Automatic Extraction of a Quadrilateral Network of NURBS Patches from Range Data Using Evolutionary Strategies

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Abstract. We propose an algorithm to produce automatically a 3-D CAD model from a set of range data, based on non-uniform rational B-splines (NURBS) surface fitting technique. Our goal is to construct automatically continuous geometric models, assuming that the topology of the surface is unknown. In the propose algorithm, the triangulated surface is partitioned in quadrilateral patches, using Morse theory. The quadrilateral regions on the mesh are then regularized using geodesic curves and B-splines to obtain an improved smooth network on which to fit NURBS surfaces. NURBS surfaces are fitted and optimized using evolutionary strategies. In addition, the patches are smoothly joined guaranteeing C^1 continuity. Experimental results are presented.

1 Introduction

Three-dimensional reconstruction is the process by which the 3D geometry of real-world objects are captured in computer memory from geometric sensors such as laser scanners, photogrammetric cameras, and tomography machines. This reconstructed models consist of two main information; first, its physical characteristics such as density, volume and shape; second, its topological structure such as the adjacency between points, surfaces, and volumes.

Finding a useful and general representation of 3D shape from 3D sensors that is useful for industrial and medical applications has proven to be a nontrivial problem. Up to recently, there are no real automated ways to perform this task, hence the flurry of surface reconstruction packages available in industry such as Polyworks, RapidForm, Geomagic to name a few. Many of these software packages can perform some of the reconstruction task automatically but many of them requires extensive user inputs especially when ones deal with high-level representation such as CAD modeling of complex industrial parts or natural shapes such as the one found in medical applications.

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There are many ways to perform this high-level surface reconstruction task from 3D sensors. In most schemes, surface reconstruction starts with the registration of various views produced by the 3D sensor. Then, each views are generally referenced to each other relative to a common central coordinate system. In some systems, this task is performed using data to data registration, other system use external positioning devices such as mechanical arms or optical tracking systems. Following this process each views are then triangulated to create a unique non-redundant 3D mesh. Many of the commercial packages can do this meshing process automatically. Because of sensor occlusions and limitations, the mesh produced by the triangulation process is frequently plagued with holes that need to be filled in order to create a leak free solid model. Most commercial systems use some sort of semi-automated algorithms but more recent new algorithms [1] based on radial basis functions were introduced to perform this task automatically. In most cases, the hole filling process keeps the original discontinuities of the real object, and generates a complete closed triangular model. Following this process many commercial software require extensive manual processing to lay on the 3D mesh a network of curves where NURBS surfaces can be fitted. This process is extremely labor intensive and require a strong experience in 3D modeling and data processing to complete the task.

In this paper, we present a possible solution to this problem using an automated NURBS extraction process where the 3D mesh is converted automatically into a smooth quadrilateral network of NURBS surfaces based on Morse's theory. Section 2, presents a literature review of the state-of-the-art of automated NURBS fitting algorithms. Section 3, describes how Morse's theory can be applied to determine critical points from an eigenvalue analysis of the Laplacian of the surface mesh. Section 4, describes how to regularize those curves joining the critical points using geodesics calculation and b-spline fitting. Section 5, describes how to fit smooth NURBS with C^1 continuity constraints for each regions using an evolutionary strategy. Section 6, presents some experimental results and compares these results to a well know algorithm. We then conclude and discuss future work.

2 Literature Review

Eck and Hoppe [2] present the first complete solution to the fitting problem of a network of B-spline surfaces of arbitrary topology on disperse and unordered points. The method builds an initial parametrization, which in turn is re-parameterized to build a triangular base, which is then used to create a quadrilateral domain. In the quadrilateral domain, the B-spline patches adjust with a continuity degree of C^1. This method, although effective, is quite complex due to the quantity of steps and process required to build the net of B-spline patches. It is also limited to B-spline as oppose to NURBS.

Krishnamurthy and Levoy [3] presented an approach to adjust NURBS surface patches on cloud of points. The method consists of building a polygonal mesh on the points set. Using this 3D mesh, a re-sampling is performed to generate a regular mesh, on which NURBS surfaces patches can be adjusted. The method has poor performance when dealing with complex surfaces and with surfaces with holes. Other limitation is the underlying difficulty on keeping continuity on the NURBS surface patches.

