A Provably Secure Ring Signature Scheme in Certificateless Cryptography *

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Abstract

Ring signature is a kind of group-oriented signature. It allows a member of a group to sign messages on behalf of the group without revealing his/her identity. Certificateless public key cryptography was first introduced by Al-Riyami and Paterson in Asiacrypt 2003. In certificateless cryptography, it does not require the use of certificates to guarantee the authenticity of users' public keys. Meanwhile, certificateless cryptography does not have the key escrow problem, which seems to be inherent in the Identity-based cryptography. In this paper, we propose a concrete certificateless ring signature scheme. The security models of certificateless ring signature are also formalized. Our new scheme is provably secure in the random oracle model, with the assumption that the Computational Diffie-Hellman problem is hard. In addition, we also show that a generic construction of certificateless ring signature is insecure against the key replacement attack defined in our security models.

1 Introduction

In Asiacrypt 2001, Rivest, Shamir and Tauman [22] introduced the concept of ring signature, which makes it possible to specify a set of possible signers without revealing which member actually produced the signature. As pointed in [22], ring signatures provide an elegant way to leak authoritative secrets in an anonymous way, to sign casual email in a special way that can

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only be verified by its intended recipient, anonymous membership authentication for ad hoc groups [5], etc. In addition, ring signatures can also be served as the building block of concurrent signatures and solve some other problems in multiparty computations.

Ring signatures can be regarded as the simplified group signatures that have only users and no managers. Group signatures are useful in the situation where the members want to cooperate, while ring signatures are useful when the members do not want to cooperate. Both group signatures and ring signatures are signer-ambiguous. However, group signatures have the additional feature that the anonymity of a signer can be revoked (i.e., the signer can be traced) by a designated group manager. Ring signatures allow greater flexibility: no centralized group manager or coordination among the various users is required (indeed, users may be unaware of each other at the time they generate their public keys); rings may be formed completely in an ad-hoc manner, do not require any coordination among the various users (indeed, users do not even need to be aware of each other) and full anonymity (unless the actual signer decides to expose himself). To produce a ring signature, the actual signer declares an arbitrary set of possible signers that must include himself, and computes the signature entirely by himself using only his private key and the others' public keys.

In traditional public key cryptosystem (PKC), each user U has a pair of cryptographic keys-a public key and a private key. The private key is kept secret by the user himself, while the public key may be widely distributed. Anyone can encrypt messages with U's public key and obtain the ciphertexts which can only be decrypted with U's private key. Similarly, one can use U's public key to verify if a signature is generated by U. Therefore, there is no need for the sender and receiver to share secret information before the communication. The biggest challenge in PKC is ensuring the authenticity of public key, that is how to bind a user and his public key. Suppose Alice wants to encrypt a message to send to Bob, and Bob is someone who Alice does not know personally, how can Alice be sure that Bob's purported public key really is Bob's key (and not Charlie's, for example). If Alice uses a "false" public key to encrypt the message and send the ciphertext to Bob, it will result that the intended receiver Bob can not obtain the message, and even worse, someone else can decrypt the ciphertext and read the message. The usual approach to solve this problem is to use a public key infrastructure (PKI), in which one or more third parties, known as certificate authorities, issue certificates to bind a user and his public key. In traditional PKC, one must first check the authenticity of the pair (U, PK)by verifying the validity of its certificate before any operation regarding the user U. History has shown that certificates in traditional PKC are generally considered to be costly to use and manage. It is even more problematic for a ring signature scheme in traditional PKC, where the signer must first verify all the certificates of group members before generating the ring signature on behalf of that group, otherwise his anonymity is jeopardized under the extreme case that all other ring members' certificates are indeed invalid. Given a ring signature, the verifier must perform the same verification as well before checking the validity of the ring signature. This will lead to the inefficiency of the whole scheme because the computation cost will increase linearly with the group size.

In 1984, Shamir [23] first proposed the Identity-Based public key cryptography (ID-PKC), in which the public key of a user is some unique public information about the identity of the user (e.g. a user's email address) [9, 17]. Therefore, the need of certification can be eliminated. A Trusted Third Party, called the Private Key Generator (PKG), generates the corresponding private keys for the users in ID-PKC. To operate, the PKG first publishes a "master" public key, and keeps the corresponding master private key as secret. Given the master public key, any party can compute a public key corresponding to an identity ID by combining the master public key with the identity value. To obtain a corresponding private key, the party authorized to use the identity ID contacts the PKG, which uses the master private key to generate the private key for the identity ID. However, this approach creates a new inherent problem, namely the key escrow of a user's private key, since PKG must be completely trusted. This is due to the knowledge of the PKG on the user's private key. For a ring signature scheme in ID-PKC, a malicious PKG can forge a ring signature on behalf of any group without being detected.

In order to enjoy the implicit certification property of ID-PKC while without suffering from its inherent key escrow problem, Al-Riyami and Paterson [2] proposed a new paradigm called certificateless public key cryptography (CL-PKC). Different from ID-PKC, a third party which we call Key Generation Center (KGC) in CL-PKC does not have the access to a user's private key. Instead, the KGC supplies a user with a partial private key, which derives from the user's identity. Then the user combines the partial private key with some secret information chosen by himself to generate his actual private key. The corresponding public key is computed from the system's public parameters and the secret information chosen by the user, which is finally published in the system. Hence, it is no longer an identity-based cryptography, since the public key needs to be provided (but in contrast to the traditional public key cryptography, the public key does

not require any certificate).

Due to the lack of certification in CL-PKC, it is conceivable that the adversary can replace anyone's public key of his choice. This key replacement attack is also called *Type I adversary* in [2]. Obviously, a secure signature signature scheme in CL-PKC must have the property that it is infeasible for *Type I adversary* to create a valid signature under the false public key chosen by the adversary himself. An assumption that must be made is that KGC does not mount a public key replacement attack to a target user since he is armed with this user's partial private key. However, KGC might engage in other adversarial activities: eavesdropping on signatures and making signing queries, which is also known as *Type II Adversary*. In this way, the level of trust is similar to the trust in a CA in a traditional PKI.

1.1 Motivations

Certificateless cryptography have some advantages over traditional PKC and ID-PKC in some aspects [28, 29]. As a useful primitive, ring signatures have been studied in traditional PKC and ID-PKC for more than five years. Even in a theoretic point of view, ring signatures should be studied in CL-PKC to rich the theories and techniques of CL-PKC. In practice, to generate a ring signature on behalf of a group in traditional PKC, the signer must first verify all the certificates of the group members, otherwise his anonymity is jeopardized and the ring signature will be rejected if he uses invalid certificates of some group members. Given a ring signature, the verifier must perform the same verification as well before checking the validity of the ring signature. These verifications inevitably lead to the inefficiency of the whole scheme since the computational cost increases linearly with the group size. Although Identity-based ring signatures eliminate such costly verifications, they suffer from a security drawback induced by the inherent key escrow problem of ID-PKC. Namely, a malicious PKG can always issue valid ring signatures on behalf of any group. As CL-PKC does not use public key certificates, and in the meantime, it removes the key escrow problem of ID-PKC, we think it supplies an appropriate environment for implementing ring signatures. So it is necessary to extend the notion and security model of ring signatures to CL-PKC. Compared with ring signature schemes in traditional PKC, in a CL-Ring scheme, both the signer and the verifier can avoid the costly verification of group members' certificates. On the other hand, in contrast to ID-based ring signatures, the KGC can no longer forge a ring signature on behalf of a group without being detected.

In application aspects, like ring signatures in traditional PKC and ID-

PKC, certificateless ring signatures can also be used in leaking authoritative secrets in an anonymous way, anonymous membership authentication for ad hoc groups [5], reports to the authorities embezzlement and corruption, certificateless designated signatures and concurrent signatures, etc.

1.2 Our Contributions

In this paper, we introduce the notion of ring signature into certificateless cryptography and propose a concrete certificateless ring signature scheme.

Firstly, we provide the security models of certificateless ring signatures. Two types of adversaries: Type I adversary \mathcal{A}_I and Type II adversary \mathcal{A}_{II} have been formally defined. The above two adversaries in our definition are "super adversaries" [33]. That is, the adversary can get valid ring signatures of the group whose public keys have been replaced, without supplying the secret values that are used to generate those public keys. In addition, our models also capture the group-changing attack [18] in the notion of ring signatures.

