An Integrated Methodology for the Rapid Transit Network Design Problem

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Abstract. The Rapid Transit System Network Design Problem consists of two intertwined location problems: the determination of alignments and that of the stations. The underlying space, a network or a region of the plane, mainly depends on the place in which the system is being constructed, at grade or elevated, or underground, respectively. For solving the problem some relevant criteria, among them cost and future utilisation, are applied. Urban planners and engineering consulting usually select a small number of corridors to be combined and then analysed. The way of selecting and comparing these alternatives is performed by the application of the four-stage transit planning model. Due to the complexity of the overall problem, during last ten years some efforts have been dedicated to modelling some aspects as optimisation problems and to provide Operations Research methods for solving them. This approach leads to the consideration of a higher number of candidates than that of the classic corridor analysis. The main aim of this paper is to integrate the steps of the transit planning model (trip attraction and generation, trip distribution, mode choice and traffic equilibrium) into an optimisation process.

Keywords: Network Design, Rapid Transit Systems.

1 Introduction

Increasing mobility, longer trips caused by the enlargement of the urbanised areas and the reduction of average ground traffic speed are some of the reasons why during last 30 years new lines of rapid rail transit systems (metro, light rail, people mover, monorail, etc.) have opened in some agglomerations, while in others, systems are being constructed or planned.

The minimum threshold of population of a city or of a metropolitan area for needing a rapid rail transit system depends on density, private vehicle availability, traffic congestion, environmental aspects and other characteristics. This figure has diminished from about two millions inhabitants during the sixties to half a million at the end of last century. As a consequence the number of cities interested in such systems is increasing.

Because of the very large cost of constructing and operating rapid transit systems, it is important to pay close attention to their efficiency and effectiveness (Karlaftis, 2004, [7]). A crucial part of the planning process is the underlying network design, which consists of two intertwined problems: the determination of alignments and the location of stations.

The methodological support for forecasting the travel demand is a multi-stage process where different techniques can be used at each stage. Once the area under consideration has been adequately broken into study zones, the classical four-stage model is applied in order to finally obtain a prediction of the travel demand for the proposed transportation system.

1. Trip Generation Analysis: computation of the number of trips starting in each zone for each particular trip purpose.
2. Trip Distribution Analysis: production of a table containing the number of trips starting in each zone and ending in each other zone.
3. Modal Choice Analysis: allocation of trips among the currently available transportation systems (bus, train, pedestrian and private vehicles).
4. Trip Assignment Analysis: assignment of trip flows for the specific routes on each transportation system that will be selected by the users.

The four-stage process for selecting a network of lines of a mass transit system leads to the identification of a list of potential rapid transit corridors, which are assessed on the basis of several factors, among which the expected future ridership, computed by taking into account the modal split, is the most important. Corridors are then ordered and those that are selected are combined into several networks giving rise to different scenarios. Note that, since the number of alternatives is reduced to a very short list of corridors, this approach does not need to apply optimisation methods. However, it is probable that good candidates would be eliminated at an early stage or not considered at all, especially when the network can be underground.

During the last ten years some aspects of the planning problem have been modelled by optimisation models and solved by Operations Research techniques, thus allowing the consideration of a higher number of candidates than for the classical four-stage corridor analysis. The main efforts in this line of research have been oriented toward the determination of a single alignment and the location of stations given an alignment. Examples of the first type of application are the papers by Dufourd, Gendreau and Laporte (1996, [3]) and Bruno, Gendreau and Laporte (2002, [2]) when it is desired to maximise the coverage, and those by Bruno, Ghiani and Improta (1999, [1]) and Laporte, Mesa and Ortega (2005, [11]) which incorporate origin-destination matrix data, thus integrating the second stage of the classic model. García and Marín (2001, [4]; 2002, [5]) studied the transit network design problem using bilevel programming. They considered the multimodal traffic assignment problem with combined mode at the lower level.

Since an objective of the paper consists of encouraging the introduction of optimisation methods in the process of designing a rapid transit system, three stages are going to be identified below:

S1. Selecting key nodes
The network design will be based on the knowledge of the location of the main sites where trips are generated as origin or as destination.

S2. Designing the core network
A short list of lines, which are supported by the key nodes previously selected, will be determined so that the system effectiveness is maximised.

S3. Locating secondary stations
Once lines are broadly decided new secondary stations will be located, along the edges determined by pairs of key stations, in order to optimally increase the total coverage of the line.