Boulanger *et al.* [4] describe a linear approximation of continuous pieces by means of trimmed NURBS surfaces. This method generates triangular meshes which are adaptive to local surface curvature. First, the surface is approximated with hierarchical quadrilaterals without considering the jagged curves. Later, jagged curves are inserted and hierarchical quadrilaterals are triangulated. The result is a triangulation which satisfies a given tolerance. The insertion of jagged curves is improved by organizing the quadrilaterals' hierarchy into a *quad-tree* structure. The quality of triangles is also improved by means of a Delaunay triangulation. Although this method produces good results, it is restricted to surfaces which are continuous and it does not accurately model fine details.

A different approach is presented by Yvart *et al.* [5], which uses triangular NURBS for dispersed points adjustment. Triangular NURBS do not require that the points-set has a rectangular topology, although it is more complex than NURBS. Similar to the previous works, it requires intermediate steps where triangular meshes are reconstructed, re-parametrize, and where continuity patches G^1 are adjusted to obtain a surface model.

Dong *et al.* [6] describe a fundamentally new approach to the quadrangulation of manifold polygon meshes using Laplacian eigenfunctions, the natural harmonics of the surface. These surface functions distribute their extrema evenly across a mesh, which connect via gradient flow into a quadrangular base mesh. An iterative relaxation algorithm simultaneously refines this initial complex to produce a globally smooth parameterization of the surface. From this, they can construct a well-shaped quadrilateral mesh with very few extraordinary vertices. The quality of this mesh relies on the initial choice of eigenfunction, for which they describe algorithms and hueristics to efficiently and effectively select the harmonic most appropriate for the intended application.

3 Determination of Morse Critical Points Using Spectral Coding of the Laplacian Matrix

The proposed procedure estimates an initial quadrilaterization of the mesh, using a spectral coding scheme based on the eigen-value analysis of the Laplacian matrix of the 3D mesh. The method is similar to the one proposed in Dong *et al.* [7]. Initially, the quadrilateral's vertices are obtained as a critical points-set of a Morse function. Morse's discrete theory [6] guarantees that, without any concerns on the topological complexity of the surface represented by triangular mesh, a complete quadrilateral description of the surface is possible.

Since one requires a scalar function for each vertex, it has been shown by [7] that the eigenvalue of the Laplacian matrix behaves like a Morse-Smale complex creating a spectral coding function that can be used to determine which vertices are Morse critical points. One advantage of eigenvalue analysis over other coding schemes is that by selecting the dimension of the eigenvector one can directly define the number of critical points on the surface as higher frequencies produce a higher number of critical points. The eigenvalues assigned to every vertex of the mesh is then analyzed to determine if the vertex is a Morse critical point. In addition, according to a value set obtained as the neighborhood of the first ring of every vertex, it is possible to classify the critical points as maximum, minimum or "saddle points." Once the critical points are obtained and classified, then they can be connected to form a quadrilateral base of the mesh using the following Algorithm 1.1:

Algorithm 1.1. Bulding method of MS cells.

```
Critical points interconnection();
begin
    Let T={F,E,V} M triangulation;
    Initialize Morse-Smale complex, M=0;
    Initialize the set of cells and paths, P=C=0;
    S=SaddlePointFinding(T);
    S=MultipleSaddlePointsDivission(T);
    SortByInclination(S);
    for every s ∈ S in ascending order do
        CalculeteAscedingPath(P);
    end
    while exists intact f ∈ F do
        GrowingRegion(f, p0, p1, p2, p3);
        CreateMorseCells(C, p0, p1, p2, p3);
    end
    M = MorseCellsConnection(C);
end
```

4 Regularization of the Quadrilateral Regions

Because the surface needs to be fitted using NURBS patches, it is necessary to regularize the quadrilateral regions boundaries connecting the critical points on the mesh. Regularization means here that we need a fix number of points (λ) on each boundary and that the boundary is described by a smoothing function such as a B-spline. The Algorithm 1.2 is proposed to regularize the path on the mesh joining the critical points. In this algorithm, the link between two Morse critical points on the mesh is defined by a geodesic trajectory between them. A geodesic trajectory is the minimum path joining two points on a manifold. To compute this geodesic path on the mesh, we use a Fast Marching Method (FMM) algorithm [8]. This algorithm compute on a discrete mesh the minimal trajectory joining two critical points in $O(nlogn)$ computational complexity. At the end of the regularization process, a B-splines curve is fitted on the geodesic path and the curve is re-sampled with λ points to obtain a grid which is used to fit the NURBS surfaces. This is a much simpler and robust algorithm that the one proposed by Dong where an uniform parameterization is computed.