Secondly, we give an analysis of a "seem-secure" generic construction of certificateless ring signatures. The generic construction of certificateless signatures was first proposed by Yum and Lee [26], which has been shown insecure in [14]. Hu et al. also presented a secure construction of certificateless signatures [14]. Using the similar methods in [14], one can also get a generic construction of certificateless ring signatures. However, as we will show later, the resulting generic construction of certificateless ring signatures is totally insecure against the key replacement attack.

Lastly, we present a concrete construction of certificateless ring signatures. The new scheme uses the bilinear pairing on elliptic curves and concretely, the signing phase requires 2 pairings and the verification requires 3 pairings. We prove its security in the random oracles, with the assumption that Computational Diffie-Hellman problem is intractable.

Organization.

The rest of the paper is organized as follows. In the next section, we review some preliminaries which are required in this paper. Section 4 defines the security models in the notion of certificateless ring signatures. We analyze a generic construction of certificateless ring signatures and show its insecurity in Section 5. The concrete construction of certificateless ring signature is proposed in Section 6. Its security proofs are given in Section 7. Finally, Section 8 comes our conclusion.

2 Related Work

Following the prior work of Rivest, Shamir and Tauman [22], a number of constructions of ring signature in traditional PKC and ID-PKC have been presented. Abe, Ohkubo, and Suzuki [1] provided a construction applicable for several categories of public keys (e.g., integer factoring based and discrete-log based). A simple ring signature using bilinear maps was given in [4]. Herranz and Saez [12] generalized the forking lemma to the ring signatures. In [27], Zhang and Kim extended the concept to Identity-Based ring signature (IDRS) schemes. Some ring signature schemes with constant-size were also presented in [10, 21].

In terms of security models for provably secure ring signature schemes, there are three models commonly used. They provide different security levels. The first and the weakest model was introduced by Rivest et al. [22]. Later Abe et al. [1] proposed a very strong model. Finally, Liu and Wong [18] presented a model whose security level is considered to be lying in between the two foregoing models. We mainly use the ideas of constructing IDRS schemes in [13], and the security models of ring signatures in [18] in this paper.

CL-PKC has got fruitful achievements since its introduction in [2, 6, 7, 11, 25, 30, 31]. Al-Riyami and Paterson presented [2] the first certificateless signature (CLS) scheme. Since then, several CLS schemes [14, 16, 19, 20, 24, 26, 32, 35] were proposed. In [15], Huang et al. defined the security model of CLS schemes. Zhang et al. [35] improved the security model of CLS schemes, and presented a secure CLS scheme. Generic ways to construct CLS schemes were investigated in [26], [14]. In [16], a certificateless proxy signature scheme was proposed. An work about certificateless ring signature was done by Chow and Yap [8]. The security of their scheme is based on the hardness of the k-CAA problem and Modified Inverse Computational Diffie-Hellman problem and is proved in a weak model that requires a type I adversary to submit the secret values corresponding to the replaced public keys to the challenger in the sign queries. The computional cost of their scheme involves a large amount of paring operations which linearly increase with the number of group members.

3 Preliminaries

In this section, we will review some fundamental backgrounds required in this paper.

3.1 Bilinear Pairings and Computational Problems

Let G_1 be an additive group of prime order q and G_2 be a multiplicative group of the same order. Let P denote a generator of G_1 . A mapping $e: G_1 \times G_1 \longrightarrow G_2$ is called a bilinear mapping if it satisfies the following properties:

- 1. Bilinear: $e(aP, bQ) = e(P, Q)^{ab}$ for all $P, Q \in G_1, a, b \in Z_a^*$.
- 2. Non-degeneracy: There exists $P, Q \in G_1$ such that $e(P,Q) \neq 1$.
- 3. Computable: There exists an efficient algorithm to compute e(P,Q) for any $P,Q \in G_1$.

For a group G of prime order, we denote the set $G^* = G \setminus \{\mathcal{O}\}$, where \mathcal{O} is the identity element of the group.

Discrete Logarithm (DL) Problem: Given a generator P of a cyclic additive group G with order q, and $Q \in G^*$ to find an integer $a \in Z_q^*$ such that Q = aP.

Computational Diffie-Hellman (CDH) Problem: Given a generator P of a cyclic additive group G with order q, and given (aP,bP) for unknown $a,b \in \mathbb{Z}_q^*$; to compute abP.

3.2 The Concept of Certificateless Ring Signature Schemes

A CL-Ring scheme is defined by seven algorithms: Setup, Partial-Private-Key-Extract, Set-Secret-Value, Set-Private-Key, Set-Public-Key, Ring-Sign and Verify. The description of each algorithm is as follows.

- Setup: This algorithm runs by the KGC that takes as input a security parameter ℓ to produce a masterkey and a list of system parameters param.
- Partial-Private-Key-Extract: This algorithm runs by the KGC that takes as input a user's identity ID, a parameter list param and a masterkey to produce the user's partial private key D_{ID} .
- Set-Secret-Value: This algorithm takes as input a parameter list param and a user's identity *ID* to produce the user's secret value *x*.
- Set-Private-Key: This algorithm takes as input a parameter list param, a user's identity ID, the user's partial private key D_{ID} and secret value x to produce a private signing key S_{ID} for this user.

- Set-Public-Key: This algorithm takes as input a parameter list param, a user's identity ID and secret value x_{ID} to produce a public key P_{ID} for the user.
- Ring-Sign: This algorithm takes as input a message $M \in \mathcal{M}, \mathcal{M}$ is the message space, a set of n group members whose identities form the set $L_{ID} = \{ID_1, ..., ID_n\}$ and their corresponding public keys form the set $L_{PK} = \{P_{ID_1}, ..., P_{ID_n}\}$, a parameter list param and a singer's signing key S_{ID_s} to produce a ring signature σ . Here S_{ID_s} is the s-th group member's private key.
- Verify: This algorithm takes as input a message M, a ring signature σ , a parameter list param, the set L_{ID} of the group members' identities and the set L_{PK} of the corresponding public keys of the group members to output True if the signature is correct, or False otherwise.

4 Security Models of Certificateless Ring Signature Schemes

There are two types of adversaries in the certificateless system: namely $Type\ I\ Adversary$ and $Type\ II\ Adversary$. A $Type\ I\ Adversary\ \mathcal{A}_I$ simulates attacks when the adversary (anyone except the KGC) replaces the user's public key with a value of his/her choice. However, \mathcal{A}_I is not given this user's partial private key D_{ID} (and system's masterkey). On the other hand, a $Type\ II\ Adversary\ \mathcal{A}_{II}$ has access to the masterkey but cannot perform public key replacement.

Combining the security notions of certificateless public key cryptography and traditional ring signature schemes, we define the security of a CL-Ring scheme via the following two games between a challenger \mathcal{C} and an adversary \mathcal{A}_I or \mathcal{A}_{II} .

Game 1: Unforgeability of CL-Ring against Type I Adversary A_I

Setup: C runs the Setup algorithm, takes as input a security parameter ℓ to obtain a masterkey and the system parameter param. C then sends param to the adversary A_I while keeping the masterkey as secret. In addition, C will maintain three lists L_1, L_2, L_3 where

• L_1 is used to record the identities which have been chosen by A_I in the Partial-Private-Key Queries.

- L_2 is used to record the identities whose public keys have been replaced by A_I .
- L_3 is used to record the identities which have been chosen by A_I in the Private-Key Queries.

All these three lists L_1, L_2, L_3 are the empty set \emptyset at the beginning of the game.

