Geometry and Computing

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Subdivision Surfaces

With 52 Figures

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Preface

Akin to B-splines, the appeal of subdivision surfaces reaches across disciplines from mathematics to computer science and engineering. In particular, subdivision surfaces have had a dramatic impact on computer graphics and animation over the last 10 years: the results of a development that started three decades ago can be viewed today at movie theaters, where feature length movies cast synthetic characters 'skinned' with subdivision surfaces. Correspondingly, there is a rich, ever-growing literature on its fundamentals and applications.

Yet, as with every vibrant new field, the lack of a uniform notation and standard analysis tools has added unnecessary, at times inconsistent, repetition that obscures the simplicity and beauty of the underlying structures. One goal in writing this book is to help shorten introductory sections and simplify proofs by proposing a standard set of concepts and notation.

When we started writing this book in 2001, we felt that the field had sufficiently settled for standardization. After all, Cavaretta, Dahmen, and Micchelli's monograph [CDM91] had appeared 10 years earlier and we could build on the habilitation of the second author as well as a number of joint papers. But it was only in the process of writing and seeing the issues in conjunction, that structures and notation became clearer. In fact, the length of the book repeatedly increased and decreased, as key concepts and structures emerged.

Chapter 2/15, for example, was a late addition, as it became clear that the differential geometry for singular parameterizations, of continuity, smoothness, curvature, and injectivity, must be established upfront and in generality to simplify the exposition and later proofs. By contrast, the key definition of subdivision surfaces as splines with singularities, in Chap. 3/39, was a part of the foundations from the outset. This point of view implies a radical departure from any focus on control nets and instead places the main emphasis on nested surface rings, as explained in Chap. 4/57. Careful examination of existing proofs led to the explicit formulation of a number of assumptions, in Chap. 5/83, that must hold when discussing subdivision surfaces in generality. Conversely, placing these key assumptions upfront, shortened the presentation considerably. Therefore, the standard examples of subdivision algorithms reviewed in Chap. 6/109 are presented with a new, shorter and simpler analysis than in earlier publications. Chapters 7/125 and 8/157 were triggered by very recent, new insights and results and partly contain unpublished material. The suitability of the major known subdivision algorithms for engineering design was at the heart of the investigations into the shape of subdivision surfaces in Chap. 7/125. The shortcomings of the standard subdivision algorithms discovered in the process forced a renewed search for an approach to subdivision capable of meeting shape and higher-order continuity requirements. Guided subdivision was devised in response. The second part of Chap. 7/125 recasts this class of subdivision algorithms in a more abstract form that may be used as a prototype for a number of new curvature continuous subdivision algorithms. The first part of Chap. 8/157 received a renewed impetus from a recent stream of publications aimed at predicting the distance of a subdivision surface from their geometric control structures after some m refinement steps. The introduction of proxy surfaces and the distance to the corresponding subdivision surface subsumes this set of questions and provides a framework for algorithm-specific optimal estimates. The second part of Chap. 8/157 grew out of the surprising observation that the Catmull–Clark subdivision can represent the same sphere-like object starting from any member of a whole family of initial control configurations. The final chapter, Chap. 9/175, shows that a large variety of subdivision algorithms is fully covered by the exposition in the book. But it also outlines the limits of our current knowledge and opens a window to the fascinating forms of subdivision currently beyond the canonical theory and to the many approaches still awaiting discovery.

As a monograph, the book is primarily targeted at the subdivision community, including not only researchers in academia, but also practitioners in industry with an interest in the theoretical foundations of their tools. It is not intended as a course text book and contains no exercises, but a number of worked out examples. However, we aimed at an exposition that is as self-contained as possible, requiring, we think, only basic knowledge of linear algebra, analysis or elementary differential geometry. The book should therefore allow for independent reading by graduate students in mathematics, computer science, and engineering looking for a deeper understanding of subdivision surfaces or starting research in the field.

Two valuable sources that complement the formal analysis of this book are the SIGGRAPH course notes [ZS00] compiled by Schröder and Zorin, and the book 'Subdivision Methods for Geometric design' by Warren and Weimer [WW02]. The notes offer the graphics practitioner a quick introduction to algorithms and their implementation and the book covers a variety of interesting aspects outside our focus; for example, a connection to fractals, details of the analysis of univariate algorithms, variational algorithms based on differential operators and observations that can simplify implementation.

We aimed at unifying the presentation, placing for example *bibliographical notes* at the end of each core chapter to point out relevant and original references. In addition to these, we included a large number of publications on subdivision surfaces in the reference section. Of course, given the ongoing growth of the field, these notes cannot claim completeness. We therefore reserved the internet site

`www.subdivision-surface.org`

for future pointers and additions to the literature and theme of the book, and, just possibly, to mitigate any damage of insufficient proof reading on our part.

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Our final thanks are reserved for our families for their support and their patience. You kept us inspired.

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