This paper primarily addresses the second stage, although some comments about the first stage are also included in order to present their associated optimisation problems. The papers by Laporte, Mesa and Ortega (2002, [10]) and Hamacher, Liebers, Schöbel, Wagner and Wagner (2001, [6]) deal with the problem of locating stations on a given alignment; these are useful references for the study of the third stage.

The paper is organised as follows. In the next section the problem of selecting the main stations is discussed. Section 3 is dedicated to the problem of deciding how to connect the selected stations. Section 4 describes some computational experience based on an example.

2 Selection of Key Station Sites

As above mentioned above, once the area under consideration has been partitioned into zones, the number of trips that each zone will produce or attract must be quantified taking into account land use activities and the socio-economic characteristics of potential users.

As a general rule, zones are designed to include city blocks relatively homogeneous with respect to their urban activities (residential, commercial or industrial) and size. For instance, Figure 1 represents the city of Sevilla (Spain) split into 47 transportation zones. The two grey bands correspond to the branches of Guadalquivir river.

Some zones produce a high number of trips since they are densily populated (e.g., darker zones in Figure 1) and situated far away from the central area of the city (trip-generator zones). On the other hand, those that provide a large number of jobs (such as office zones or industrial areas) or contain important facilities (such as commercial zones, universities and hospitals) must be considered as trip-attractor zones.

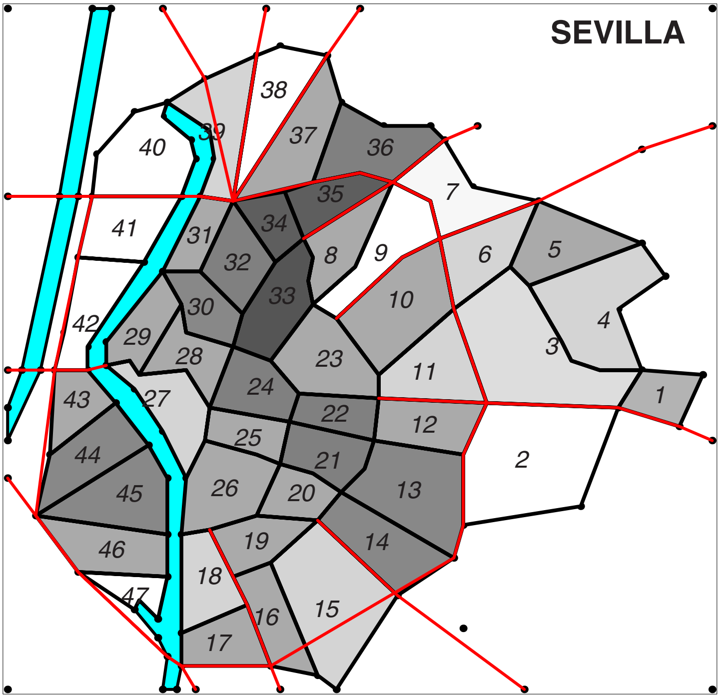

Fig. 1. Transportation zones in Sevilla

Figure 2 illustrates those sites in Sevilla which produce a high number of trips. Moreover, the most used roads for getting into the city from the towns of the metropolitan area have been drawn with a triangle indicating an access flow. Figure 2 also shows one line of the Metro of Sevilla (currently under construction; more details in http://www.urbanrail.net/eu/sev/sevilla.htm) based on the key node set which was selected.

Therefore, this simplified scenario with nineteen sites which produce a high number of trips contains:

- three centroids of zones densely populated (e.g., node WEST);
- four filled triangles point out those roads where the density of trips by public bus is higher. The number associated to each triangle is measured in thousands of users;
- twelve key nodes where the main facilities of the city are located: universities (nodes labelled by US1, US2, US3, US4 and UPO), hospitals (nodes HOS1 and HOS2), train stations (ST1,ST2), etc;
- an approach to the real drawing of Line 1 based on the earlier nodes.