Training: The adversary A_I can adaptively issue a polynomially bounded number of queries as defined below:

- Partial-Private-Key Queries PPK(ID): A_I can request the partial private key of any user whose identity is ID. In respond,
 - 1. C first resets $L_1 = L_1 \cup \{ID\}$.
 - 2. C then runs the algorithm Partial-Private-Key-Extract and outputs the partial private key D_{ID} .
- Public-Key Queries PK(ID): A_I can request the public key of a user whose identity is ID. In respond,
 - 1. C first runs the algorithm Set-Secret-Value and obtains the secret value x_{ID} .
 - 2. C then runs the algorithm Set-Public-Key and obtains the public key P_{ID} . C outputs the public key P_{ID} as the answer.
- Public-Key-Replacement Queries $PKR(ID, P'_{ID})$: For any user whose identity is ID, \mathcal{A}_I can choose a new public key P'_{ID} . \mathcal{A}_I then sets P'_{ID} as the new public key of this user and submits (ID, P'_{ID}) to \mathcal{C} . On receiving a query $PKR(ID, P'_{ID})$, \mathcal{C} resets $L_2 = L_2 \cup \{ID\}$ and updates the public key of this user to the new value P'_{ID} .
- Private-Key Queries PrK(ID): A_I can request the private key of a user whose identity is ID. In respond,
 - 1. C first checks the set L_2 . If $ID \in L_2$ (that is, the public key of the user ID has been replaced), C will return the symbol \bot which means C cannot output the private key of an identity whose public key has been replaced.
 - 2. Otherwise, $ID \notin L_2$ and \mathcal{C} resets $L_3 = L_3 \cup \{ID\}$. \mathcal{C} then runs the algorithm Set-Private-Key and outputs the private key S_{ID} .

• Ring-Sign Queries $RS(M, L_{ID}, L_{PK})$: \mathcal{A}_I can request the ring signature of a message M on behalf of a group whose identities are listed in the set L_{ID} and the corresponding public keys are in the set L_{PK} . In respond, \mathcal{C} outputs a ring signature σ for the message M. It is required that the algorithm Verify will output True for the input $(M, \sigma, \mathsf{param}, L_{ID}, L_{PK})$.

Forgery: Finally, \mathcal{A}_I outputs a tuple $(M^*, \sigma^*, L_{ID}^*, L_{PK}^*)$ as the forgery. We say \mathcal{A}_I wins the game if the forgery satisfies all the following requirements:

- 1. The algorithm Verify outputs True for the input $(M^*, \sigma^*, \mathsf{param}, L^*_{ID}, L^*_{PK})$.
- 2. $L_{ID}^* \cap L_1 \cap L_2 = \emptyset$ and $L_{ID}^* \cap L_3 = \emptyset$.
- 3. $(M^*, L_{ID}^*, L_{PK}^*)$ has never been queried during the Ring-Sign Queries.

Game 2: Unforgeability of CL-Ring against Type II Adversary A_{II}

Setup: C runs the Setup algorithm, takes as input a security parameter ℓ to obtain the system parameter list param and also the system's masterkey. C then sends param and masterkey to the adversary A_{II} . C will maintain two lists L_1, L_2 where

- L_1 is used to record the identities whose public keys have been replaced by A_{II} .
- L_2 is used to record the identities which have been chosen by \mathcal{A}_{II} in the Private-Key Queries.

Both two lists L_1, L_2 are empty at the beginning of the game.

Training: As defined in **Game 1**, the type II adversary \mathcal{A}_{II} can issue a polynomially bounded number of Public Key Queries, Private-Key Queries, Public-Key-Replacement Queries and Ring-Sign Queries. \mathcal{C} will answer those queries as same in **Game 1**. Note that \mathcal{A}_{II} does not need to issue Partial-Private-Key queries because he has already known the system's masterkey.

Forgery: Finally, \mathcal{A}_{II} outputs a tuple $(M^*, \sigma^*, L_{ID}^*, L_{PK}^*)$ as the forgery. We say \mathcal{A}_{II} wins the game if the forgery satisfies all the following requirements:

- 1. The algorithm Verify outputs True for the input $(M^*, \sigma^*, \mathsf{param}, L^*_{ID}, L^*_{PK})$.
- 2. $L_{ID}^* \cap L_1 = \emptyset$ and $L_{ID}^* \cap L_2 = \emptyset$.

3. $(M^*, L_{ID}^*, L_{PK}^*)$ has never been queried during the Ring-Sign Queries.

Definition 4.1 A CL-Ring scheme is existentially unforgeable under adaptively chosen-message attack iff the success probability of any polynomially bounded adversary in the above two games is negligible.

Definition 4.2 A CL-Ring scheme is said to have the unconditional signer anonymity if for any group of n users whose identities form the set L_{ID} and their corresponding public keys form the set L_{PK} , any message M and any ring signature $\sigma = Ring-Sign(M, L_{ID}, L_{PK}, S_{ID_s})$, any verifier \mathcal{V} cannot identify the actual signer with probability better than a random guess. That is, \mathcal{V} can only output the actual signer with probability no better than $\frac{1}{n}$ ($\frac{1}{n-1}$ when \mathcal{V} is in the signers' ring).

5 Analysis of A Generic Construction of CL-Ring

In [26], Yum and Lee presented a generic way to construct a certificateless signature scheme. However, Hu et al. [14] pointed out that their construction is flawed and proposed a new one. It seems at first glance that the methods in [14] can also be used to obtain a generic construction of CL-Ring signatures. However, as we will show later, the resulting scheme is not secure in our security model defined in Section 4.

5.1 A Generic Construction of CL-Ring

Let $\prod_{PK} = (Gen_{PK}, Ring-Sign_{PK}, Ver_{PK})$ be a traditional public keybased ring signature scheme which is existentially unforgeable under adaptively chosen-message attack. Gen_{PK} takes a security parameter as input and generates a public/secret pair (pk_{PK}, sk_{PK}) ; $Ring-Sign_{PK}$ takes a private signing key, a set of public keys and a message as inputs, and generates a ring signature σ_{PK} ; and Ver_{PK} is the corresponding ring signature verification algorithm.

Let $\prod_{ID} = (Gen_{ID}, KGen_{ID}, Ring-Sign_{ID}, Ver_{ID})$ be an identity-based ring signature scheme that is existentially unforgeable under adaptively chosen-message and identities attacks. Gen_{ID} takes a security parameter as input and generates a master secret key masterkey and a list of system parameters param; $KGen_{ID}$ is an identity-based secret key generation algorithm which takes masterkey and an identity ID and generates a secret key denoted by D_{ID} ; $Ring-Sign_{ID}$ takes a private signing key, a set of identities and a message as inputs, and generates an identity-based ring signature

denoted by σ_{ID} ; and Ver_{ID} is the corresponding ring signature verification algorithm.

As defined in Section 4, a CL-Ring signature scheme consists of seven algorithms. Using the similar methods in [14], we can obtain a generic construction of CL-Ring as described in Fig 1.

5.2 Security Analysis of the Generic Construction

In this section, we will show that the generic construction described in Fig 1 is not secure under the definition in Section 4.

We firstly show that a type I adversary A_I can forge a valid ring signature of any message M. The attack algorithm is described as below:

• A_I first chooses n identities $(ID_1, ID_2, \cdots, ID_n)$ and sets

$$L_{ID} = \{ID_1, ID_2, \cdots, ID_n\}.$$

- As defined in the Game 1 in Section 4, A_I then issues n Public-Key queries to obtain the corresponding public keys $(P_{ID_1}, P_{ID_2}, \dots, P_{ID_n})$.
- \mathcal{A}_I runs the algorithm Set-Secret-Value to generate a secret value x_{ID_i} for the user $ID_i \in L_{ID}$. It also runs the algorithm Set-Public-Key to obtain a public key P'_{ID_i} . Finally, it replaces ID_i 's public key with P'_{ID_i} and sets

$$L_{PK} = \{P_{ID_1}, P_{ID_2}, \cdots, P'_{ID_i}, \cdots, P_{ID_n}\}.$$

- \mathcal{A}_I then submits a partial private key query for an identity $ID_j \in L_{ID}$ and obtains the partial private key D_{ID_j} , with the only requirement that $ID_i \neq ID_i$.
- For any message M, A_I sets $M' = M||param||L_{ID}||L_{PK}$ and uses x_{ID_i} to compute

$$\sigma_{PK} = Ring - Sign_{PK}(x_{ID_i}, L_{PK}, M').$$

• It then sets $M'' = M||param||L_{ID}||L_{PK}||\sigma_{PK}$ and uses D_{ID_j} to compute

$$\sigma_{ID} = Ring - Sign_{ID}(D_{ID_i}, L_{ID}, M'')$$

• A_I outputs $(M, \sigma = \sigma_{PK} || \sigma_{ID}, L_{ID}, L_{PK})$ as the forgery.

