tag{9}$$

Intuitively, $\Psi_\sigma(\widetilde{\Sigma}_{f_i})$ is the set of all physically possible traces that end in a event on which the possibility of failure of type f_i occurring is not less than $\widetilde{\Sigma}_{f_i}(\sigma)$.

When a string of events occurs in a system, the events sequence is filtered by a projection based on their degrees of observability.

Definition 2: For $\sigma \in \Sigma$, the *σ-projection* $P_\sigma : \Sigma^* \to \Sigma^*$ is defined as: For any $a \in \Sigma$ and $s \in \Sigma^*$,

$$P_\sigma(a) = \begin{cases} a, & \text{if } a \in \Sigma_{mo} \text{ or } \widetilde{\Sigma}_o(a) > \widetilde{\Sigma}_o(\sigma), \\ \epsilon, & \text{otherwise}, \end{cases} \tag{10}$$

and $P_\sigma(\epsilon) = \epsilon, \quad P_\sigma(sa) = P_\sigma(s)P_\sigma(a)$.

The inverse projection operator is given by:

$$\begin{aligned} P_\sigma^{-1}(y) \ &= \{s \in \Sigma^* : (\exists q \in Q) \\ &[\delta(q_0, s) = q] \wedge [\mathcal{L}(s) > 0] \wedge [P_\sigma(s) = y]\}. \end{aligned}$$

The purpose of σ-projection is to erase the events whose degree of observability is not greater than $\widetilde{\Sigma}_o(\sigma)$ in a string. Especially, when a deterministic or nondeterministic finite automaton is regarded as a special form of fuzzy finite automaton, then all σ-projections are equal, and, all of them degenerate to projection $P : \Sigma^* \to \Sigma_o^*$ in the usual manner, which simply erases the unobservable events [31, 32].

Remark 2: In order to avoid the case that the event set of the diagnoser constructed later is null, we introduce the maximal observable set Σ_{mo} in the definition of σ-projection P_σ, since it is impossible to diagnose the failure using a diagnoser with a null event set.

For the sake of simplicity, we make the following two assumptions about the fuzzy automaton G, which are similar to those in [31, 32, 36].

(*A1*): Language $\mathcal{L}_G$ is live. This means that system cannot reach a state without transitions.

(*A2*): For any $\sigma \in \Sigma$ and state $q \in Q$, there exists $n_0 \in N$ such that $\parallel t \parallel \leq n_0$ for every $t \in \mathcal{L}(q, \sigma)$.

Intuitively, assumption (*A1*) indicates that there is a transition defined at each state, and (*A2*) means that for any event $\sigma \in \Sigma$, before generating an event whose observability degree is the greatest among Σ or greater than $\widetilde{\Sigma}_o(\sigma)$, G does not generate arbitrarily long sequences in which each event's degree of observability is less than $\widetilde{\Sigma}_o(\sigma)$.

In order to compare diagnosability for FDESs with that for classical DESs, we recall the definition of diagnosability for classical DESs presented by Sampath *et al* [31].

Definition 3 [31]: A language L are said to be *diagnosable* with respect to the projection P and the partition Π_f on Σ_f, if the following holds:

$$\begin{aligned} &(\forall i \in \Pi_f)(\exists n_i \in \mathbf{N})[\forall s \in \Psi(\Sigma_{f_i})] \\ &(\forall t \in L/s)[\parallel t \parallel \geq n_i \Rightarrow D) \end{aligned} \tag{11}$$

where the diagnosability condition function D is

$$\omega \in P^{-1}[P(st)] \Rightarrow \Sigma_{f_i} \in \omega. \tag{12}$$

The objective of diagnosis for classical DESs is to detect the unobservable failures from the record of the observed events. As mentioned above, in FDESs, the failures may occur on every fuzzy event, only their possibilities of failure occurring are different. Therefore, the purpose of diagnosis for FDESs is to detect the failures from the sequence of the observed events, based on the degree of observability and the possibility of failure occurring. Now let us give the definition of diagnosability for FDESs.

Definition 4: Let $\mathcal{L}$ be a language generated by a fuzzy finite automaton $G = (Q, \Sigma, \delta, q_0)$ and $\sigma \in \Sigma$. $\mathcal{L}$ is said to be *F_i-diagnosable with respect to* σ, if there exists $n_i \in N$ such that for any $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_i})$ and any $t \in \mathcal{L}/s$ where $\| t \| \geq n_i$, the following holds:

$$\widetilde{\Sigma}_{f_i}(\sigma) \leq \min\left\{\widetilde{\Sigma}_{f_i}(\omega) : \omega \in P_\sigma^{-1}(P_\sigma(st))\right\}. \quad (13)$$

Denote $\Sigma_{fail_i} = \left\{\sigma \in \Sigma : \widetilde{\Sigma}_{f_i}(\sigma) > 0\right\}$. If for each $\sigma \in \Sigma_{fail_i}$, $\mathcal{L}$ is F_i-diagnosable with respect to σ, then $\mathcal{L}$ is said to be *F_i-diagnosable*.

Intuitively, $\mathcal{L}$ being F_i-diagnosable with respect to σ means that, for any physically possible trace s where the possibility that failure of type f_i occurs on s_f is not less than that on σ, any sufficiently long continuation t of s, and any trace ω, if ω produces the same record by the σ-projection as the trace st, then the possibility that failure of type f_i occurs on ω must be not less than that on σ, too. In other words, if the failure type f_i has occurred on event s_f, then f_i must also occur on every trace ω whose observed record is the same as st.

Remark 3: If the observability and possibility of failure occurring of each event are crisp, i.e., $\widetilde{\Sigma}_o(\sigma)$, $\widetilde{\Sigma}_{f_i}(\sigma) \in \{0, 1\}$, then the definition of diagnosability for FDESs reduces to Definition 3, the diagnosability for classical DESs presented by Sampath *et al* [31].

We present an example to explain the definition of diagnosability for FDESs, and the real-world application example will be given in Example 2 later.