Often, the trip-producing zones cannot be represented as a point in the model (as, for instance, a large university campus or a distant and dense area);

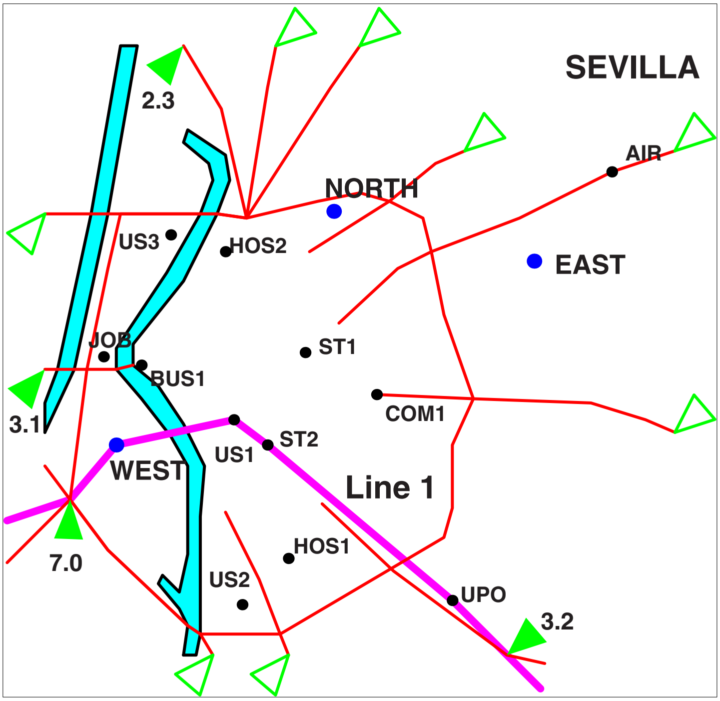

Fig. 2. Key node set and Line 1 of the Metro of Sevilla

therefore, some of these connecting centroids will be considered in the model as candidate sites for locate future stations of the new rapid transit network, channelling the zone demand total or partially. In order to obtain the most effective placement of the stations, a location problem on the zone should be solved.

Hence, let each demand node be denoted by $\mathbf{a}_k,\ k \in \mathcal{K} = \{1, \ldots, M\}$ in area A. We assume that each node $\mathbf{a}_k,\ k \in \mathcal{K} = \{1, \ldots, M\}$ is weighted with an average number w_k of inhabitants or visitors attracted by the service. Let $d_A(\mathbf{x}, \mathbf{y})$ be a planar distance measure, which is the best one fitted to the existing urban structure in zone A with $\| \cdot \|_A$ as associated norm: $d_A(\mathbf{x}, \mathbf{y}) = \| \mathbf{x} - \mathbf{y} \|_A$. Let $B_A(\mathbf{x}, r)$ denote the set of points in the plane whose distance to the station $\mathbf{x}$ is not greater than r (usually, called *ball* of radius r).

1. If the attraction model is assumed to be all-or-nothing and the solution set is continuous (all points in A are candidate sites), then a maximal covering location problem with fixed capture radius $r > 0$ must be solved to determine the best location for concentrating the area demand. For this purpose, let $\mathcal{I}(\mathbf{x}, r) = \{k \in \mathcal{K} :\ \mathbf{a}_k \in B_A(\mathbf{x}, r)\}$ be the index set necessary to formulate the maximal covering problem with fixed radius:

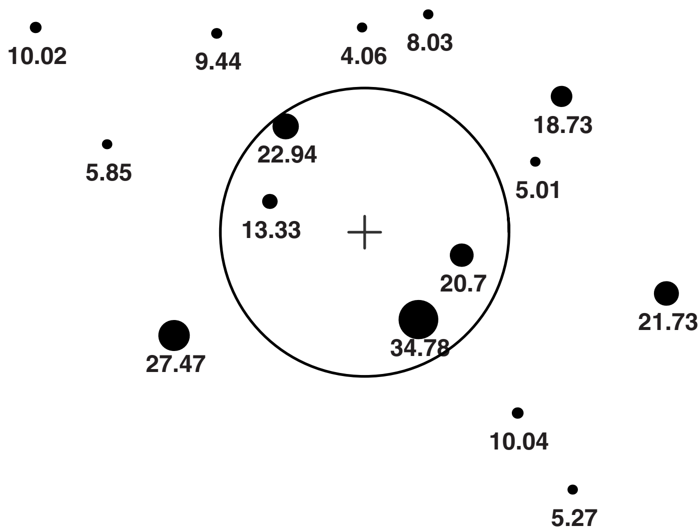

Fig. 3. A solution of the maximal covering problem

$$\max_{\mathbf{x}\in A} \sum_{k\in\mathcal{I}(\mathbf{x},r)} w_k.$$

Figure 3 illustrates a solution for the maximal covering problem where building blocks have been replaced by centroids; the radii of nodes are proportional to their weights which appear below.