Figure 1: A Generic Construction of CL-Ring

- Setup: On input 1^{ℓ} , the KGC runs $Gen_{ID}(1^{\ell})$ to produce a masterkey and a list of system parameters param.
- Partial-Private-Key-Extract: This algorithm accepts param, masterkey, a user's identity $ID \in \{0,1\}^*$ and runs $KGen_{ID}(param, masterkey, ID)$ to output the user's partial private key D_{ID} .
- Set-Secret-Value: This algorithm accepts param, a user's identity ID and runs $Gen_{PK}(1^{\ell})$ to generate a public/secret (P_{ID}, x_{ID}) pair and returns x_{ID} as user's secret value.
- Set-Private-Key: This algorithm takes as input a parameter list param, a user's identity ID, the user's partial private key D_{ID} and secret value x_{ID} to produce a private signing key $S_{ID} = (x_{ID}, D_{ID})$ for this user.
- Set-Public-Key: This algorithm takes as input a parameter list param, a user's identity ID, (P_{ID}, x_{ID}) and outputs public key P_{ID} for the user.
- Ring-Sign: This algorithm takes as input a message $M \in \mathcal{M}$, a set of n group members whose identities form the set $L_{ID} = \{ID_1,...,ID_n\}$ and their corresponding public keys form the set $L_{PK} = \{P_{ID_1},...,P_{ID_n}\}$, a parameter list param and a singer's signing key $S_{ID_s} = (x_{ID_s},D_{ID_s})$. Here x_{ID_s} and D_{ID_s} is the s-th group member's secret value and partial private key respectively. To generate a ring signature σ , the signer does the following.
 - Set $M' = M||param||L_{ID}||L_{PK};$
 - Compute $\sigma_{PK} = Ring\text{-}Sign_{PK}(x_{ID_s}, L_{PK}, M');$
 - Set $M'' = M||param||L_{ID}||L_{PK}||\sigma_{PK};$
 - Compute $\sigma_{ID} = Ring\text{-}Sign_{ID}(D_{ID_s}, L_{ID}, M'');$
 - Set $\sigma = \sigma_{PK} || \sigma_{ID}$.

Verify: If $Ver_{PK}(\sigma_{PK}, L_{PK}, M') = True$ and $Ver_{ID}(\sigma_{ID}, L_{ID}, M'') = True$ then the algorithm outputs True, otherwise outputs False.

As we can see, σ is a valid ring signature of M under L_{ID} and L_{PK} . This is because \mathcal{A}_I runs all the algorithms as same as defined in the generic construction in Section 4. We note that this attack is a strong attack that belongs to the no-message attack classes, where no signing oracle is required.

The generic construction given in Section 4 only guarantees that the singer of a valid ring signature possesses a secret value x_{ID_i} of a user $ID_i \in L_{ID}$ and a partial private key D_{ID_j} of a user $ID_j \in L_{ID}$, instead of proving that the signer must know the private key of one user (i.e., $ID_i = ID_j$). This is the reason why a Type I adversary can forge a valid signature for any message. How to give a provably secure generic construction of certificateless ring signature is still an open problem.

6 A Concrete Certificateless Ring Signatures Scheme

In this section, we will give the concrete construction of certificateless ring signature.

6.1 Description of Our CL-Ring Scheme

Our CL-Ring scheme consists of the following concrete algorithms:

- Setup: Given a security parameter ℓ , the algorithm works as follows.
 - 1. Specify G_1, G_2, e , as described in Section 3.1.
 - 2. Arbitrarily choose a generator $P \in G_1$ and set g = e(P, P).
 - 3. Choose a random masterkey $\kappa \in \mathbb{Z}_q^*$ and set $P_0 = \kappa P$.
 - 4. Choose cryptographic hash functions $H_1: \{0,1\}^* \longrightarrow G_1, H_2: \{0,1\}^* \longrightarrow Z_q^*$ and $H_3: \{0,1\}^* \longrightarrow G_1$.

The system parameters $param=(G_1, G_2, e, P, g, P_0, H_1, H_2, H_3)$. The message space is $\mathcal{M} = \{0, 1\}^*$.

- Partial-Private-Key-Extract: This algorithm accepts param, masterkey and a user's identity $ID_i \in \{0,1\}^*$ to output the user's partial private key $D_i = \kappa Q_i$. Where $Q_i = H_1(ID_i)$.
- Set-Secret-Value: Given param, this algorithm selects a random $x_i \in \mathbb{Z}_q^*$ as the user's (whose identity is ID_i) secret value.
- Set-Private-Key: This algorithm takes as input param, a user's identity ID_i , the user's partial private key D_i and the user's secret value $x_i \in \mathbb{Z}_q^*$. The output of the algorithm is the user's private key $S_i = (x_i, D_i)$.

- Set-Public-Key: This algorithm accepts param, a user's identity ID_i and his secret value $x_i \in \mathbb{Z}_q^*$ to produce the user's public key $P_i = x_i P$.
- Ring-Sign: Suppose there's a group of n users whose identities form the set $L_{ID} = \{ID_1, ..., ID_n\}$, and their corresponding public keys form the set $L_{PK} = \{P_1, ..., P_n\}$. To sign a message $M \in \mathcal{M}$ on behalf of the group, the actual signer, indexed by s using the private key $S_s = (x_s, D_s)$, performs the following steps.
 - 1. For each $i \in \{1, ..., n\} \setminus \{s\}$, select $r_i \in Z_q^*$ uniformly at random, compute $y_i = g^{r_i}$.
 - 2. Compute $h_i = H_2(M||L_{ID}||L_{PK}||y_i)$ for all $i \in \{1, ..., n\} \setminus \{s\}$.
 - 3. Choose random $r_s \in Z_q^*$, compute $U = H_3(M||L_{ID}||L_{PK})$, $y_s = g^{r_s}e(-P_0, \sum_{i\neq s}h_iQ_i)e(-U, \sum_{i\neq s}h_iP_i)$. If $y_s = 1_{G_2}$ or $y_s = y_i$ for some $i\neq s$, then redo this step.
 - 4. Compute $h_s = H_2(M||L_{ID}||L_{PK}||y_s)$.
 - 5. Compute $V = (\sum_{i=1}^{n} r_i)P + h_s(D_s + x_sU)$.
 - 6. Output the ring signature on M as $\sigma = \{(y_1, ..., y_n), V\}$.
- Verify: To verify a ring signature $\sigma = \{(y_1, ..., y_n), V\}$ on a message M with identities in L_{ID} and corresponding public keys in L_{PK} , the verifier performs the following steps.
 - 1. Compute $h_i = H_2(M||L_{ID}||L_{PK}||y_i)$ for all $i \in \{1,...,n\}$, compute $U = H_3(M||L_{ID}||L_{PK})$.
 - 2. Verify $e(V, P) \stackrel{?}{=} y_1 \cdot \dots \cdot y_n e(\sum_{i=1}^n h_i Q_i, P_0) e(\sum_{i=1}^n h_i P_i, U)$ holds with equality.
 - 3. Accept the ring signature as valid and output True if the above equation holds, otherwise, output False.

6.2 Efficiency

We only consider the costly operations including the pairing operation (Pairing), scalar multiplication in G_1 (G_1 SM), exponentiation in G_2 (G_2 E) and MapToPoint hash operation [31] (Hash). The numbers of these operations in our scheme are shown in **Table 1**.

Table 1. Efficiency

	Pairing	G_1 SM	$G_2 \to$	Hash
Sign	2	2n+3	n	n+1
Verify	3	2n	0	n+1
Total	5	4n+3	n	2n+2

Pairing operation is the most time consuming operation. Our CL-Ring scheme only requires 5 pairing operations which is independent of the group size.

7 Analysis of the Proposed CL-Ring Scheme

In this section, we will analyze our proposed scheme in detail.