Example 1. Consider the fuzzy automaton $G = (Q, \Sigma, \delta, q_0)$ represented in Fig.1,

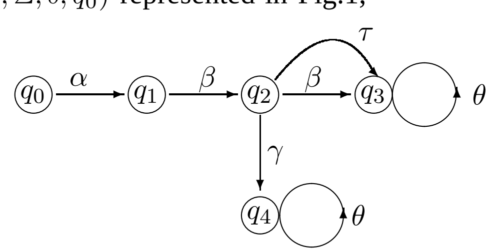

Fig.1. The fuzzy automaton of Example 1.

where $Q = \{q_0, q_1, \ldots, q_4\}$, $q_0 = [0.8, 0.2]$, and $\Sigma = \{\alpha, \beta, \gamma, \tau, \theta\}$ is defined as follows:

$$\alpha = \begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0.4 & 0.8 \\ 0.8 & 0.6 \end{bmatrix},$$
$$\gamma = \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}, \quad \tau = \begin{bmatrix} 0.6 & 0.4 \\ 0.8 & 0.6 \end{bmatrix},$$
$$\theta = \begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.9 \end{bmatrix}.$$

Note that δ is defined with *max-min* operation, we can calculate the other fuzzy states: $q_1 = [0.8, 0.4]$, $q_2 = [0.4, 0.8]$, $q_3 = [0.8, 0.6]$, and $q_4 = [0.4, 0.4]$.

Suppose that the degree of observability and the possibility of failure occurring on each fuzzy event are defined as follows:

$$\widetilde{\Sigma}_o(\alpha) = 0.8, \ \widetilde{\Sigma}_o(\beta) = 0.5, \ \widetilde{\Sigma}_o(\gamma) = 0.3,$$
$$\widetilde{\Sigma}_o(\theta) = 0.7, \ \widetilde{\Sigma}_o(\tau) = 0.3; \ \widetilde{\Sigma}_{f_1}(\alpha) = 0.2,$$
$$\widetilde{\Sigma}_{f_1}(\beta) = 0.4, \ \widetilde{\Sigma}_{f_1}(\gamma) = 0.3, \ \widetilde{\Sigma}_{f_1}(\theta) = 0.3,$$
$$\widetilde{\Sigma}_{f_1}(\tau) = 0.6; \ \widetilde{\Sigma}_{f_2}(\alpha) = 0.1, \ \widetilde{\Sigma}_{f_2}(\beta) = 0.3,$$
$$\widetilde{\Sigma}_{f_2}(\gamma) = 0.4, \ \widetilde{\Sigma}_{f_2}(\theta) = 0.2, \ \widetilde{\Sigma}_{f_2}(\tau) = 0.5.$$

In the following, we will use Definition 4 to verify two conclusions: (1) the language $\mathcal{L}$ generated by G is not F_1-diagnosable with respect to τ, but (2) $\mathcal{L}$ is F_2-diagnosable with respect to β.

In fact, when $\sigma = \tau$, for $\forall n_i \in N$, we take $s = \alpha\beta\tau$, $t = \theta^{n_i+1}$, and take $\omega = \alpha\beta\gamma\theta^{n_i+1}$. Obviously, $\omega \in P_\sigma^{-1}(P_\sigma(st))$, but $\widetilde{\Sigma}_{f_1}(\sigma) = 0.6$, while $\widetilde{\Sigma}_{f_1}(\omega) = 0.4$. Therefore, Ineq.(13) does not hold, so $\mathcal{L}$ is not F_1-diagnosable with respect to τ.

When $\sigma = \beta$, we take $n_i = 2$, then for any $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_2})$, (i.e.,$s = \alpha\beta$, $\alpha\beta\beta$, $\alpha\beta\tau$, or $\alpha\beta\gamma$), and any $t \in \mathcal{L}/s$, where $\| t \| \geq n_i$, we have

$$P_\sigma^{-1}(P_\sigma(st)) = \{\alpha\beta\tau\theta^k, \alpha\beta\beta\theta^k, \alpha\beta\gamma\theta^k : k \geq 1\}.$$

Due to each element in $P_\sigma^{-1}(P_\sigma(st))$ containing β, therefore, for any $\omega \in P_\sigma^{-1}(P_\sigma(st))$, we have $\widetilde{\Sigma}_{f_2}(\sigma) \leq \widetilde{\Sigma}_{f_2}(\omega)$, that is, $\mathcal{L}$ is F_2-diagnosable with respect to β.

IV. NECESSARY AND SUFFICIENT CONDITION OF DIAGNOSABILITY FOR FDESS

In this section, through the construction of observability-based diagnosers of FDESs, we investigate some main properties of the diagnosers. In particular, we present a necessary and sufficient condition for diagnosability of FDESs. Our results not only generalize the significant consequences in classical DESs introduced by Sampath *et al* [31], but also may better deal with the problems of vagueness in real-world situation. Example 2 in Section V verifies this view to a certain degree.

A. Construction of the Diagnosers

We firstly present the construction of the observability-based diagnoser, which is a finite automaton built on fuzzy finite automaton G.

Denote the set of possible failure labels as $\triangle = \{N\} \cup 2^{\triangle_f}$, where N stands for "normal", and $2^{\triangle_f}$ denotes the power set of $\triangle_f = \{F_1, \cdots, F_m\}$ [31]. For $\sigma \in \Sigma$, we define a subset of Q as

$$\begin{aligned} Q_\sigma \quad &= \{q_0\} \cup \{q \in Q : (\exists q^{'} \in Q)(\exists a \in \Sigma) \\ &[\delta(q^{'}, a) = q \wedge a \in \mathcal{L}_1(q, \sigma)]\}, \end{aligned} \tag{14}$$

i.e., Q_σ is composed of the initial state q_0 and the states reachable from one event whose degree of observability is either the greatest among Σ or greater than $\widetilde{\Sigma}_o(\sigma)$.

Definition 5: Let $G = (Q, \Sigma, \delta, q_0)$ be a fuzzy finite automaton and $\sigma \in \Sigma_{fail_i}$. The *diagnoser with respect to* σ is the finite automaton

$$G_d = (Q_d, \Sigma_d, \delta_d, \chi_0), \tag{15}$$

where the initial state $\chi_0 = \{(q_0, \{N\})\}$, means that the automaton G is normal to start with. The set of events of the diagnoser is

$$\Sigma_d = \left\{a \in \Sigma : (a \in \Sigma_{mo}) \vee [\widetilde{\Sigma}_o(a) > \widetilde{\Sigma}_o(\sigma)]\right\}. \tag{16}$$

The state space $Q_d \subseteq Q_\sigma \times \triangle$ is composed of the states reachable from χ_0 under δ_d. A state χ of Q_d is of the form

$$\chi = \{(q_1, \ell_1), (q_2, \ell_2), \ldots, (q_n, \ell_n)\}, \tag{17}$$

where $q_i \in Q_\sigma$ and $\ell_i \in \triangle$, i.e., ℓ_i is the form $\ell_i = \{N\}$, or $\ell_i = \{F_{i_1}, F_{i_2}, \ldots, F_{i_k}\}$. And δ_d is the partial transition function of the diagnoser, which will be constructed in Definition 7.