The corresponding method which leads to the solution requires a number of steps is directly related to set cardinality M. A discussion on the continuous covering location problems can be found in Plastria (2002, [12]). The distance between the potential users and the station must be mathematically formulated by accurately fitting a representative sample of real travel times in the area under study. The distance type used in the approach can determine the solution to be adopted; see Laporte, Mesa and Ortega (2002, [10]) for an application arising from the planning of the Sevilla metro.

A more realistic approach to assess the catchment level around each station of an alignment, consists of establishing concentric geometrical shapes with decreasing attraction factors (Figure 4) which can vary following continuous or discrete models. A discrete version of this approach was developed by Dufourd, Gendreau and Laporte (1996, [3]) and by Bruno, Gendreau and Laporte (2002, [2]). On the other hand, a continuous model for the catchment problem, using a gravity model in order to maximize coverage, was applied by Laporte, Mesa and Ortega (2002, [10]) to determine the most effective pair of sites to locate two stations in a section of the metro of Sevilla.

2. If the set $\{\mathrm{s}_1, \mathrm{s}_2, \ldots, \mathrm{s}_L\}$ of candidate sites for the station is considered as discrete, then a discrete version of the covering problem can be used. Namely,

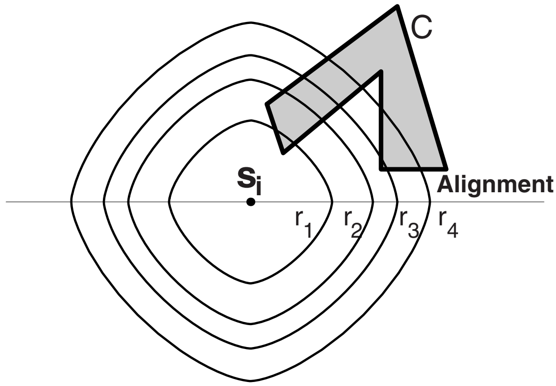

Fig. 4. Catchment area in census tract C from station S_i

$$\max_{l=1,\dots,L} \sum_{k\in\mathcal{I}(\mathrm{S}_l,r)} w_k.$$

Since the location of candidate sites is previously known, the attractiveness for the users can be better estimated by taking other additional considerations into account (like the transportation cost along the street network, the cost of establishing a facility, or the penalty cost for providing poor service to the users). This requires sophisticated integer linear programming models. Useful references on this topic are Kolen and Tamir (1990, [8]) and Schilling, Jayaraman and Barkhy (1993, [13]).

3 Core Network Design

After having located the stations that must belong to some of the lines of the network, the problem of connecting them with a small number of alignments $\mathcal{A} = \{A_l : \ l = 1, \dots, L\}$, with origins o_l and destinations d_l given, in competition with the private mode PRIV, is tackled. For this purpose and as a first approach, some geometric models like the Minimum Spanning Tree, the Minimum-Diameter Spanning Tree and the Steiner Tree problems could be applied. However, these approaches do not take into account the mode and route user decisions. For this reason an integer network design formulation considering the user and location decisions will be developed in this section.

3.1 Data

First of all, we will assume that the data required for the model are known, namely:

- the set $N = \{n_i, i = 1, 2, \dots, I\}$ of potential locations for key stations. Typically for a medium size agglomeration, $5 \le I \le 20$;

- the matrix $d = (d_{ij})$ of distances between pairs of points of N. Note that the entries of matrix d could correspond to (almost) Euclidean distances if the system were designed to be underground; otherwise, i.e. for a grade or elevated system, the data of matrix d should reflect the street network distances;
- the travel patterns given by the origin-destination matrix: $F = (f_p)$; $p = (q, r) \in P$, where P is the set of ordered pairs of demand points.

3.2 Formulating the Model

Let then E be the set of feasible edges linking the key stations. Therefore, we have a network $\mathcal{N} = (N, E)$ from which the core network is to be selected. For each node $n_i \in N$ denote by $N(i)$ the set of nodes adjacent to n_i. Let c_{ij} and c_i denote the costs of constructing a section of an alignment on edge ij and that of constructing a station at node n_i. The generalised routing cost (under demand point of view) of satisfying the demand of pair p through the private and the public network are c_p^{PRIV} and c_p^{PUB}, respectively. The first one is a given value, but the latter cost depends on the final topology of the public network and therefore on the edges that are selected; for this reason a generalised cost c_{ij}^{PUB} is given for each edge. This value is taken equal to the distance between i and j. Depending on the available budget for the total construction cost of each alignment, bounds $c_{\min}^l$, $c_{\max}^l$, $l = 1, \ldots, L$, on the construction cost of each alignment and bounds $c_{\min}$ and $c_{\max}$ on the total construction network cost are known.