7.1 Correctness

The correctness of the proposed scheme can be easily verified with the following:

$$e(V, P) = e((\sum_{i=1}^{n} r_i)P + h_s(D_s + x_s U), P)$$

$$= e((\sum_{i=1}^{n} r_i)P, P)e(h_s(D_s + x_s U), P)$$

$$= y_1 \cdot \dots \cdot y_n e(\sum_{i \neq s} h_i Q_i, P_0)e(\sum_{i \neq s} h_i P_i, U)e(h_s D_s, P)e(h_s x_s U, P)$$

$$= y_1 \cdot \dots \cdot y_n e(\sum_{i \neq s} h_i Q_i, P_0)e(\sum_{i \neq s} h_i P_i, U)e(h_s Q_s, P_0)e(h_s P_s, U)$$

$$= y_1 \cdot \dots \cdot y_n e(\sum_{i=1}^{n} h_i Q_i, P_0)e(\sum_{i=1}^{n} h_i P_i, U)$$

7.2 Unconditional Anonymity

Let $\sigma = \{(y_1,...,y_n), V\}$ be a valid ring signature of a message M on behalf of a group of n members specified by identities in L_{ID} and public keys in L_{PK} . Since all the $r_i, i \in \{0,...,n\} \setminus \{s\}$ are randomly generated, hence all $y_i, i \in \{0,...,n\} \setminus \{s\}$ are also uniformly distributed. The randomness of r_s chosen by the signer implies $y_s = g^{r_s} e(-P_0, \sum_{i \neq s} h_i Q_i) e(-U, \sum_{i \neq s} h_i P_i)$ is also uniformly distributed. So $(y_1,...,y_n)$ in the signature reveals no information about the signer.

It remains to consider whether $V = (\sum_{i=1}^{n} r_i)P + h_s(D_s + x_s U)$ leaks information about the actual signer. From the construction of V, it is obvious to see that $D_s + x_s U = h_s^{-1}(V - (\sum_{i=1}^{n} r_i)P)$. To identify whether ID_s is the identity of the actual signer, the only way is to check $e(Q_s, P_0)e(P_s, U) \stackrel{?}{=} e(D_s + x_s U, P)$. Namely, $e(Q_s, P_0)e(P_s, U) \stackrel{?}{=} e(h_s^{-1}(V - (\sum_{i=1}^{n} r_i)P), P)$. If ID_s is the identity of the actual signer, it should hold

$$y_s = g^{r_s} e(-P_0, \sum_{i \neq s} h_i Q_i) e(-U, \sum_{i \neq s} h_i P_i).$$

It remains to check

$$e(Q_s, P_0)e(P_s, U) \stackrel{?}{=} \left(\frac{e(V, P)}{y_1 \cdot \dots \cdot y_n e(P_0, \sum_{i \neq s} h_s Q_s) e(U, \sum_{i \neq j} h_s P_i)}\right)^{h_s^{-1}}$$

However, we have for each $j \in \{1, 2, ..., n\}$

$$(\frac{e(V,P)}{y_1 \cdot \dots \cdot y_n e(P_0, \sum_{i \neq j} h_i Q_i) e(U, \sum_{i \neq j} h_i P_i)})^{h_j^{-1}}$$

$$= (\frac{e(\sum_{i=1}^n r_i) P + h_s(D_s + x_s U), P)}{y_1 \cdot \dots \cdot y_n e(P_0, \sum_{i \neq s} h_i Q_i) e(U, \sum_{i \neq s} h_i P_i) e(h_s Q_s, P_0) e(h_s P_s, U) w})^{h_j^{-1}}$$

$$= (\frac{e((\sum_{i=1}^n r_i) P, P)}{e((\sum_{i \neq s} r_i) P, P) y_s e(P_0, \sum_{i \neq s} h_i Q_i) e(U, \sum_{i \neq s} h_i P_i) w})^{h_j^{-1}}$$

$$= w^{-h_j^{-1}} = e(Q_j, P_0) e(P_j, U)$$

where $w = e(-h_j Q_j, P_0)e(-h_j P_j, U)$, and ID_s is the identity of the actual signer. This fact shows that V in the signature does not leak any information about the identity of the actual signer. And hence, the unconditional anonymity of our CL-Ring scheme is proved.

7.3 Unforgeability

Assuming that the CDH problem is hard, we now show the unforgeability of our CL-Ring scheme.

Theorem 7.1 In the random oracle model [3], if A_I can win the Game 1, with an advantage $\epsilon \geq 7P_n^{q_{H_1}}/2^{\ell}$ within a time span t for a security parameter ℓ ; and asking at most q_K Partial-Private-Key queries, at most q_P Public-Key queries, at most q_{P_T} Private-Key queries, at most q_{H_1} H_1 queries, at most q_{H_2} H_2 queries, at most q_{H_3} H_3 queries, q_S Ring-Sign

queries. Then the CDH problem in G_1 can be solved within time $2(t+q_{H_1}T_1+q_{H_2}T_2+q_{H_3}T_3+q_KT_K+q_PT_P+q_PT_{Pr}+q_ST_S)$ and with probability $\geq ((\frac{q_K+q_{Pr}}{q_K+q_{Pr}+n})^{q_K+q_{Pr}+n}(\frac{n}{q_K+q_{Pr}+n})^n\epsilon)^2/66P_n^{q_{H_1}}$ where n is the ring scale, $P_n^{q_{H_1}}$ is defined as the number of n-permutations of q_{H_1} elements i.e. $P_n^{q_{H_1}}=q_{H_1}\cdot\ldots\cdot(q_{H_1}-n+2)\cdot(q_n-n+1)$, T_1 (resp. T_2,T_3,T_K,T_P,T_{Pr} and T_S) is the time cost of an H_1 (resp. H_2,H_3 , Partial-Private-Key, Public-Key, Private-Key and Ring-Sign) query.

Please refer to Appendix A.

Theorem 7.2 In the random oracle model, if A_{II} can win the Game 2, with an advantage $\epsilon \geq 7P_n^{q_{H_1}}/2^\ell$ within a time span t for a security parameter ℓ ; and asking at most q_P Public-Key queries, at most q_K Private-Key queries, at most q_{H_1} H₁ queries, at most q_{H_2} H₂ queries, at most q_{H_3} H₃ queries, at most q_S Ring-Sign queries. Then the CDH problem in G_1 can be solved within time $2(t+q_{H_1}T_1+q_{H_2}T_2+q_{H_3}T_3+q_KT_{Pr}+q_PT_P+q_ST_S)$ and with probability $\geq ((\frac{q_K}{q_K+n})^{q_K+n}(\frac{n}{q_K+n})^n\epsilon)^2/66P_n^{q_{H_1}}$.

Please refer to Appendix B.

8 Conclusion

In this paper, we proposed a concrete construction of certificateless ring signature scheme from the bilinear pairing. The security models of certificateless ring signatures are also formalized. The models capture the essence of the possible adversaries in the notion of certificateless system and ring signatures. In the random oracle models, the unforgeability of our scheme is based on the hardness of Computational Diffie-Hellman problem. We note that the number of pairing computation in our scheme is constant and does not grow with the number of group members.

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A Proof of Theorem 7.1

Proof. Let \mathcal{C} be a CDH attacker, \mathcal{A}_I be a type I adversary of our CL-Ring scheme who interacts with \mathcal{C} following Game 1 and can forge a valid ring signature. Suppose \mathcal{C} receives a random instance (P, aP, bP) of the CDH problem in G_1 . We show how \mathcal{C} can use \mathcal{A}_I to solve the CDH problem, i.e. to compute abP.

Training: A_I can ask C H_1 , H_2 , H_3 , Partial-Private-Key, Public-Key, Private-Key, Public-Key-Replacement and Ring-Sign queries. In order to maintain consistency and avoid conflict, C keeps four lists $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$, and \mathbf{K} to store the answers used, where $\mathbf{H_1}$ includes items of the form (ID, α, Q_{ID}, c) , $\mathbf{H_2}$ includes items of the form $(M, L_{ID}, L_{PK}, y, h)$, $\mathbf{H_3}$ includes items of the form $(M, L_{ID}, L_{PK}, \beta, U, c'')$, and \mathbf{K} includes items of the form $(ID, x, D_{ID}, P_{ID}, c')$. All of these four lists are initially empty. C also maintains three lists L_1, L_2, L_3 , the function of these three lists are the same as mentioned in Game 1 Section 4.

