

# Multilevel Thresholding Method for Image Segmentation Based on an Adaptive Particle Swarm Optimization Algorithm

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**Abstract.** The multilevel thresholding method with maximum entropy is one of the most important image segmentation methods in image processing. However, its time-consuming computation is often an obstacle in real time application systems. Particle swarm optimization (PSO) algorithm is a class of heuristic global optimization algorithms which appeared recently. In this paper, the maximum entropy is obtained through an adaptive particle swarm optimization (APSO) algorithm. The APSO algorithm is shown to obtain the maximum entropy of multilevel thresholding effectively on experiments of image segmentation.

## 1 Introduction

Thresholding is a popular tool for image segmentation which is essentially a pixels classification problem which can be classified into two groups: global and local [5]. Global thresholding methods select one threshold value for the entire image based on different criteria, such as minimum error thresholding [6] and Otsu's method [7]. Entropy is a useful criterion in communications [8]. It was firstly introduced into thresholding by Pun [9] and then many methods based on entropy are developed. Local thresholding methods select different threshold values for different regions, or even for each pixel [10]. To segment complex images, a multilevel thresholding method is required. However, its time-consuming computation is often an obstacle in real time application systems.

The particle swarm optimization algorithm (PSO) has been proven to be a powerful competitor to other heuristic algorithms, such as genetic algorithm, tabu search and simulated annealing algorithm for global optimization problems [11, 12]. Clerc improved the standard PSO algorithm by introducing a constriction factor (CPSO) [13]. Du et al. segmented infrared image with 2-D maximum entropy based on PSO [14].

In this paper, considering the complexity of multilevel thresholding method with maximum entropy, we take use of an adaptive particle swarm optimization (APSO) algorithm [15], which introduces two adaptive acceleration factors and a new weight function in terms of the convergence speed and global search capability of the PSO algorithm, to search the maximum entropy of image segmentation.

## 2 Maximum Entropy Thresholding

In image segmentation, the most commonly method is to segment the image into two parts by one threshold. For an image with  $n$  gray-levels, let  $p_1, p_2, \dots, p_n$  be the probability distribution of the levels. From the distribution we derive two probability distributions given a threshold value  $t$ , one for the object  $A_1$  and another for the background  $A_2$ . The probability distributions of the object  $A_1$  and background  $A_2$  are given by

$$A_1: \frac{p_1}{P_{A_1}}, \frac{p_2}{P_{A_1}}, \dots, \frac{p_t}{P_{A_1}}, \quad A_2: \frac{p_{t+1}}{P_{A_2}}, \frac{p_{t+2}}{P_{A_2}}, \dots, \frac{p_n}{P_{A_2}}, \quad (1)$$

where  $P_{A_1} = \sum_{i=1}^t p_i$ ,  $P_{A_2} = \sum_{i=t+1}^n p_i$ .

The Shannon entropy for each distribution is defined as

$$H(A_1) = -\sum_{i=1}^t \frac{p_i}{P_{A_1}} \log \frac{p_i}{P_{A_1}}, \quad H(A_2) = -\sum_{i=t+1}^n \frac{p_i}{P_{A_2}} \log \frac{p_i}{P_{A_2}}. \quad (2)$$

Let  $\varphi(t) = H(A_1) + H(A_2)$ . When  $\varphi(t)$  is maximized the luminance level  $t$  is considered to be the optimal threshold value. This can be achieved with a cheap computational effort.

$$t^* = \arg \max_t (\varphi(t)). \quad (3)$$

It is not difficult to extend this principle to multilevel thresholding. Let

$$\varphi(t_1, t_2, \dots, t_k) = H(A_1) + H(A_2) + \dots + H(A_k), \quad (4)$$

where  $k$  is the number of the thresholds which segment the image into  $k+1$  parts, and

$A_j = \sum_{i=t_{j-1}+1}^{t_j} p_i$ . Then the optimal threshold vector can be described as

$$(t_1^*, t_2^*, \dots, t_k^*) = \arg \max_{(t_1, t_2, \dots, t_k)} (\varphi(t_1, t_2, \dots, t_k)). \quad (5)$$

In this paper, we use the multilevel thresholding to segment images.

## 3 APSO Algorithm

Let  $i$  indicate a particle index in the swarm that includes  $m$  particles. Each particle flies through an  $n$ -dimensional search space. Let  $X_i = (x_{i1}, x_{i2}, \dots, x_{in})$  and