Definition 6: The *label propagation function* $LP : Q_\sigma \times \triangle \times \Sigma^* \to \triangle$ is defined as follows: For $q \in Q_\sigma, \ell \in \triangle$, and $s \in \mathcal{L}(q, \sigma)$,

$$\begin{aligned} &LP(q, \ell, s) \\ &= \begin{cases} \{N\}, \text{if } \ell = \{N\} \text{ and } \forall i[\widetilde{\Sigma}_{f_i}(s) < \widetilde{\Sigma}_{f_i}(\sigma)], \\ \left\{F_i : F_i \in \ell \vee \widetilde{\Sigma}_{f_i}(s) \geq \widetilde{\Sigma}_{f_i}(\sigma)\right\}, \text{otherwise.} \end{cases} \end{aligned} \tag{18}$$

The label propagation function is due to describe the changes of label from one state of diagnoser to another. Obviously, label F_i is added whenever the possibility of the ith type failure occurring on the string s is not less than $\widetilde{\Sigma}_{f_i}(\sigma)$, and once this label is appended, it cannot be removed in the successor states of the diagnoser.

Definition 7: The *transition function* of the diagnoser $\delta_d : Q_d \times \Sigma_d \to Q_d$ is defined as

$$\delta_d(\chi, a) = \bigcup_{(q_i, \ell_i) \in \chi} \bigcup_{s \in \mathcal{L}_a(q_i, \sigma)} \{(\delta(q_i, s), LP(q_i, \ell_i, s))\}. \tag{19}$$

For example, $\delta_d(\chi_0, \alpha) = \{(q_1, \{N\}), (q_5, \{F_1\})\}$ in Fig. 4 of Example 2.

B. Some Properties of the Diagnosers

In this subsection, we present some main properties of the diagnoser, which will be used to prove the condition of the diagnosability for FDESs.

Property 1: Let $G = (Q, \Sigma, \delta, q_0)$ be a fuzzy finite automaton, and let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ, where $\sigma \in \Sigma_{fail_i}$. For $\chi_1, \chi_2 \in Q_d$, $s \in \Sigma^*$, if $(q_1, \ell_1) \in \chi_1$, $(q_2, \ell_2) \in \chi_2$,

$\delta(q_1, s) = q_2$, $\delta_d(\chi_1, P_\sigma(s)) = \chi_2$, then $F_i \in \ell_1$ implies $F_i \in \ell_2$.

Proof: It can be directly verified from Definitions 6 and Definitions 7. ■

Property 2: If $\chi \in Q_d$, then (q_1, ℓ_1), $(q_2, \ell_2) \in \chi$ if and only if there exist $s_1, s_2 \in \Sigma^*$ such that $(s_1)_f = (s_2)_f \in \Sigma_d$, $P_\sigma(s_1) = P_\sigma(s_2)$, $\delta_d(\chi_0, P_\sigma(s_1)) = \chi$, and for $k = 1, 2$, $\mathcal{L}(s_k) > 0$,

$$\delta(q_0, s_k) = q_k, \ \ LP(q_0, \{N\}, s_k) = \ell_k.$$

Proof: *Necessity:* If $\chi \in Q_d$, then there are $a_1, \ldots, a_j \in \Sigma_d$ and $\chi_1, \ldots, \chi_{j-1} \in Q_d$, such that $\delta_d(\chi_i, a_{i+1}) = \chi_{i+1}$, where $0 \le i \le j-1$ and $\chi_j = \chi$. From the assumption that (q_1, ℓ_1), $(q_2, \ell_2) \in \chi$, there exist $(q_1^k, \ell_1^k) \in \chi_{j-1}$, and $t_1^k \in \mathcal{L}_{a_j}(q_1^k, \sigma)$ $(k = 1, 2)$ such that for $k = 1, 2$,

$$q_k = \delta(q_1^k, t_1^k), \quad \ell_k = LP(q_1^k, \ell_1^k, t_1^k).$$

Similarly, note that $\delta_d(\chi_{j-2}, a_{j-1}) = \chi_{j-1}$, hence, there are $(q_2^k, \ell_2^k) \in \chi_{j-2}$, and $t_2^k \in \mathcal{L}_{a_{j-1}}(q_2^k, \sigma)$ $(k = 1, 2)$ satisfying for $k = 1, 2$,

$$q_1^k = \delta(q_2^k, t_2^k), \quad \ell_1^k = LP(q_2^k, \ell_2^k, t_2^k).$$

$$\cdots\cdots$$

With the analogous process, there are $(q_{j-1}^k, \ell_{j-1}^k) \in \chi_1$, $t_j^k \in \mathcal{L}_{a_1}(q_0, \sigma)$ $(k = 1, 2)$ such that for $k = 1, 2$,

$$q_{j-1}^k = \delta(q_0, t_j^k), \quad \ell_{j-1}^k = LP(q_0, \{N\}, t_j^k).$$

We take

$$s_k = t_j^k t_{j-1}^k \ldots t_2^k t_1^k, \quad (k = 1, 2). \tag{20}$$

Obviously, $\delta_d(\chi_0, P_\sigma(s_1)) = \chi$, $(s_1)_f = (s_2)_f = a_j \in \Sigma_d$ and for $k = 1, 2$, we have $\mathcal{L}(s_k) > 0$, $\delta(q_0, s_k) = q_k$, $LP(q_0, \{N\}, s_k) = \ell_k$. Moreover

$$P_\sigma(s_1) = a_1 a_2 \ldots a_j = P_\sigma(s_2).$$

Sufficiency: Assume that there exist $s_1, s_2 \in \Sigma^*$ satisfying $\mathcal{L}(s_1) > 0$, $\mathcal{L}(s_2) > 0$ and $P_\sigma(s_1) = P_\sigma(s_2)$. From $\delta_d(\chi_0, P_\sigma(s_1)) = \chi$, we denote

$$P_\sigma(s_1) = a_1 a_2 \ldots a_j,$$

then we can obtain a state sequence $\chi_1, \chi_2, \ldots, \chi_{j-1} \in Q_d$ such that $\delta_d(\chi_i, a_{i+1}) = \chi_{i+1}$, where $0 \le i \le j-1$ and $\chi_j = \chi$. Furthermore, from $\delta(q_0, s_k) = q_k$, and $LP(q_0, \{N\}, s_k) = \ell_k$, $(k = 1, 2)$, we have that (q_1, ℓ_1), $(q_2, \ell_2) \in \chi$ by Definition 7. ■

Remark 4: In the proof of Necessity, it is possible that (q_h^1, ℓ_h^1) is the same as (q_h^2, ℓ_h^2) for some h, but it does not concern the proof.