 H_1 Queries: On receiving a query $H_1(ID)$, \mathcal{C} does as follows.

- 1. If there exists an item (ID, α, Q_{ID}, c) in $\mathbf{H_1}$, then \mathcal{C} returns Q_{ID} as answer.
- 2. Otherwise, C first flips a coin $c \in \{0,1\}$ that yields 0 with probability δ and 1 with probability 1δ (δ will be determined later), then picks a random element α (has not been used before) in Z_q^* . If c = 0, C computes $Q_{ID} = H_1(ID) = \alpha P$; otherwise c = 1, it computes $Q_{ID} = H_1(ID) = \alpha b P$. C then adds (ID, α, Q_{ID}, c) to $\mathbf{H_1}$ and returns Q_{ID} as answer.

 H_2 Queries: On receiving a query $H_2(M||L_{ID}||L_{PK}||y)$, C first checks if there exists an item $(M, L_{ID}, L_{PK}, y, h)$ in $\mathbf{H_2}$, if so, returns h as answer. Otherwise, C picks a random $h \in Z_q^*$ which has not been used in the answers of the former H_2 Queries, then returns h as answer and adds $(M, L_{ID}, L_{PK}, y, h)$ to $\mathbf{H_2}$.

 H_3 Queries: On receiving a query $H_3(M||L_{ID}||L_{PK})$, \mathcal{C} first checks if there exists an item $(M, L_{ID}, L_{PK}, \beta, U, c'')$ in $\mathbf{H_3}$, if so, returns U as answer. Otherwise, \mathcal{C} first flips a coin $c'' \in \{0, 1\}$ that yields 0 with probability δ and 1 with probability $1-\delta$ then picks a random $\beta \in \mathbb{Z}_q^*$ which has not been used in the answers of the former H_3 Queries. If c'' = 0, compute $U = \beta P$; while c'' = 1, compute $U = \beta bP$. In both cases, \mathcal{C} will add $(M, L_{ID}, L_{PK}, \beta, U, c'')$ to $\mathbf{H_3}$ and return U as answer.

Partial-Private-Key Queries: Whenever C receives a query PPK(ID)

- 1. If there exists an item $(ID, x, D_{ID}, P_{ID}, c')$ in \mathbf{K}, \mathcal{C} does the following:
 - (a) If $D_{ID} \neq \perp$, C returns D_{ID} as answer.
 - (b) Else, if there's an item (ID, α, Q_{ID}, c) exists in $\mathbf{H_1}$, \mathcal{C} sets $L_1 = L_1 \cup \{ID\}$, $D_{ID} = \alpha P_0$ and returns D_{ID} as answer when c = 0; while c = 1, \mathcal{C} aborts.
 - (c) Otherwise, C first makes an $H_1(ID)$ query to obtain an item (ID, α, Q_{ID}, c) . If c = 1, C aborts; while c = 0, C sets $L_1 = L_1 \cup \{ID\}$, $D_{ID} = \alpha P_0$ and returns D_{ID} as answer.
- 2. Otherwise \mathcal{C} does the following:
 - (a) If there exists an item (ID, α, Q_{ID}, c) in $\mathbf{H_1}$, \mathcal{C} sets $L_1 = L_1 \cup \{ID\}$, computes $D_{ID} = \alpha P_0$, sets $x = \bot$, $P_{ID} = \bot$, adds $(ID, x, D_{ID}, P_{ID}, c')$ to \mathbf{K} and returns D_{ID} as answer when c = 0; while c = 1 \mathcal{C} aborts.
 - (b) Otherwise, C first makes an $H_1(ID)$ query to obtain an item (ID, α, Q_{ID}, c) in $\mathbf{H_1}$, then proceeds as in (a).

Public-Key Queries: Whenever C receives a query PK(ID)

- 1. If there exists an item $(ID, x, D_{ID}, P_{ID}, c')$ in \mathbf{K}, \mathcal{C} does the following:
 - (a) If $P_{ID} \neq \bot$, C returns P_{ID} as answer;
 - (b) Otherwise, C first flips a coin $c' \in \{0,1\}$ that yields 0 with probability δ and 1 with probability $1-\delta$, then picks a random $x \in Z_q^*$. If c' = 0, C sets $P_{ID} = xP$; otherwise c = 1, it

computes $P_{ID} = xaP$. C then updates $(ID, x, D_{ID}, P_{ID}, c')$ with new values and returns P_{ID} as answer.

2. Otherwise, C first flips a coin $c' \in \{0,1\}$ that yields 0 with probability δ and 1 with probability $1 - \delta$, then picks a random $x \in Z_q^*$. If c' = 0, C sets $P_{ID} = xP$; otherwise c = 1, it computes $P_{ID} = xaP$. C then sets $D_{ID} = \bot$, returns P_{ID} as answer and adds $(ID, x, D_{ID}, P_{ID}, c')$ to K.

Public-Key-Replacement Queries: On receiving a query $PKR(ID, P'_{ID})$ (C sets $L_2 = L_2 \cup \{ID\}$), C first makes a PPK(ID) query to obtain an item $(ID, x, D_{ID}, P_{ID}, c')$, then sets $x = \bot$, $P_{ID} = P'_{ID}$, and updates the item $(ID, x, D_{ID}, P_{ID}, c')$ in **K** to record this replacement.

Private-Key Queries: Whenever receives a query PrK(ID), if $ID \in L_2$ C returns \perp , otherwise

- 1. When there exists an item $(ID, x, D_{ID}, P_{ID}, c')$ in **K**
 - (a) If $x = \bot$, C first makes a PK(ID) query. If $c' \ne 1$, C sets $L_3 = L_3 \cup \{ID\}$, returns (x, D_{ID}) as answer; otherwise C aborts.
 - (b) Else if $D_{ID} = \bot$, \mathcal{C} first makes a PPK(ID) query, if \mathcal{C} does not abort and $c' \neq 1$ then sets $L_3 = L_3 \cup \{ID\}$ and (x, D_{ID}) will be returned as answer. Otherwise \mathcal{C} aborts.
 - (c) Otherwise, when c' = 1 \mathcal{C} aborts, while c' = 0 \mathcal{C} sets $L_3 = L_3 \cup \{ID\}$ and returns (x, D_{ID}) as answer.
- 2. Otherwise, C first makes PK(ID) and PPK(ID) queries. If C does not abort and $c' \neq 1$, then sets $L_3 = L_3 \cup \{ID\}$, returns (x, D_{ID}) as answer and adds $(ID, x, D_{ID}, P_{ID}, c')$ to \mathbf{K} ; otherwise, C aborts.

Ring-Sign Queries: A_I chooses a group of n users whose identities form the set $L_{ID} = \{ID_1, ..., ID_n\}$ and their corresponding public keys form the set $L_{PK} = \{P_1, ..., P_n\}$, and may ask a ring signature on a message M of this group. On receiving a Ring-Sign query $RS(M, L_{ID}, L_{PK})$, C creates a ring signature as follows:

- 1. Choose a random index $s \in \{1, ..., n\}$.
- 2. For all $i \in \{1, ..., n\} \setminus \{s\}$, choose $r_i \in Z_q^*$ uniformly at random, compute $y_i = g^{r_i}$.
- 3. For all $i \in \{1, ..., n\} \setminus \{s\}$, compute $h_i = H_2(M||L_{ID}||L_{PK}||y_i)$.

- 4. Choose $h_s \in Z_q^*, V \in G_1$ at random.
- 5. Compute $y_s = e(V (\sum_{i \neq s} r_i)P, P)e(\sum_{i=1}^n h_i Q_i, -P_0)e(\sum_{i=1}^n h_i P_i, -U)$ (Where $U = H_3(M||L_{ID}||L_{PK}), Q_i = H_1(ID_i)$). If $y_s = 1_{G_2}$ or $y_s = y_i$ for some $i \neq s$, then goto step 4.
- 6. Set $H_2(M||L_{ID}||L_{PK}||y_s) = h_s$.
- 7. Return $(M, L_{ID}, L_{PK}, \sigma = \{(y_1, ..., y_n), V\})$ as answer.