$V_i = (v_{i1}, v_{i2}, \dots, v_{in})$  represent the current position and velocity of particle  $i$  respectively. They are dynamically adjusted according to the particle's previous best position  $P_i = (p_{i1}, p_{i2}, \dots, p_{in})$  and the best position of the entire swarm  $P_g = (p_{g1}, p_{g2}, \dots, p_{gn})$ . In APSO [15], the evolving equations of particles can be described as follows:

$$v_{ij}(t+1) = w(t)v_{ij}(t) + w_1 r_{1j} A_{i1}(p_{ij}(t) - x_{ij}(t)) + w_2 r_{2j} A_{i2}(p_{gj}(t) - x_{ij}(t)) , \quad (6)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad i=1, 2, \dots, m; \quad j=1, 2, \dots, n , \quad (7)$$

where  $r_{1j}, r_{2j} \in [0, 1]$  are random numbers. The weight function  $w(t) \sim U(w_0, 2w_0)$ , where  $w_0$  is a constant parameter, the suggested range for  $w_0$  is  $[0.3, 0.6]$ , and  $U$  presents a uniform probability distribution.  $w_1$  and  $w_2$  are as shown in (8), and the two adaptive acceleration factors  $A_{i1}$  and  $A_{i2}$  are as shown in (9) respectively.

$$w_1 = c_1^2 / (c_1 + c_2) , \quad w_2 = c_2^2 / (c_1 + c_2) \quad (8)$$

where  $c_1$  and  $c_2$  are the same as in the standard PSO.

$$A_{i1} = \ln\left(\left|\frac{f(P_i(t)) - f(X_i(t))}{\overline{PF}}\right| + e - 1\right) , \quad A_{i2} = \ln\left(\left|\frac{f(P_g(t)) - f(X_i(t))}{\overline{GF}}\right| + e - 1\right) \quad (9)$$

where  $f(x)$  is the fitness function,  $e$  is the base of the natural logarithm, i.e.  $\ln(e)=1$ ,

$$\overline{PF} = \frac{1}{m} \sum_{i=1}^m (f(P_i(t)) - f(X_i(t))) , \quad \text{and} \quad \overline{GF} = \frac{1}{m} \sum_{i=1}^m (f(P_g(t)) - f(X_i(t))) .$$

## 4 Image Segmentation Based on APSO Algorithm

Two images are segmented in the experiments to illustrate the validation of APSO algorithm. The original images Lena and Fruit contain  $256 \times 256$  pixels. After getting the gray histogram of two images, we set the number of threshold  $k=3$  for each image. The optimal threshold vector of Lena is (61, 109, 159) with the maximum Shannon entropy value 15.6632, and the optimal threshold vector of Fruit is (57, 111, 166) with the maximum Shannon entropy value 15.9421. That can be obtained by exhaustive search which needs to do  $256 \times 255 \times 254 = 16,581,120$  times computation of entropy function.

In both experiments, standard PSO, CPSO and APSO algorithm are used to search the best thresholding. For the standard PSO algorithm, a linearly decreasing weight function is used which starts at 0.9 and ends at 0.4 with  $c_1=c_2=2.0$ . For CPSO and APSO algorithm, we set  $c_1=2.8$ ,  $c_2=1.3$  under the suggestion of Clerc [13]. For all algorithms we set the initial particles  $m=30$  and the maximum generation  $G_{\max} = 10000$ . The final result takes from the mean value of 50 runs for each

algorithm. All the algorithms are written in C and executed in Visual C++ 6.0 on PC with a CPU of Pentium IV and 256M RAM. Table 1 shows the results of experiments.

**Table 1.** The comparison of efficiency between Exhaustive search, PSO, CPSO and APSO

		Algorithm			
		Exhaustive search	PSO	CPSO	APSO
Lena	Time-consuming (ms)	406,609	12,055	6,232	1,103
	Calls for entropy function	16,581,120	45,472	22,168	921
	Success rate	—	100%	100%	100%
Fruit	Time-consuming (ms)	656,296	13,918	6,548	1,067
	Calls for entropy function	16,581,120	62,736	29,673	731
	Success rate	—	100%	100%	100%

From Table 1, we can see it clearly that APSO algorithm performances much better than standard PSO, CPSO algorithm and exhaustive search. For example, the time-consuming of PSO for Lena is 12,055ms, comparing with exhaustive search method 606,609ms, the efficiency is improved by  $606,609/12,055 \approx 50.32$  times; The time-consuming of CPSO for Lena is 6,232ms, comparing with exhaustive search method 606,609ms, the efficiency is improved by  $606,609/6,232 \approx 97.34$  times; and the time-consuming of APSO for Lena is 1,103ms, comparing with exhaustive search method 606,609ms, the efficiency is improved by  $606,609/1,103 \approx 549.96$  times. Moreover, it is shown that PSO, CPSO and APSO can all reach the global optimal solution 100% within a smaller  $G_{\max}$ .

## 5 Conclusion

Multilevel thresholding with maximum entropy is a good method to do image segmentation except its complex computation. And PSO algorithm is a heuristic global optimization algorithm which appeared recently. PSO has been widely concerned by researcher because of its feasibility and effectiveness. In this paper, with the help of APSO algorithm, the maximum entropy vector can be obtained easily with high efficiency. The multilevel thresholding method for image segmentation based on APSO algorithm is a simple and effective method to do image segmentation. Especially, it is full of importance in the system where real time processing is needed.

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