# Lecture Notes in Artificial Intelligence4869Edited by J. G. Carbonell and J. Siekmann

## Subseries of Lecture Notes in Computer Science

Francisco Botana Tomas Recio (Eds.)

# Automated Deduction in Geometry

6th International Workshop, ADG 2006 Pontevedra, Spain, August 31-September 2, 2006 Revised Papers



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Library of Congress Control Number: 2007941260

CR Subject Classification (1998): I.2.3, I.3.5, F.4.1, I.5, G.2

LNCS Sublibrary: SL 7 - Artificial Intelligence

ISSN	0302-9743
ISBN-10	3-540-77355-X Springer Berlin Heidelberg New York
ISBN-13	978-3-540-77355-9 Springer Berlin Heidelberg New York

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Typesetting: Camera-ready by author, data conversion by Scientific Publishing Services, Chennai, India Printed on acid-free paper SPIN: 12206292 06/3180 5 4 3 2 1 0

#### Preface

After five successful editions (Toulouse, 1996; Beijing, 1998; Zurich, 2000; Linz, 2002, Gainesville, Fl., 2004), the series of international workshops on Automated Deduction in Geometry (ADG) has consolidated its fundamental role concerning the scientific community working on the interaction between geometry and automated deduction. From August 31 to September 2, 2006, a new ADG meeting took place at the Pontevedra (Galicia, Spain) campus of the University of Vigo, as a satellite event of the International Congress of Mathematicians (Madrid, August 22–30, 2006). We acknowledge the financial support for ADG 2006, provided by the University of Vigo and the Spanish Ministerio de Educación y Ciencia under grant MTM2005-24580-E.

It was a fruitful meeting – made possible by the work of the Organizing Committee (see next page) – for exchanging ideas and for the presentation of original results and software novelties – 21 contributions in total – under the scientific guidance of the Program Committee (listed on the next page). Moreover, it was a privilege to receive the lectures of our distinguished guest speakers, Thomas Hales (U. Pittsburgh) and Martin Peternell (T.U. Wien), dealing with the so-called Flyspeck project, i.e., the automatization of Hales' solution to Kepler's conjecture, and with rational offset surfaces and related issues in CAGD, respectively.

Shortly after the meeting, a call for papers – within the scope of ADG, but with content not necessarily related to a presentation at ADG 2006 – was launched. After a long and detailed process of peer review and revision, we – the editors – have selected the 13 papers of this volume, as a testimony of the current state of the art concerning automated deduction in geometry.

This volume includes a paper by X. Chen and D. Wang proposing a system in the form of a textbook – an electronic geometry textbook, to be more precise - for managing geometric knowledge dynamically, effectively, and interactively. The contribution by T. Hales, in the context of the "Flyspeck" project, describes an algorithm that decides whether a region in three dimensions, described by quadratic constraints, is equidecomposable with a collection of primitive regions and, when a decomposition exists, finds the volume of the given region. P. Janičić and P. Quaresma present an application of automatic theorem proving in the verification of constructions made with dynamic geometry software. The paper by P. Lebmeir and J. Richter-Gebert proposes an algorithm for automated recognition of computationally constructed curves and discusses several aspects of the recognition problem. R. H. Lewis and E. Coutsias deal with polynomial systems, flexibility of three-dimensional objects, computational chemistry, and computer algebra. D. Lichtblau's contribution on computational real enumerative geometry discusses the number and reality of the cylinders generically determined by five points in  $\mathbb{R}^3$ . D. Michelucci and S. Foufou address the detection of dependences in geometric constraints solving, and propose to use the recently published witness method. The paper by A. Montes and T. Recio merges two techniques (automatic discovery and minimal canonical comprehensive Gröbner systems) to discover missing hypotheses in generally false statements. J. Narboux describes the mechanization of the proofs of the first eight chapters of the classic book "Metamathematische Methoden in der Geometrie" by Schwabäuser, Szmielew and Tarski. The paper by P. Pech deals with the problem, posed by Chou long ago, of finding a natural geometry problem where hypotheses are not described by a radical ideal, such as the existence of regular polygons (pentagons, heptagons) of even dimension. E. Roanes-Macías and E. Roanes-Lozano present a Maple package, on the interaction of computer algebra and dynamic geometry, for investigating problems about configuration theorems in 3D geometry and performing mechanical theorem proving and discovery. P. Todd presents an interactive symbolic geometry package, "Geometry Expressions," generating algebraic formulas from geometry in an interactive style which is convenient not only for high school students, but also for mechanical engineers, graphics programmers, architects, surveyors, machinists, and many more. Finally, the paper by L. Yang and Z. Zeng, employing a method of distance geometry, achieves the symbolic solution to the following problem: express the edge-lengths of a tetrahedron in terms of its heights and widths.

We, the editors, would like to thank the efforts of so many anonymous referees involved in the process of selection and improvement of the submitted papers. We think that, as a consequence of their work, this collection of papers, although necessarily incomplete, shows the lively variety of topics and methods and the current applicability of ADG to different branches of mathematics and to other sciences and technologies.

October 2007

Francisco Botana Tomas Recio

### Organization

#### **Invited Speakers**

Thomas Hales (University of Pittsburgh, USA) Martin Peternell (Vienna University of Technology, Austria)

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