Forgery: Finally, \mathcal{A}_I outputs a tuple $(M^*, L_{ID}^* = \{ID_1^*, ..., ID_n^*\}, L_{PK}^* = \{P_1^*, ..., P_n^*\}, \sigma^* = \{(y_1^*, ..., y_n^*), V^*\}$) which means σ^* is a ring signature on message M^* on behalf of the group specified by identities in L_{ID}^* and the corresponding public keys in L_{PK}^* . It is required that \mathcal{C} does not know the private key of any member in this group, $L_{ID}^* \cap ((L_1 \cap L_2) \cup L_3) = \emptyset$ and the ring signature σ^* on message M^* on behalf of the group must be valid (**Event 1**). Now, applying the 'ring forking lemma' [13], if \mathcal{A}_I succeeds in outputting a valid ring signature σ^* with probability $\epsilon \geq 7P_n^{q_{H_1}}/2^{\ell}$ in a time span t in the above interaction, then within time 2t and probability e0 can get two valid ring signatures e1 (e2) and e3. From these two valid ring signatures, e3 obtains

$$e(V^*, P) = y_1^* \cdot \dots \cdot y_n^* e(\sum_{i=1}^n h_i^* P_i^*, U^*) e(\sum_{i=1}^n h_i^* Q_i^*, P_0)$$

and

$$e(V'^*, P) = y_1^* \cdot \dots \cdot y_n^* e(\sum_{i=1}^n h_i'^* P_i^*, U^*) e(\sum_{i=1}^n h_i'^* Q_i^*, P_0)$$

Where $U^* = H_3(M^*||L_{ID}^*||L_{PK}^*)$, $Q_i^* = H_1(ID_i^*)$, $h_i^* = H_2(M^*, L_{ID}^*, L_{PK}^*, y_i^*)$, $h_i'^* = H_2'(M^*, L_{ID}^*, L_{PK}^*, y_i^*)$, and for some $s \in \{1, ..., n\}$, $h_s^* \neq h_s'^*$, while for $i \in \{1, ..., n\} \setminus \{s\}$, $h_i^* = h_i'^*$. From the above two equations we have

$$e(V^*-V'^*,P)=e((h_s^*-h_s'^*)P_s^*,U^*)e((h_s^*-h_s'^*)Q_s^*,P_0)$$

At this stage, \mathcal{C} may find the item $(M^*, L_{ID}^*, L_{PK}^*, \beta^*, U^*, c''^*)$ from $\mathbf{H_3}$, $(ID_s^*, \alpha_s^*, Q_s^*, c_s^*)$ from $\mathbf{H_1}$, $(ID_s^*, x_s^*, D_s^*, P_s^*, c_s'^*)$ from \mathbf{K} . There are three cases in which \mathcal{C} can successfully solve the CDH problem.

• Case 1: $c_s^* = 1, c''^* = 0$, this means $Q_s^* = \alpha_s^* b P, U^* = \beta^* P$. In this case, $e(V^* - V'^*, P) = e((h_s^* - h_s'^*)(\beta^* P_s^* + \alpha_s^* a b P), P)$. So, \mathcal{C} can get $abP = \alpha_s^{*-1}((h-h')^{-1}(V^* - V'^*) - \beta^* P_s^*)$.

- Case 2: $c'_s^* = 1, c''^* = 1, c_s^* = 0, x_s^* \neq \bot$, and $P_s^* = x_s^* a P, U^* = \beta^* b P, Q_s^* = \alpha_s^* P, \mathcal{C}$ can get $abP = (x_s^* \beta^*)^{-1} ((h h')^{-1} (V^* V'^*) \alpha_s^* P_0)$.
- Case 3: $c'_s^* = 1, c''^* = 1, c_s^* = 1, x_s^* \neq \bot$, and $P_s^* = x_s^* a P, U^* = \beta^* b P, Q_s^* = \alpha_s^* b P, \mathcal{C}$ can get $abP = (x_s^* \beta^* + \alpha_s^*)^{-1} (h h')^{-1} (V^* V'^*)$ (Note the probability that $x_s^* \beta^* + \alpha_s^* = 0$ is negligible).

Probability of Success: Now we determine the value of δ and consider the probability for $\mathcal C$ to successfully solve the given CDH problem. The probability that $\mathcal C$ does not abort in all the q_K Partial-Private-Key Queries and q_{Pr} Private-Key Queries is at least $\delta^{q_K+q_{Pr}}$. The probability that the forged ring signature is helpful for $\mathcal C$ to solve the CDH problem is $Pr[(\mathbf{Case1} \vee \mathbf{Case2} \vee \mathbf{Case3}) \wedge \mathbf{Event 1}] \leq (1-\delta)^n$. So the combined probability is $\frac{q_K+q_{Pr}}{q_K+q_{Pr}+n}$ and the maximized probability is $(\frac{q_K+q_{Pr}}{q_K+q_{Pr}+n})^{q_K+q_{Pr}}(\frac{n}{q_K+q_{Pr}+n})^n$. Based on the bound from the ring forking lemma [13], if $\mathcal A_I$ succeeds in time $\leq t$ with probability $\epsilon \geq 7P_n^{q_{H_1}}/2^\ell$, then the CDH problem in G_1 can be solved by $\mathcal C$ within time $2(t+q_{H_1}T_1+q_{H_2}T_2+q_{H_3}T_3+q_KT_K+q_PT_P+q_{Pr}T_{Pr}+q_ST_S)$ and with probability $\geq ((\frac{q_K+q_{Pr}}{q_K+q_{Pr}+n})^{q_K+q_{Pr}}\cdot (\frac{n}{q_K+q_{Pr}+n})^n\epsilon)^2/66P_n^{q_{H_1}}$.

B Proof of Theorem 7.2

Proof. Let \mathcal{A}_{II} be our type II adversary, \mathcal{C} be a CDH attacker who receives a random instance (P, aP, bP) and has to compute the value of abP.

Setup: \mathcal{C} generates the KGC's masterkey $\kappa \in Z_q^*$ and system parameters $\mathsf{param} = (G_1, G_2, e, P, g, P_0, H_1, H_2, H_3)$. When the simulation is started, \mathcal{A}_{II} is provided with param and the masterkey κ .

Training: \mathcal{A}_{II} can ask \mathcal{C} H_1, H_2, H_3 , Public-Key, Private-Key, and Ring-Sign queries. Since \mathcal{A}_{II} has access to the masterkey κ , he can do Partial-Private-Key-Extract himself. \mathcal{C} also maintains four lists, namely $\mathbf{H_1}$ contains items of the form (ID, Q_{ID}) , $\mathbf{H_2}$ contains items of the form $(M, L_{ID}, L_{PK}, y, h)$, $\mathbf{H_3}$ contains items of the form $(M, L_{ID}, L_{PK}, \beta, U)$ and \mathbf{K} contains items of the form (ID, x, P_{ID}) to store the answers used. All of these four lists are initially empty. \mathcal{C} also maintains two lists L_1, L_2 , the function of these two lists are the same as mentioned in Game 2 Section 4.

 H_1 Queries: On receiving a query $H_1(ID)$. If (ID, Q_{ID}) exists in $\mathbf{H_1}$ then \mathcal{C} returns Q_{ID} as answer. Otherwise, \mathcal{C} picks a random $Q_{ID} \in G_1^*$ which has

not been used in the former H_1 Queries, then returns Q_{ID} as answer and adds (ID, Q_{ID}) to $\mathbf{H_1}$.

 H_2 Queries: On receiving a query $H_2(M||L_{ID}||L_{PK}||y)$, C first checks whether there exists an item $(M, L_{ID}, L_{PK}, y, h)$ in $\mathbf{H_2}$, if so, returns h as answer. Otherwise, C picks a random $h \in \mathbb{Z}_q^*$ which has not been used in the former H_2 Queries, then returns h as answer and adds $(M, L_{ID}, L_{PK}, y, h)$ to $\mathbf{H_2}$.

 H_3 Queries: Whenever receives a query $H_3(M||L_{ID}||L_{PK})$, C first checks whether there exists an item $(M, L_{ID}, L_{PK}, \beta, U)$ in $\mathbf{H_3}$, if so, returns U as answer. Otherwise, C picks a random $\beta \in \mathbb{Z}_q^*$ which has not been used in the former H_3 Queries, computes $U = \beta aP$, then adds $(M, L_{ID}, L_{PK}, \beta, U)$ to $\mathbf{H_3}$ and returns U as answer.