The problem we are dealing with consists of choosing a low number L of lines (typically $1 \leq L \leq 5$) covering as much as possible the travel demand between the points of N, subject to constraints on the construction cost. The model has four decision variables:

1. the first variable is the binary variable which represents the selection of stations for each alignment: $y_i^l = 1$ if $n_i \in A_l$; $y_i^l = 0$ otherwise;
2. the second one is the binary variable associated to the selection of the specific edge used to build the alignment: $x_{ij}^l = 1$, if edge $ij \in E$ is selected for the alignment l; $x_{ij}^l = 0$ otherwise;
3. since edge ij will be included in the design of the public network depending on if the demand between pairs of origin-destination nodes is satisfied, a third variable is used for modelling that decision: $u_{ij}^p = 1$, if the demand of pair p of origin-destination nodes would use edge ij in the public network; $u_{ij}^p = 0$ otherwise;
4. the objective consists of deciding what particular pair of origin-destination nodes will be included in the final design. For assessing that decision $z_p = 1$ if the generalised cost for the demand of node pair p, through the public network $\mathcal{A}$, is less than that of the private mode; $z_p = 0$ otherwise.

Thus the problem can be stated in the following terms:

- Objective function: Trip covering
$\max \sum_{p\in P} f_p\, z_p$

- Cost constraints
$\sum_{ij\in E} c_{ij}\, x^l_{ij} + \sum_{i\in N} c_i\, y^l_i \in [c^l_{\min}, c^l_{\max}], \quad l = 1, 2, \ldots, L$ [1]
$\sum_{l=1}^{L} \sum_{ij\in E} c_{ij}\, x^l_{ij} + \sum_{i\in N} c_i \sum_{l=1}^{L} y^l_i \in [c_{\min}, c_{\max}]$ [2]

- Alignment location constraints
$\sum_{j\in N(o_l)} x^l_{o_l j} = 1, \quad l = 1, 2, \ldots, L$ [3]
$\sum_{i\in N(d_l)} x^l_{i d_l} = 1, \quad l = 1, 2, \ldots, L$ [4]
$x^l_{ij} = x^l_{ji} \quad ij \in E, \quad l = 1, 2, \ldots, L$ [5]
$y^l_{o_l} = y^l_{d_l} = 1, \; l = 1, \ldots, L$ [6]
$\sum_{j\in N(i)} x^l_{ij} = 2y^l_i, \quad i \in N \setminus \{o_l, d_l\}, \; l = 1, \ldots, L$ [7]

- Routing demand constraints
$\sum_{j\in N(q)} u^p_{qj} = 1, \quad p = (q, r) \in P$ [8]
$\sum_{i\in N(q)} u^p_{iq} = 0, \quad p = (q, r) \in P$ [9]
$\sum_{i\in N(r)} u^p_{ir} = 1, \quad p = (q, r) \in P$ [10]
$\sum_{j\in N(r)} u^p_{rj} = 0, \quad p = (q, r) \in P$ [11]
$\sum_{i\in N(j)} u^p_{ij} - \sum_{k\in N(j)} u^p_{jk} = 0, \quad j \in N, \; p = (q, r) \in P$ [12]

- Splitting demand constraints
$\sum_{ij\in E} c^{PUB}_{ij}\, u^p_{ij} - c^{PRIV}_p - M\,(1 - z_p) \le 0, \quad p \in P$ [13]

- Location-Allocation constraints
$u^p_{ij} + z_p - 1 \le \sum_{l=1}^{L} x^l_{ij}, \quad p \in P, \; ij \in E$ [14]
$x^l_{ij}, y^l_i, u^p_{ij}, z_p \in \{0, 1\}$

3.3 Description of the Constraints

Constraints [1] and [2] impose lower and upper bounds on the cost of each line and on the overall network, respectively. They could be simplified by considering only the individual and the total line lengths instead of costs.