Definition 8: Let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ. A state $\chi \in Q_d$ is said to be F_i*-certain* if either $F_i \in \ell$ for all $(q, \ell) \in \chi$, or $F_i \notin \ell$ for all $(q, \ell) \in \chi$. And χ is said to be F_i*-uncertain* , if there are $(q_1, \ell_1), (q_2, \ell_2) \in \chi$ such that $F_i \in \ell_1$ and $F_i \notin \ell_2$.

For example, $\chi_1 = \{(q_1, \{F_2\}), (q_5, \{F_1, F_2\})\}$ and $\chi_2 = \{(q_2, \{F_2\}), (q_6, \{F_1, F_2\})\}$ in Fig.8 are both F_2-certain and F_1-uncertain states.

Property 3: Let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ and $\delta_d(\chi_0, u) = \chi$. If χ is F_i-certain, then either $\widetilde{\Sigma}_{f_i}(s) \ge \widetilde{\Sigma}_{f_i}(\sigma)$ for all $s \in P_\sigma^{-1}(u)$, or $\widetilde{\Sigma}_{f_i}(s) < \widetilde{\Sigma}_{f_i}(\sigma)$ for all $s \in P_\sigma^{-1}(u)$, where $s_f \in \Sigma_d$.

Proof: By contradiction, suppose there exist $s_1, s_2 \in P_\sigma^{-1}(u)$ such that

$$\widetilde{\Sigma}_{f_i}(s_1) \ge \widetilde{\Sigma}_{f_i}(\sigma) > \widetilde{\Sigma}_{f_i}(s_2)$$

where $(s_1)_f, (s_2)_f \in \Sigma_d$. Denote

$$LP(q_0, \{N\}, s_1) = \ell_1, \ \ LP(q_0, \{N\}, s_2) = \ell_2,$$

then from Definition 6, we know that $F_i \in \ell_1$, but $F_i \notin \ell_2$. By Property 2, we have (q_1, ℓ_1), $(q_2, \ell_2) \in \chi$, where $\delta(q_0, s_1) = q_1$ and $\delta(q_0, s_2) = q_2$. That is, χ is F_i-uncertain. ■

Property 4: Let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ and $\delta_d(\chi_0, u) = \chi$. If χ is F_i-uncertain, then there exist $s_1, s_2 \in \Sigma^*$ such that $(s_1)_f = (s_2)_f \in \Sigma_d$, $P_\sigma(s_1) = P_\sigma(s_2)$, $\delta_d(\chi_0, P_\sigma(s_1)) = \chi$, and

$$\widetilde{\Sigma}_{f_i}(s_1) \ge \widetilde{\Sigma}_{f_i}(\sigma) > \widetilde{\Sigma}_{f_i}(s_2). \tag{21}$$

Proof: It is straight obtained by Property 3. ■

Property 5: Let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ. If the set of states in Q_d forms a cycle in G_d, then all states in the cycle have the same failure label.

Proof: It is easy to prove since any two states in a cycle of G_d are reachable from each other, and once a failure label is appended, it cannot be removed in all successors. ■

C. Necessary and Sufficient Condition of Diagnosability for FDESs

In this subsection, we present an approach of failure diagnosis in the framework of FDESs, and a necessary and sufficient condition of the diagnosability for FDESs is obtained.

We may define an F_i-indeterminate cycle in diagnosers for FDESs, just as for classical DESs.

Definition 9: Let $G_d = (Q_d, \Sigma_d, \delta_d, \chi_0)$ be the diagnoser with respect to σ. A set of F_i-uncertain states $\chi_1, \chi_2, \ldots, \chi_k \in Q_d$ is said to form an F_i-*indeterminate cycle* if

(1) $\chi_1, \chi_2, \ldots, \chi_k$ form a cycle in G_d, i.e., there is $\sigma_j \in \Sigma_d$ such that $\delta_d(\chi_j, \sigma_j) = \chi_{(j+1) \bmod k}$, where $j = 1, \ldots, k$.

(2) $\exists\, (x_j^h, \ell_j^h), (y_j^r, d_j^r) \in \chi_j$ ($j \in [1,k]$; $h \in [1,m]$; $r \in [1,n]$) such that

1) $F_i \in \ell_j^h$ but $F_i \notin d_j^r$ for all j, h, r;
2) The sequences of states $\left\{x_j^h\right\}$ and $\left\{y_j^r\right\}$ form cycles respectively in G with

$$\delta(x_j^h, s_j^h\sigma_j) = x_{j+1}^h, (j \in [1, k-1]; h \in [1,m]),$$
$$\delta(x_k^h, s_k^h\sigma_k) = x_1^{h+1}, (h \in [1, m-1]),$$

and $\delta(x_k^m, s_k^m\sigma_k) = x_1^1$;

$$\delta(y_j^r, t_j^r\sigma_j) = y_{j+1}^r, (j \in [1, k-1]; r \in [1,n]),$$
$$\delta(y_k^r, t_k^r\sigma_k) = y_1^{r+1}, (r \in [1, n-1]),$$

and $\delta(y_k^n, t_k^n\sigma_k) = y_1^1$,

where $s_j^h\sigma_j \in \mathcal{L}(x_j^h, \sigma)$, $t_j^r\sigma_j \in \mathcal{L}(y_j^r, \sigma)$.

Intuitively, an F_i-indeterminate cycle in G_d is a cycle composed of F_i-uncertain states where, corresponding to this cycle, there exist two sequences $\left\{x_j^h\right\}$ and $\left\{y_j^r\right\}$ forming cycles of G, in which one carries and the other does not carry failure label F_i.

Now we can present a necessary and sufficient condition of the diagnosability for FDEs.