Public-Key Queries: On receiving a query PK(ID)

- 1. If there is an item (ID, x, P_{ID}, c) exists in **K**, then C returns P_{ID} as answer.
- 2. Otherwise, C first flips a coin $c \in \{0,1\}$ that yields 0 with probability δ and 1 with probability 1δ , then picks a random $x \in Z_q^*$. If c = 0, C sets $P_{ID} = xP$, returns P_{ID} as answer and adds (ID, x, P_{ID}, c) to \mathbf{K} . Else c = 1, C sets $P_{ID} = xbP$, returns P_{ID} as answer and adds (ID, x, P_{ID}, c) to \mathbf{K} .

Public-Key-Replacement Queries: On receiving a query $PKR(ID, P'_{ID})$ (C sets $L_1 = L_1 \cup \{ID\}$), C first makes a PPK(ID) query to obtain an item (ID, x, P_{ID}, c) , then sets $x = \bot$, $P_{ID} = P'_{ID}$, and updates the item (ID, x, P_{ID}, c) in K to record this replacement.

Private-Key Queries: On receiving a query PrK(ID), if $ID \in L_1$ C returns \bot , otherwise

- 1. If there is an item (ID, x, P_{ID}, c) in **K**, when c = 0 \mathcal{C} sets $L_2 = L_2 \cup \{ID\}$, returns (x, D_{ID}) as answer (where $D_{ID} = \kappa H_1(ID)$ is the partial private key of the user whose identity is ID); while c = 1, \mathcal{C} aborts.
- 2. Otherwise, C first makes a PK(ID) query to obtain an item (ID, x, P_{ID}, c) ; when c = 0, C sets $L_2 = L_2 \cup \{ID\}$, returns (x, D_{ID}) as answer; while c = 1, C aborts.

Ring-Sign Queries: A_{II} chooses a group of n users whose identities form the set $L_{ID} = \{ID_1, ..., ID_n\}$ and their corresponding public keys form the

set $L_{PK} = \{P_1, ..., P_n\}$. On receiving a Ring-Sign query $RS(M, L_{ID}, L_{PK})$, C creates a ring signature as follows:

- 1. Choose a random index $s \in \{1, ..., n\}$;
- 2. For all $i \in \{1, ..., n\} \setminus \{s\}$, choose $r_i \in Z_q^*$ uniformly at random, compute $y_i = g^{r_i}$.
- 3. For all $i \in \{1, ..., n\} \setminus \{s\}$, set $h_i = H_2(M||L_{ID}||L_{PK}||y_i)$.
- 4. Randomly choose $h_s \in \mathbb{Z}_q^*, V \in G_1$.
- 5. Compute $y_s = e(V (\sum_{i \neq s} r_i)P, P)e(\sum_{i=1}^n h_i Q_i, -P_0)e(\sum_{i=1}^n h_i P_i, -U)$ (Where $U = H_3(M||L_{ID}||L_{PK}), Q_i = H_1(ID_i)$). If $y_s = 1_{G_2}$ or $y_s = y_i$ for some $i \neq s$, then goto step 4.
- 6. Set $H_2(M||L_{ID}||L_{PK}||y_s) = h_s$.
- 7. Return $(M, L_{ID}, L_{PK}, \sigma = \{(y_1, ..., y_n), V\})$ as answer.

Forgery: Finally, \mathcal{A}_{II} outputs a tuple $(M^*, L_{ID}^* = \{ID_1^*, ..., ID_n^*\}, L_{PK}^* = \{P_1^*, ..., P_n^*\}, \sigma^* = \{(y_1^*, ..., y_n^*), V^*\}$) which implies that σ^* is a ring signature on message M^* on behalf of the group specified by identities in L_{ID}^* and the corresponding public keys in L_{PK}^* . It is required that (M^*, σ^*) is a valid message and ring signature pair, $L_{ID}^* \cap (L_1 \cup L_2) = \emptyset$ and \mathcal{C} does not know the private key of any member in the group specified by L_{ID}^* and L_{PK}^* (**Event 1**). Now, Applying the 'ring forking lemma' [13], \mathcal{C} gets two valid ring signatures $(M^*, L_{ID}^*, L_{PK}^*, \sigma^* = \{(y_1^*, ..., y_n^*), V^*\})$ and $(M^*, L_{ID}^*, L_{PK}^*, \sigma'^* = \{(y_1^*, ..., y_n^*), V'^*\})$. From these two ring signatures, \mathcal{C} obtains

$$e(V^*, P) = y_1^* \cdot \dots \cdot y_n^* e(\sum_{i=1}^n h_i^* P_i^*, U^*) e(\sum_{i=1}^n h_i^* Q_i^*, P_0)$$

and

$$e(V'^*, P) = y_1^* \cdot \dots \cdot y_n^* e(\sum_{i=1}^n h_i'^* P_i^*, U^*) e(\sum_{i=1}^n h_i'^* Q_i^*, P_0)$$

Where $U^* = H_3(M^*||L^*_{ID}||L^*_{PK}), Q^*_i = H(ID^*_i), h^*_i = H_2(M^*, L^*_{ID}, L^*_{PK}, y^*_i)$ and $h'^*_i = H'_2(M^*, L^*_{ID}, L^*_{PK}, y^*_i)$. The hash functions H_2 and H'_2 satisfy: for some $s \in \{1, ..., n\}, h^*_s \neq h'^*_s$, while $i \in \{1, ..., n\} \setminus \{s\}, h^*_i = h'^*_i$. From the above two equations we have

$$e(V^*-V'^*,P)=e((h_s^*-h_s'^*)P_s^*,U^*)e((h_s^*-h_s'^*)Q_s^*,P_0)$$

At this point, \mathcal{C} may find the item $(M^*, L_{ID}^*, L_{PK}^*, \beta^*, U^*)$ from $\mathbf{H_3}$, (ID_s^*, Q_s^*) from $\mathbf{H_1}$ and $(ID_s^*, x_s^*, P_s^*, c^*)$ from \mathbf{K} . Since $U^* = \beta^* a P$, $P_s^* = x_s^* b P$, \mathcal{C} has the following

$$e(V^* - V'^*, P) = e((h_s^* - h_s'^*)(x_s^*\beta^*abP + \kappa Q_s^*), P)$$

This implies

$$V^* - V'^* = (h_s^* - h_s'^*)(x_s^* \beta^* abP + \kappa Q_s^*)$$

Hence, C can obtain $abP = (x_s^*\beta^*)^{-1}((h_s^* - h_s'^*)^{-1}(V^* - V'^*) - \kappa Q_s^*)$.

Probability of Success: Now we determine the value of δ and consider the probability for \mathcal{C} to successfully solve the given CDH problem. The probability that \mathcal{C} does not abort in all the q_K Private-Key Queries is δ^{q_K} . The probability that \mathcal{A}_{II} forged a valid ring signature which \mathcal{C} does not know any private key of the group members' involved in the ring signature is $(1-\delta)^n$. So the combined probability $(Pr[\mathbf{Event 1}])$ is $\delta^{q_K}(1-\delta)^n$. We can find the value of δ that maximize this probability is $\frac{q_K}{q_K+n}$ and the maximized probability is $(\frac{q_K}{q_K+n})^{q_K}(\frac{n}{q_K+n})^n$.

probability is $(\frac{q_K}{q_K+n})^{q_K}(\frac{n}{q_K+n})^n$. Based on the bound from the ring forking lemma [13], if \mathcal{A}_{II} succeeds in time $\leq t$ with probability $\epsilon \geq 7P_n^{q_{H_1}}/2^\ell$, then the CDH problem in G_1 can be solved by $\mathcal C$ within time $2(t+q_{H_1}T_1+q_{H_2}T_2+q_{H_3}T_3+q_KT_{Pr}+q_PT_P+q_ST_S)$ and with probability $\geq ((\frac{q_K}{q_K+n})^{q_K}(\frac{n}{q_K+n})^n\epsilon)^2/66P_n^{q_{H_1}}$.