- Length constraints
$\sum_{ij\in E} d_{ij}\, x^l_{ij} \in [\mathrm{length}^l_{\min}, \mathrm{length}^l_{\max}], \quad l = 1, 2, \ldots, L$ [1′]
$\sum_{l=1}^{L} \sum_{ij\in E} d_{ij}\, x^l_{ij} \in [\mathrm{Tlength}_{\min}, \mathrm{Tlength}_{\max}]$ [2′]

Constraints [3] and [4] guarantee that each line starts and ends at its specified origin and destination. Note that, although the trip flow is directed, the network in undirected; therefore, the decision of connecting two nodes must involve flows in both directions, as constraints [5] express. Constraints [6] ensure that all origins and destinations belong to $\mathcal{A}$. Constraints [7] impose that each line must be a path between the corresponding origin and destination.

Constraints [8], [9], [10], [11] and [12] guarantee demand conservation. Constraints [13] were introduced to ensure that $z_p = 1$ when the demand of pair p goes through the public network and $z_p = 0$ if it uses the private network. Finally, constraints [14] guarantee that a demand is routed on an edge only if this edge belongs to the public system.

Note that this formulation does not include the common subtour elimination constraints, but when a solution contains a cycle, then such a constraint is imposed. However well developed networks (e.g. Paris, London, Moscow, Tokyo and Madrid) often contain circular lines. It has also been proved by Laporte, Mesa and Ortega (1997, [9]) that the inclusion of a circle line often increases the effectiveness of the network and thus the inclusion of cycles may be interesting.

4 Computational Experiments and Conclusions

The integer model of the latter section was implemented in GAMS 2.0.27.7, which calls CPLEX 9.0, and tested on the 6-node network $\mathcal{N} = (N, E)$ of Figure 5.

Each node n_i has an associated cost c_i and each edge ij has a pair (c_{ij}, d_{ij}) of associated weights: the first coordinate is the cost c_{ij} of constructing the edge and the second is the distance d_{ij} (which in the experiments is also the generalised public cost of using the edge) between nodes n_i and n_j.

The origin-destination demands $f_p;\ p = (q, r) \in P$ and the private cost c_p^{PRIV} for each demand pair $p \in P$ are given in the following matrices.

$$F = \begin{pmatrix} - & 9 & 26 & 19 & 13 & 12 \\ 11 & - & 14 & 26 & 7 & 18 \\ 30 & 19 & - & 30 & 24 & 8 \\ 21 & 9 & 11 & - & 22 & 16 \\ 14 & 14 & 8 & 9 & - & 20 \\ 26 & 1 & 22 & 24 & 13 & - \end{pmatrix};\ C^{PRIV} = \begin{pmatrix} - & 1.6 & 0.8 & 2 & 2.6 & 2.5 \\ 2 & - & 0.9 & 1.2 & 1.5 & 2.5 \\ 1.5 & 1.4 & - & 1.3 & 0.9 & 2 \\ 1.9 & 2 & 1.9 & - & 1.8 & 2 \\ 3 & 1.5 & 2 & 2 & - & 1.5 \\ 2.1 & 2.7 & 2.2 & 1 & 1.5 & - \end{pmatrix}$$

Although the levels of congestion are not uniform in a real context (in fact, they depend on the time of the day, the day of the week, the month, the weather condition as well as the area of the city), different congestion coefficients (which are contributing factors to the private costs) have been considered. The following tables show, for each case, the lines that compose the optimal network and the corresponding values of the objective function. For Table 1, the total length of the network is an active constraint, whereas a maximum location cost is the additional constraint considered for Table 2.

Times spent in the transfers was not taken into account in the model. Location cost for multiple stations (stations with line transfers) was considered proportional to the location cost of a single station, with a proportionality factor equal to the number of lines crossing the corresponding station.