Theorem 1: A fuzzy language $\mathcal{L}$ generated by a fuzzy finite automaton G is F_i-diagnosable if and only if for any $\sigma \in \Sigma_{fail_i}$, the diagnoser G_d with respect to σ satisfies the condition: There are no F_i-indeterminate cycles in G_d.

Proof: Necessity: We prove it by contradiction. Assume that $\mathcal{L}$ is F_i-diagnosable, and there is an F_i-indeterminate cycle $\chi_1, \chi_2, \ldots, \chi_k$ in diagnoser G_d with respect to σ, where $\sigma \in \Sigma_{fail_i}$. By Definition 9, the corresponding sequences of states $\left\{x_j^h\right\}$ and $\left\{y_j^r\right\}$ form two cycles in G, and the corresponding strings $s_j^h\sigma_j$ and $t_j^r\sigma_j$ satisfy condition 2) of Definition 9, where $(x_j^h, \ell_j^h), (y_j^r, d_j^r) \in \chi_j$, and $F_i \in \ell_j^h$ but $F_i \notin d_j^r$ for all $j = 1, \cdots, k$; $h = 1, \cdots, m$; $r = 1, \cdots, n$.

Since $(x_1^1, \ell_1^1), (y_1^1, d_1^1) \in \chi_1$, from Property 2, there exist $s_0, t_0 \in \Sigma^*$ such that $P_\sigma(s_0) = P_\sigma(t_0)$, $\delta(q_0, s_0) = x_1^1$, and $\delta(q_0, t_0) = y_1^1$. Notice that $F_i \in \ell_1^1$ and $F_i \notin d_j^r$ for all j, r. Therefore, we have $\widetilde{\Sigma}_{f_i}(t_0) < \widetilde{\Sigma}_{f_i}(\sigma)$, and

$$\widetilde{\Sigma}_{f_i}(s_0) \geq \widetilde{\Sigma}_{f_i}(\sigma) \geq \widetilde{\Sigma}_{f_i}(t_j^r\sigma_j). \quad (22)$$

Let l be arbitrarily large. We consider the following two traces

$$\omega_1 = s_0(s_1^1\sigma_1 \ldots s_k^1\sigma_k \ldots s_1^m\sigma_1 \ldots s_k^m\sigma_k)^{ln}, \quad (23)$$
$$\omega_2 = t_0(t_1^1\sigma_1 \ldots t_k^1\sigma_k \ldots t_1^n\sigma_1 \ldots t_k^n\sigma_k)^{lm}. \quad (24)$$

Then $\mathcal{L}(\omega_1) > 0,\ \mathcal{L}(\omega_2) > 0$ and

$$P_\sigma(\omega_1) = P_\sigma(\omega_2) = P_\sigma(s_0)(\sigma_1\sigma_2 \ldots \sigma_k)^{lmn}. \quad (25)$$

Because $\widetilde{\Sigma}_{f_i}(s_0) \geq \widetilde{\Sigma}_{f_i}(\sigma)$, there is a prefix s of s_0 such that $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_i})$. Take $t \in \mathcal{L}/s$ where $\omega_1 = st$, then from (25), we know $\omega_2 \in P_\sigma^{-1}(P_\sigma(st))$. But from Ineqs.(22), and

$$\widetilde{\Sigma}_{f_i}(\omega_2) = \max\{\widetilde{\Sigma}_{f_i}(t_0), \widetilde{\Sigma}_{f_i}(t_j^r\sigma_j) : j = 1, \cdots, k; r = 1, \cdots, n\},$$

we have $\widetilde{\Sigma}_{f_i}(\omega_2) < \widetilde{\Sigma}_{f_i}(\sigma)$. That is, $\mathcal{L}$ is not F_i-diagnosable, which contradicts the assumption.

Sufficiency: Assume that there are no F_i-indeterminate cycles in diagnoser G_d with respect to σ, where $\sigma \in \Sigma_{fail_i}$. The proof of sufficiency will be completed by following two steps: (1) χ_0 can reach an F_i-certain state after a finite number of transitions; (2) $\mathcal{L}$ is F_i-diagnosable with respect to σ.

(1) Firstly, we verify that χ_0 can reach an F_i-certain state after a finite number of transitions.

For simplicity, if $(q, \ell), (q', \ell') \in \chi$, and $F_i \in \ell$, $F_i \notin \ell'$, we shall denote q as "x-state" of χ and q' as "y-state" of χ, respectively. Let $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_i})$ and $\delta(q_0, s) = q$. From Assumption (A2), there exists $n_0 \in N$ such that $\parallel t_1 \parallel \leq n_0$ for any $t_1 \in \mathcal{L}(q, \sigma)$. Denote $\delta(q_0, st_1) = q_1$, $\delta_d(\chi_0, P_\sigma(st_1)) = \chi_1$, then q_1 is an "x-state" since $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_i})$ implies $\widetilde{\Sigma}_{f_i}(st_1) \geq \widetilde{\Sigma}_{f_i}(\sigma)$.

The desired result is obtained if χ_1 is F_i-certain. So the following is to prove the desired result under the assumption that χ_1 is F_i-uncertain. Since there are no F_i-indeterminate cycles in G_d, one of the following is true: (i) there are no cycles of F_i-uncertain states in G_d, or (ii) there is one or more cycles of F_i-uncertain states in G_d but corresponding to such cycle, there do not exist two sequences of "x-states" and of "y-states" forming cycles in G.

Case (i): Suppose that there are no cycles of F_i-uncertain states in G_d, which means F_i-uncertain states will reach an F_i-certain state by Assumption (A1) and Property 1. Therefore, there is sufficiently long $t_2 \in \mathcal{L}_G(q_1)$ such that $\delta_d(\chi_0, P_\sigma(st_1t_2))$ is an F_i-certain state.

Case (ii): Suppose that there is a cycle of F_i-uncertain states $\chi_1, \chi_2, \ldots, \chi_k$ in G_d, but correspondingly to such cycle, there do not exist two sequences of "x-states" and of "y-states" forming cycles in G. The following will prove that this case is impossible. In fact, there is an "x-state" q_2 of χ_2 such that q_2 is a successor of q_1 since q_1 is an "x-state" of χ_1. Similarly, there is an "x-state" q_3 of χ_3 such that q_3 is a successor of q_2. .... So, we obtain a sequence $\{q_1, q_2, \cdots\}$ of "x-states" which forms cycles in G. With the analogous process, we can obtain a sequence of "y-states" which forms cycles in G, too. That is, Case (ii) is impossible.