Algorithm 1.2. Quadrilateral path regularization algorithm.

`Regularization();`
begin
1. Quadrilateral selection;
2. Determination of common paths between regions by computing the geodesics on the mesh connecting both points;
   3.1 Smoothing of the geodesic path using B-splines fitting functions;
3. Determination of common boundary points by interpolating λ points on the path;

end

5 Surface Fitting Using Optimized NURBS Patches

In order to fit smoothly NURBS surfaces on the quadrilateral network a method based on an evolutionary strategy (ES) is proposed. In order to fit a NURBS surface onto a grid, one needs to determine the weights of control points of a NURBS surface, without modifying the location of sampled points of the original surface. The main goal of this algorithm is to reduce the error between the NURBS surfaces and the data points inside the quadrilateral regions. In addition, the algorithm make sure that the C^1 continuity condition is preserved for all optimized NURBS patches. The proposed algorithm is composed of two parts: first an optimization of the NURBS patches parameters is performed, and second a NURBS patch intersections is computed.

5.1 Optimization of the NURBS Patches Parameters

A NURBS surface is completely determined by its control points $\boldsymbol{P}_{i,j}$ and by its weight factors $w_{i,j}$. The main difficulty in fitting NURBS surface locally is in finding an adequate parametrization for the NURBS and the ability to automatically choose the number of control points and their positions.

Weight factors $w_{i,j}$ of NURBS surfaces determine the local influence degree of a point on the surface topology. Generally, as in Dong [6], weights of control points for a NURBS surface are assigned in an homogeneous way and are set equal to 1 in most common algorithms, reducing NURBS to simple B-spline surface. The reason for this simplification is that control points weights determination for arbitrarily curved surfaces adjustment is a complex non-linear problem. This restricts fitting NURBS to a regular points-set. It is necessary that every row has the same number of points, making it impossible to fit the surface on a disperse unordered points-cloud. When a surface of explicit function is fitted using NURBS, the following Equation is normally minimized:

$$\delta = \sum_{l=1}^{np} \left\| \boldsymbol{Z}_l - \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}\boldsymbol{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}} \right\|^2 \qquad (1)$$

where $N_{i,p}(u)$ and $N_{j,q}(v)$ are base B-spline functions of p and q degree in the parametrical directions u and v respectively, $w_{i,j}$ are the weights, $\boldsymbol{P}_{i,j}$ the control points, and np the number of control points. If the number of knots and their positions are fixed, same as the weights set, and only the control points $\left(\{\{\boldsymbol{P}_{i,j}\}_{i=1}^{n}\}_{j=1}^{m} \in \mathbb{R}^3\right)$ are considered during minimization of Equation 1, then we have a linear mean square problem. If knots or the weights are considered unknown it is necessary to solve a non-linear problem during the fitting process. In many applications, the optimal position of knots is not necessary. Hence, the knots location problem is solved by using heuristics.

In the proposed algorithm, multiple ES of type "+" are used. These generally are denoted as follows: $(\gamma,^+ \mu)$, where γ is the size of the population and μ is the size of the descendence. Symbol ",$^+$" is used to indicate the existence of two replacement possibilities: deterministic replacement by inclusion (or type "+") or deterministic replacement by insertion (or type ","). The optimization process can be described as follows: Let $\boldsymbol{P} = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_n\}$ a points-set in $\mathbb{R}^3$ sampled from the surface of a physical object, our problem consists of:

$$E(s) = \frac{1}{n} \sum_{i=1}^{n} d_{P_i,S_i} < \delta \tag{2}$$

where d_{P_i,S_i} represents the distance between a point of the set $\boldsymbol{P}$ of sampled points of the original surface S, and a point on the approximated surface S'. To get the configuration of surface S', E is minimized to a tolerance lower than the given δ. Manipulation is performed by means of an evolution strategy $(\mu + \lambda)$ configured as follows:

- **Representation Criteria:** Representation is performed using pairs of real vectors. Representation using triples is often used, where the last vector controls the correlation between mutations of each component, but, because of the expense of the method, we decided to use only duplets.
- **Treatment criteria of non-feasible individuals:** A filtering of individuals is performed ignoring non-feasible individuals.
- **Genetic Operators:**
  - **Individual:** is composed of the weights of the control points belonging to the original points-cloud and the parameters of mutation step-adaptation. The initial values w_i, δ_i of every individual are uniformly distributed in interval [0.01, 1.0]. This range is chosen because it is not possible to set the weight to zero.
  - **Mutation:** Individuals mutation will not be correlated with n $\sigma' s$ (mutation steps) as established in individual configuration, and it is performed as indicated in the following equations:

$$\sigma_i^{'} = \sigma_i e^{(c_0.N(0,1)+c_i.N_i(0,1))}, \quad x_i^{'} = x_i + \sigma_i^{'}.N_i(0,1) \tag{3}$$

    where $N(0,1)$ is a normal distribution with expected value 0 and variance 1, c_0, c_i are constants which control the size of the mutation step. This refers to the change in mutation step σ. Once the mutation step has been updated, the mutation of individuals is generated w_i.
- **Selection Criteria:** The best individuals in each generation are selected according to the fitness function given by Equation (2).

- **Replacement criteria:** In ES, the replacement criteria is always deterministic, which means that μ or γ best members are chosen. In this case, the replacement by inclusion was used (type "+"), in which the μ descendants are joined with the γ parents into a single population, and from it, the γ best members and are taken for the new population.
- **Recombination operator:** Two types of recombination are applied whether object variables w_i or strategy parameters σ_i are being recombined. For object variables, an intermediate global recombination is used:

$$b_i^{'} = \frac{1}{\rho}\sum_{k=1}^{\rho} b_{k,i} \tag{4}$$

where $b_i^{'}$ is the new value of i, and ρ is the number of individuals within the population. For strategy parameters, an intermediate local recombination is used:

$$b_i^{'} = u_i b_{k_1,i} + (1-u_i) b_{k_2,i} \tag{5}$$

where $b_i^{'}$ is the new value of i, and u_i is a real number which is distributed uniformly within the interval $[0,1]$.

5.2 NURBS Patch Intersections

Several authors use complex schemes to guarantee continuity of normals in reconstructed models. Loop [9] proposes a continuity schema in which three different types of patches are used for special cases. In neighborhoods with big curves Bi-quadratic Bezier patches are used. At corners with triangular neighborhoods, cubic Bezier patches are used, and at regular zones, bi-quadratic spline patches are used. In a similar way Eck and Hoppe [2] use a model in which bi-quadratic B-splines functions and bi-cubic Bezier functions are fused to guarantee continuity between patches.

Continuity in regular cases (4 patches joined at one of the vertexes) is a solved problem [2]. However, in neighborhoods where the neighbors' number is different than 4 ($v \geqslant 3 \rightarrow v \neq 4$), continuity must be adjusted to guarantee a soft transition of the implicit surface function between patches of the partition. Continuity C^0 shows that a vertex continuity between two neighboring patches must exist. This kind of continuity only guarantees that holes at the assembling limit between two parametric surfaces does not exists. C^1 shows that continuity in normals between two neighboring patches must exist. This continuity also guarantees a soft transition between patches, offering a correct graphical representation.

In this algorithm, C^1 continuity between NURBS patches is guaranteed, using Peters continuity model [10] which guarantees continuity of normals between bicubical Spline functions. Peters proposes a regular and general model of bi-cubic NURBS functions with regular nodes vectors and the same number of control points at both of the parametric directions. In such a way, Peter's model was adapted by choosing generalizing NURBS functions, with the same control points number at both of the parametric directions, bi-cubic basis functions and regular expansions in their node vectors.

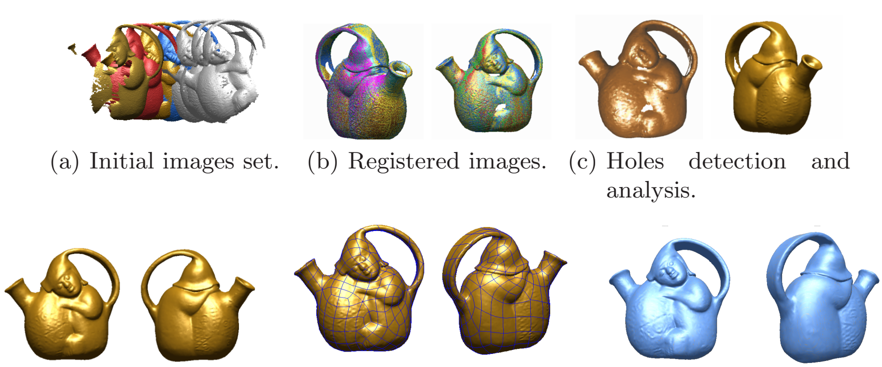
6 Experimental Results

The tests were performed using a 3.0 GHz processor, with 1.0 GB of RAM, running Microsoft Windows XP operating system. The methods were implemented using C++ and MATLAB, and graphics programs used OpenGL 1.1. The 3D data used were digitized with a Minolta Vivid 9i. The precision of the measurements were in the order of 0.05 mm.