The results are quite similar for the two different constraints considered: the optimal network is composed of a single line when the restriction level is higher,

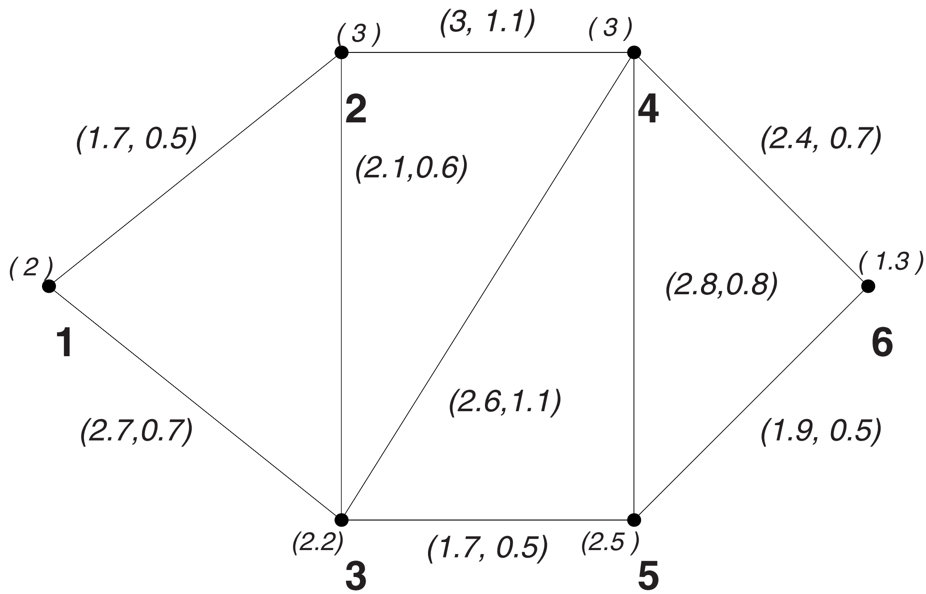

Fig. 5. Network

or composed of two lines if the constraint is less restrictive. In some cases more than one configuration reaches the maximum covering for the demand given. The accuracy of the model can be confirmed by observing that when congestion increases, the value of the objective function also increases, giving rise to a more important use of the public transit system.

The experiment was carried out on a Pentium IV laptop computer at 2.56 Mghz. Since all the results were obtained within seconds we hope the model would be useful for larger instances. In particular, for a 9-node network the computation time, once the origin and destination nodes have been previously fixed, is 27.19 seconds, whereas the exploration for all possible solutions for takes 195905.91 seconds (more than five hours).

Further computational experiments should be done for real-world contexts. In order to obtain valuable results, good data are required and some relaxation on the 0/1 allocation of the demand should be done. However, there exist several factors such as security, convenience and availability of private cars, all having some influence on the potential ridership, which can hardly be taken into account in models.

On the other hand, an interesting extension of the previous maximal covering problems with fixed radius, dealt with in Section 2, arises when an attraction gravity model is combined in the objective:

$$\max_{\mathbf{x}\in A} \sum_{k\in\mathcal{I}(\mathbf{x},r)} \frac{\rho w_k}{d^*(\mathbf{x},\mathbf{a}_k)^2}$$

where parameter $\rho > 0$ represents some proportional constant and $d^*(\cdot,\cdot)$ is an hyperbolic approach to the Euclidean distance, in order to avoid the singularities of the function. As far as the authors are aware, no work on this gravitational version of the maximal covering problem with fixed radius has been carried out.

Table 1. Maximum total length constraint [2'] active

TABLE I	$\text{Tlength}_{\text{max}} = 2$		$\text{Tlength}_{\text{max}} = 4$	
Congestion	Line(s)	Value	Line(s)	Value
0.75	$n_2 - n_3 - n_5 - n_6$	168	$n_1 - n_3 - n_5 - n_6 - n_4$ $n_2 - n_3 - n_5$	319
1	$n_1 - n_3 - n_5 - n_6$	216	$n_1 - n_2 - n_3 - n_5 - n_6$ $n_2 - n_4 - n_5$	446
1.5	$n_1 - n_3 - n_5 - n_4$	227	$n_1 - n_2 - n_4 - n_5 - n_6$ $n_2 - n_3 - n_5$	496

Table 2. Maximum location cost [2] active

TABLE II	$\text{c}_{\text{max}} = 15$		$\text{c}_{\text{max}} = 30$	
Congestion	Line(s)	Value	Line(s)	Value
0.75	$n_2 - n_3 - n_5 - n_6$	168	$n_1 - n_2 - n_3 - n_4 - n_6$ $n_3 - n_5 - n_6$	327
1	$n_1 - n_3 - n_5 - n_6$	216	$n_1 - n_3 - n_2$ $n_4 - n_3 - n_5 - n_6$	430
1.5	$n_1 - n_3 - n_5 - n_6$	216	$n_3 - n_4 - n_6$ $n_1 - n_2 - n_3 - n_5 - n_6$	496

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