Above inference indicates that χ_0 must reach an F_i-certain state within a finite steps (denoted by m_0) of transitions, no matter whether χ_1 is F_i-certain or not.

(2) From (1), we take $n_i = m_0$, then for any $s \in \Psi_\sigma(\widetilde{\Sigma}_{f_i})$ and any $t \in \mathcal{L}/s$ where $\parallel t \parallel \geq n_i$, χ_0 must lead to an F_i-certain state. That is, whenever $\omega \in P_\sigma^{-1}(P_\sigma(st))$, it always holds that $\widetilde{\Sigma}_{f_i}(\sigma) \leq \widetilde{\Sigma}_{f_i}(\omega)$. Therefore, $\mathcal{L}$ is F_i-diagnosable with respect to σ. ■

From the proof of Theorem 1, we know that Theorem 1 can be precisely described as follows.

Theorem 2: A fuzzy language $\mathcal{L}$ generated by a fuzzy finite automaton G is F_i-diagnosable with respect to $\sigma \in \Sigma_{fail_i}$ if and only if the diagnoser G_d with respect to σ satisfies the condition: There are no F_i- indeterminate cycles in G_d.

Proof: It has been shown in the proof of Theorem 1. ■

V. EXAMPLES OF DIAGNOSABILITY FOR FDESS

In this section, we will give some examples to illustrate the process of testing the necessary and sufficient condition for the diagnosability of FDESs presented above, which may be viewed as an applicable background of diagnosability for FDESs. Examples 2 and 3 are diagnosability for FDESs with single failure type: one is diagnosable but the other is not diagnosable. Example 4 is considered as an FDES with multiple failure types. For simplicity, the fuzzy events (matrices) used are all upper or lower triangular matrices.

Example 2. Let us use a fuzzy automaton $G = (Q, \Sigma, \delta, q_0)$ to model a patient's body condition. For simplicity, we consider patient's condition roughly to be three cases, i.e., "poor" , "fair", and "excellent". Suppose that patient's initial condition (initial fuzzy state) is $q_0 = [0.9,\ 0.1,\ 0]$, which means that the patient is in a state with possibility of 0.9 for "poor", 0.1 for "fair" and 0 for "excellent". Suppose that there are three treatments to choose for doctor, denoted as α, β and γ, which are defined as follows:

$$\alpha = \begin{bmatrix} 0.4 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{bmatrix}, \beta = \begin{bmatrix} 0.4 & 0 & 0 \\ 0.9 & 0.4 & 0 \\ 0.4 & 0.4 & 0.4 \end{bmatrix},$$

$$\gamma = \begin{bmatrix} 0.9 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{bmatrix}.$$

In general, it is possible that patient's condition turns better or worse after each treatment, which may be evaluated by means of experience and medical theory. For instance, fuzzy event α means that, after this treatment, the possibilities that patient's status changes from "poor" to "poor", "fair" and "excellent" are 0.4, 0.9 and 0.4; the possibilities from "fair" to "poor", "fair" and "excellent" are 0, 0.4 and 0.4; and the possibilities from "excellent" to "poor", "fair" and "excellent" are 0, 0 and 0.4, respectively. Fuzzy events β and γ have similar interpretations.

Assume that doctor's strategy for patient's treatment is described by Fig.2. From $q_0 = [0.9,\ 0.1,\ 0]$, we can calculate the other fuzzy states using the transition function δ as: $q_1 = [0.4,\ 0.9,\ 0.4]$,

$$q_2 = [0.9,\ 0.4,\ 0.4], \quad q_3 = [0.9,\ 0.9,\ 0.4],$$
$$q_4 = [0.4,\ 0.1,\ 0], \qquad q_5 = [0.4,\ 0.4,\ 0.4].$$

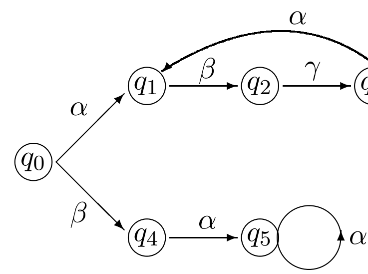

Fig.2. The fuzzy automaton of Example 2.

Fig.2 means that, if the patient obtains the first treatment being α or β, then his (or her) state changes into q_1 or q_4. After treatment β in condition q_1, the state will change from q_1 to q_2. And then, the patient will turn into state q_3 after treatment γ. If treatment α is adopted in state q_3, then the patient returns to condition q_1. Similarly, when the patient obtains treatment α in q_4, the state will turn to q_5. And the patient's condition will be unchanged if he or she obtains treatment α in q_5.

As mentioned above, for each treatment (fuzzy event), some effects are observable, but some are unobservable, even if some are undesired failures (for example, some potential side effects). Therefore, each fuzzy event has certain degrees of observable and unobservable, and, also, each fuzzy event may possess different possibility of failure occurring. Assume that the degree of observability and the possibility of failure occurring for each fuzzy event are defined:

$$\widetilde{\Sigma}_o(\alpha) = 0.6, \quad \widetilde{\Sigma}_o(\beta) = 0.4, \quad \widetilde{\Sigma}_o(\gamma) = 0.7;$$
$$\widetilde{\Sigma}_{f_1}(\alpha) = 0.1, \quad \widetilde{\Sigma}_{f_1}(\beta) = 0.2, \quad \widetilde{\Sigma}_{f_1}(\gamma) = 0.3.$$

Now, in order to detect the occurrence of failure, we construct the diagnosers with respect to each $\sigma \in \Sigma_{fail_i}$, where $\Sigma_{fail_i} = \{\alpha, \beta, \gamma\}$.

(1). When $\sigma = \alpha$, the σ-projection P_σ is determined by $P_\sigma(\alpha) = P_\sigma(\beta) = \epsilon$, $P_\sigma(\gamma) = \gamma$, and the set of events for the diagnoser is $\Sigma_d = \{\gamma\}$. According to Definition 5, the diagnoser G_d with respect to α is constructed in Fig.3. Obviously, there are no F_1-indeterminate cycles in G_d. Therefore, by Theorem 2, $\mathcal{L}$ is F_1-diagnosable with respect to α.

In fact, due to $\widetilde{\Sigma}_{f_1}(\alpha)$ being the smallest among $\{\widetilde{\Sigma}_{f_1}(a) : a \in \Sigma\}$, Ineq.(13) naturally holds with $n_i = 0$.