Figure 1 shows the result of NURBS extraction on a pre-colombian ceramic object. It was necessary to integrate 18 range images to produce a complete model, as shown in Figure 1(a). In Figure 1(b) the registered and triangulated model of the object is shown, which is composed of 22217 points, with an average error of 0.0254. The surface has two topological anomalies associated to occlusions, which were corrected using a local radial base function interpolation scheme describe in [11]. This technique guarantee that the new reconstructed region adjusts smoothly with the other and also keeps the sampling density of the original mesh intact. The final model is obtained with 391 patches of optimized NURBS surfaces with a fitting error of 1.80×10^{-4} (see Figures 1(e), 1(f)). The reconstruction model of the object took an average computing time of 21 minutes.

6.1 Comparison Between the Proposed Method and Eck and Hoppe's Method

The work by Eck and Hoppe [2] performs a similar adjustment using a network of B-spline surface patches which are iteratively refined until they achieve a



(a) Initial images set. (b) Registered images. (c) Holes detection and analysis.

(d) Holes filling. (e) Extracted quadrilateral regions. (f) Final model obtained using NURBS patches.

Fig. 1. Reconstruction of a pre-colombian object using a quadrilateral network of NURBS patches

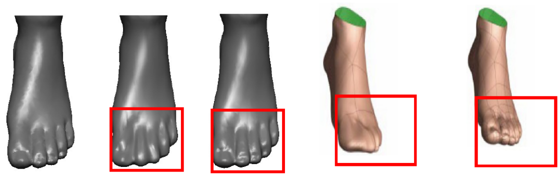


Fig. 2. Comparison between the proposed method and Eck and Hoppe's method. a) triangulated model, b) 27 patches model (proposed method without optimization), c) 27 patches model (proposed method with optimization), d) 29 patches model (Eck and Hoppe's method without optimization), e) 156 patches model (Eck and Hoppe's method with optimization).

preset error tolerance. The process of optimization performed by Eck and Hoppe reduces the error by generating new patches, which considerably augments the number of patches which represent the surface. The increment of the number of patches reduces the error because the regions to be adjusted are smaller and more geometrically homogeneous. In the method proposed in this paper, the optimization process focus on improving fitting for every patch by modifying only its parameterizations (control points and weights). For this reason, the number of patches does not increase after the optimization process. The final number of patches which represent every object is determined by the number of critical points obtained in an eigenvector associated with the eigenvalue selected from the solution system of the Laplacian matrix, and it does not change at any stage of the process. Figure 2 shows two objects (foot and skidoo part) reported by Eck and Hoppe. The model created with the proposed method , is composed of 27 and 25 patches, while Eck and Hoppe use 156 and 94 patches for the same precision. This represent a reduction of 82% and 73% less patches respectively.

7 Conclusion

The methodology proposed in this paper for the automation of reverse engineering of free-form three-dimensional objects has a wide application domain, allowing to approximate surfaces regardless of topological complexity of the original objects.

A novel method for fitting triangular mesh using optimized NURBS patches has been proposed. This method is topologically robust and guarantees that the complex base is always a quadrilateral network of NURBS patches which is compatible with most commercial CAD systems. This algorithm is simpler and robust and do not require an extensive optimization of the surface parameterization as in Dong.

In the proposed algorithm, the NURBS patches are optimized using multiple evolutionary strategies to estimate the optimal NURBS parameters. The resulting NURBS are then joined, guaranteeing C^1 continuity. An other advantage of this algorithm over Dongs is that the formulation of C^1 continuity presented in this paper can be generalized, because it can be used to approximate regular and irregular neighborhoods which present model processes regardless of partitioning and parametrization.

In the future, we are planning to explore other spectral coding functions that are more intrinsic and invariant to the way the object is immersed in 3-D space. One possible avenue is the use the eigenvalue of the curvature matrix instead of the Laplacian.

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