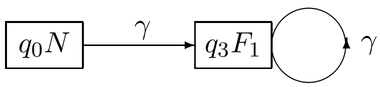

Fig.3. The diagnoser G_d w.r.t α in Example 2.

(2). When $\sigma = \beta$, we have $P_\sigma(\alpha) = \alpha$, $P_\sigma(\beta) = \epsilon$, $P_\sigma(\gamma) = \gamma$ and $\Sigma_d = \{\alpha, \gamma\}$. And the diagnoser G_d with respect to β is constructed in Fig.4. Obviously, $\mathcal{L}$ is F_1-diagnosable with respect to β for no F_1-indeterminate cycles in G_d. In fact, Ineq.(13) holds with $n_i = 1$.

(3). When $\sigma = \gamma$, we have $P_\sigma(\alpha) = P_\sigma(\beta) = \epsilon$, $P_\sigma(\gamma) = \gamma$ and $\Sigma_d = \{\gamma\}$. For no F_1-indeterminate cycles in the diagnoser G_d with respect to γ constructed in Fig.5, $\mathcal{L}$ is F_1-diagnosable with respect to γ.

Therefore, $\mathcal{L}$ is F_1-diagnosable. That is, the occurrence of failure can be detected within finite delay.

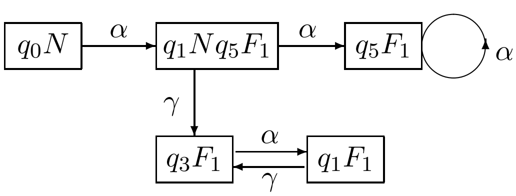

Fig.4. The diagnoser G_d w.r.t β in Example 2.

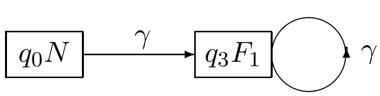

Fig.5. The diagnoser G_d w.r.t γ in Example 2.

Example 3. Consider the fuzzy automaton $G = (Q, \Sigma, \delta, q_0)$ represented in Fig.6, where $Q = \{q_0, q_1, \ldots, q_7\}$ is defined as:

$$
\begin{aligned}
q_0 &= [0.9,\ 0.1,\ 0], & q_1 &= [0.4,\ 0.9,\ 0.4],\\
q_2 &= [0.9,\ 0.4,\ 0.4], & q_3 &= [0.9,\ 0.9,\ 0.4],\\
q_4 &= [0.5,\ 0.1,\ 0], & q_5 &= [0.4,\ 0.5,\ 0.4],\\
q_6 &= [0.5,\ 0.4,\ 0.4], & q_7 &= [0.5,\ 0.5,\ 0.4].
\end{aligned}
$$

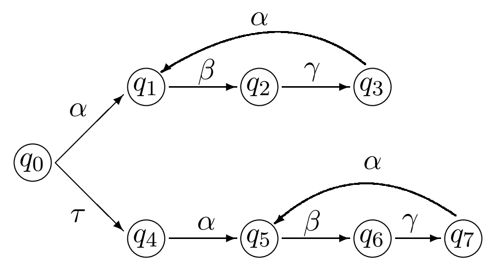

Fig.6. The fuzzy automaton of Example 3.

The set of fuzzy events $\Sigma = \{\tau, \alpha, \beta, \gamma\}$, where $\tau, \alpha, \beta, \gamma$ are defined as follows:

$$
\tau = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.1 & 0.1 & 0 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}, \alpha = \begin{bmatrix} 0.4 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{bmatrix},
$$

$$
\beta = \begin{bmatrix} 0.4 & 0 & 0 \\ 0.9 & 0.4 & 0 \\ 0.4 & 0.4 & 0.4 \end{bmatrix}, \gamma = \begin{bmatrix} 0.9 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{bmatrix}.
$$

Suppose that $\widetilde{\Sigma}_o$ and $\widetilde{\Sigma}_{f_1}$ are defined as follows:

$$
\begin{aligned}
&\widetilde{\Sigma}_o(\tau) = 0.3, \quad \widetilde{\Sigma}_o(\alpha) = 0.5, \quad \widetilde{\Sigma}_o(\beta) = 0.4,\\
&\widetilde{\Sigma}_o(\gamma) = 0.6; \quad \widetilde{\Sigma}_{f_1}(\tau) = 0.4, \quad \widetilde{\Sigma}_{f_1}(\alpha) = 0.1,\\
&\widetilde{\Sigma}_{f_1}(\beta) = 0.2, \quad \widetilde{\Sigma}_{f_1}(\gamma) = 0.3.
\end{aligned}
$$

We can verify that the language $\mathcal{L}$ is not F_1-diagnosable. In fact, when $\sigma = \tau$, for arbitrary $n_i \in N$, we take $s = \tau$, $t = \alpha(\beta\gamma\alpha)^{n_i}$, and $\omega = \alpha(\beta\gamma\alpha)^{n_i}$, and then $\omega \in P_\sigma^{-1}(P_\sigma(st))$, but

$$
\widetilde{\Sigma}_{f_1}(\sigma) = 0.4 > 0.3 \geq \widetilde{\Sigma}_{f_1}(\omega).
$$

Therefore, by Definition 4, we know that $\mathcal{L}$ is not F_1-diagnosable with respect to τ. Of course, the result can also be obtained by the diagnoser G_d with respect to τ, which is constructed in Fig.7, since there does exist an F_1-indeterminate cycle in G_d.

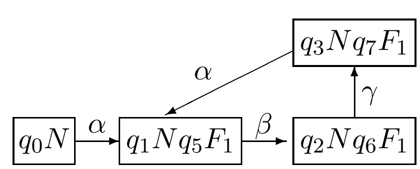

Fig.7. The diagnoser G_d w.r.t τ in Example 3.

The following is an example of diagnosability for an FDES with multiple failure types.

Example 4. Consider the fuzzy automaton $G = (Q, \Sigma, \delta, q_0)$ described in Example 3. The definition of $\widetilde{\Sigma}_o$ is the same as that in Example 3, but $\widetilde{\Sigma}_f = \widetilde{\Sigma}_{f_1} \widetilde{\cup} \widetilde{\Sigma}_{f_2}$, which is defined as follows:

$$\widetilde{\Sigma}_{f_1}(\tau) = 0.4, \quad \widetilde{\Sigma}_{f_1}(\alpha) = 0.1, \quad \widetilde{\Sigma}_{f_1}(\beta) = 0.2,$$
$$\widetilde{\Sigma}_{f_1}(\gamma) = 0.3; \quad \widetilde{\Sigma}_{f_2}(\tau) = 0.1, \quad \widetilde{\Sigma}_{f_2}(\alpha) = 0.2,$$
$$\widetilde{\Sigma}_{f_2}(\beta) = 0.3, \quad \widetilde{\Sigma}_{f_2}(\gamma) = 0.4.$$

The following is to verify that $\mathcal{L}$ is not F_1-diagnosable but F_2-diagnosable through constructing the diagnosers.

(1). If $\sigma = \tau$, then $P_\sigma(\tau) = \epsilon$, $P_\sigma(\alpha) = \alpha$, $P_\sigma(\beta) = \beta$, $P_\sigma(\gamma) = \gamma$ and $\Sigma_d = \{\alpha, \beta, \gamma\}$. Note that in the diagnoser G_d with respect to τ constructed as Fig.8, there exists an F_1-indeterminate cycle but there do not exist F_2-indeterminate cycles. Therefore, $\mathcal{L}$ is not F_1-diagnosable but F_2-diagnosable with respect to τ. Of course, this result can be verified by Definition 4, too. For failure type f_1, we take $s = \tau$, $t = \alpha(\beta\gamma\alpha)^{n_i}$ and $\omega = \alpha(\beta\gamma\alpha)^{n_i}$, then $\omega \in P_\sigma^{-1}(P_\sigma(st))$, but

$$\widetilde{\Sigma}_{f_1}(\sigma) = 0.4 > 0.3 \geq \widetilde{\Sigma}_{f_1}(\omega).$$

For failure type f_2, since $\widetilde{\Sigma}_{f_2}(\tau)$ is the least among $\{\widetilde{\Sigma}_{f_2}(a) : a \in \Sigma\}$, Ineq.(13) holds with $n_i = 0$.

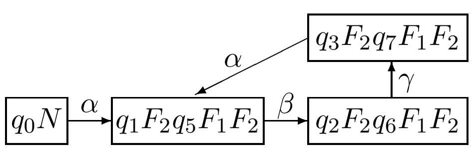

Fig.8. The diagnoser G_d w.r.t τ in Example 4.

(2). If $\sigma = \alpha$, then $P_\sigma(\tau) = P_\sigma(\alpha) = P_\sigma(\beta) = \epsilon$, $P_\sigma(\gamma) = \gamma$ and $\Sigma_d = \{\gamma\}$. Note that there do not exist F_1-indeterminate cycles or F_2- indeterminate cycles in the diagnoser with respect to α constructed in Fig.9, and $\mathcal{L}$ is both F_1-diagnosable and F_2-diagnosable with respect to α. In fact, Ineq.(13) holds for failure type f_1 with $n_i = 0$ and for f_2 with $n_i = 2$.

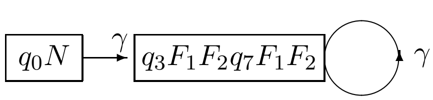

Fig.9. The diagnoser G_d w.r.t α in Example 4.

(3). If $\sigma = \beta$, then $P_\sigma(\tau) = P_\sigma(\beta) = \epsilon$, $P_\sigma(\alpha) = \alpha$, $P_\sigma(\gamma) = \gamma$, and $\Sigma_d = \{\alpha, \gamma\}$. There do not exist F_1-indeterminate cycles or F_2-indeterminate cycles in the diagnoser with respect to β, which is constructed as Fig.10, so $\mathcal{L}$ is both F_1-diagnosable and F_2-diagnosable with respect to β. In fact, Ineq.(13) holds for failure types f_1 and f_2 with $n_i = 1$.

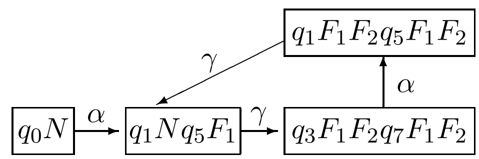

Fig.10. The diagnoser G_d w.r.t β in Example 4.

(4). If $\sigma = \gamma$, then $P_\sigma(\tau) = P_\sigma(\alpha) = P_\sigma(\beta) = \epsilon$, $P_\sigma(\gamma) = \gamma$, and $\Sigma_d = \{\gamma\}$. Since there do not exist F_1-indeterminate cycles or F_2- indeterminate cycles in the diagnoser with respect to γ constructed in Fig.11, $\mathcal{L}$ is both F_1-diagnosable and F_2-diagnosable with respect to γ. In fact, Ineq.(13) holds for failure type f_1 with $n_i = 3$ and for f_2 with $n_i = 0$.

Therefore, by Theorem 1, we know that $\mathcal{L}$ is not F_1-diagnosable but F_2-diagnosable.

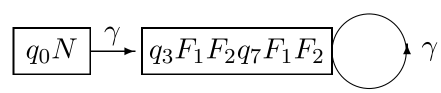

Fig.11. The diagnoser G_d w.r.t γ in Example 4.

VI. Concluding Remarks

In this paper, we dealt with the diagnosability in the framework of FDESs. We formalized the definition of diagnosability for FDESs, in which the observable set and the failure set of events are fuzzy. Then we constructed the observability-based diagnosers and investigated its some basic properties. In particular, we presented a necessary and sufficient condition for diagnosability of FDESs. Our results generalized the important consequences in classical DESs introduced by Sampath *et al* [30,31]. Moreover, the approach proposed in this paper may better deal with the problems of fuzziness, impreciseness and subjectivity in the failure diagnosis. As well, some examples serving to illuminate the applications of the diagnosability of FDESs were described.

As pointed out above, FDESs have been applied to biomedical control for HIV/AIDS treatment planning by Lin *et al* [20,21] and also to intelligent sensory information processing for robotics by R. Huq *et al* recently [10, 11]. The potential of applications of the results in this paper may be used in those systems. Moreover, with the results obtained in this paper, a further issue worthy of consideration is the I-diagnosability and the AA-diagnosability of FDESs, as those investigated in the frameworks of DESs [30] and stochastic DESs [36]. Another important issue is how to detect the failures in decentralized FDESs. Furthermore, FDESs modeled by fuzzy Petri nets [22] still have not been dealt with. We would like to consider them in subsequent work.

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