Non-maturing Deposits, Convexity and Timing Adjustments

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1 Introduction

One key driver of a bank's total interest rate risk is the position of non-maturing deposits. Several papers such as [6], [8], and [7] value non-maturing deposits in an arbitrage-free framework and analyze their risk profile. All these models consist of three major components: first, the short rate process, i.e. the dynamics of the default-free interest rate term structure; second, the interest rate pass-through, i.e. the link between the development of the deposit rates and the development of default-free interest rates, in general the short rate; third, the development of the deposit volume over time. In this paper, we concentrate on the interest rate pass-through. We provide some term structure model-free results on the valuation of deposits, when the deposit rates are linearly linked to some long-term swap rate (rather than a short-term interest rate) as the reference rate with an unnatural time lag.

2 Deposits

2.1 Preliminaries

Let $(\Omega, (F_t)_{t\in\{0,T'\}}, P)$ be a filtered probability space that fulfills the usual conditions. Like the aforementioned articles, we assume markets to be arbitrage-free, frictionless and complete. Let $P(t,T), 0 \leq t \leq T \leq T'$, denote the value in t of a default-free zero bond with face value 1 maturing in T. The filtration is assumed to be generated by these zero bonds. The M-year swap rate $SR(t,M)$, $M \in \mathbb{N}$, in t and the corresponding today's M-year forward swap rate $FSR(t,M)$ are given by

$$SR(t, M) = \frac{1 - P(t, t+M)}{\sum_{j=1}^{M} P(t, t+j)}, \tag{1}$$

$$FSR(t, M) = \frac{P(0, t) - P(0, t+M)}{\sum_{j=1}^{M} P(0, t+j)}. \tag{2}$$

Q^t denotes the unique equivalent t-forward martingale measure (see [3]) and $E_0^t()$ the respective expectation operator conditional on today. Under Q^T, the value of a non-dividend paying security discounted by $P(t, T)$ is a martingale. As a special case, the expected value for the point in time t of a zero bond maturing in T under the t-forward measure equals its forward price:

$$E_0^t(P(t, T)) = \frac{P(0, T)}{P(0, t)}. \tag{3}$$

2.2 Valuation

Define $0 = t_0, t_1, t_2, ..., t_N = T$ with $t_i - t_{i-1} = \Delta t = 1$.[1] For simplicity, we consider a deposit with a constant face value 1 and maturity date T. The deposit rate for the period $]t_{i-1}, t_i]$ paid in t_i is given by $DR(t_{i-1})$. We assume that $DR(t_{i-1})$ is fixed at $t_{i-1}^k := t_{i-1} + k$ with $-1 \leq k \leq 1$. The 'shift' k has a straightforward interpretation: if $k = 0$ the deposit rate is fixed at the beginning of the period $]t_{i-1}, t_i]$. In this case we have a 'natural' time lag between the fixing date and the date at which the deposit rate is paid. If $k < 0$ the deposit rate is fixed before the beginning of the respective period. If $k > 0$ it is fixed within the period. In both cases there is an 'unnatural' time lag. We assume that the deposit rate for the period $]t_{i-1}, t_i]$ is linearly linked to the M-year swap rate $SR(t_{i-1}^k, M)$ observed in t_{i-1}^k as the reference rate:[2]

$$DR(t_{i-1}) = b_1 + b_2\ SR(t_{i-1}^k, M). \tag{4}$$

By construction, $DR(t_{i-1})$ is measurable with respect to F_{t_i}. It can be interpreted as a European derivative on the term structure that is due in t_i. Therefore, we obtain the following representation of the present value PV of the deposit:

$$PV = b_1 \sum_{i=1}^{N} P(0, t_i) + P(0, t_N) + b_2 \sum_{i=1}^{N} P(0, t_i)\ E_0^{t_i}(SR(t_{i-1}^k, M)). \tag{5}$$

[1] The assumption $\Delta t = 1$ can easily be relaxed.

[2] Obviously, the deposit is close to a portfolio consisting of a money market floater and a constant maturity swap. See, e.g., [1] and [4] for the valuation of constant maturity swaps.

Clearly, the key to the calculation of PV is the determination of the present value of $SR(t_{i-1}^k, M)$ paid in t_i as the other components of (5) can be calculated easily. Define

$$PV(SR(t_{i-1}^k, M), t_i) = P(0, t_i)\ E_0^{t_i}(SR(t_{i-1}^k, M)). \tag{6}$$

In the following, we aim to calculate an adjustment $AD(t_i)$ on the respective forward swap rate, so that

$$PV(SR(t_{i-1}^k, M), t_i) = P(0, t_i)\ (FSR(t_{i-1}^k, M) + AD(t_i)) \tag{7}$$

is fulfilled. As the present value of $SR(t_{i-1}^k, M)$ paid in t_i equals the present value of $P(t_{i-1}^k, t_i)\ SR(t_{i-1}^k, M)$ paid in t_{i-1}^k

$$PV(SR(t_{i-1}^k, M), t_i) = P(0, t_{i-1}^k)\ E_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i)\ SR(t_{i-1}^k, M)) \tag{8}$$

must hold. By equating (6) and (8) and using the definition of the covariance we obtain[3]

$$\begin{aligned} E_0^{t_i}(SR(t_{i-1}^k, M)) &= \frac{P(0, t_{i-1}^k)}{P(0, t_i)}\ E_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i)\ SR(t_{i-1}^k, M)) \\ &= \frac{P(0, t_{i-1}^k)}{P(0, t_i)}\ E_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i))\ E_0^{t_{i-1}^k}(SR(t_{i-1}^k, M)) \\ &\quad + \frac{P(0, t_{i-1}^k)}{P(0, t_i)}\ CoVar_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i), SR(t_{i-1}^k, M)). \end{aligned} \tag{9}$$

Substituting (3) into (9) and rearranging terms leads to

$$\begin{aligned} &E_0^{t_i}(SR(t_{i-1}^k, M)) \\ &= E_0^{t_{i-1}^k}(SR(t_{i-1}^k, M)) \\ &\quad + \frac{P(0, t_{i-1}^k)}{P(0, t_i)}\ CoVar_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i), SR(t_{i-1}^k, M)) \\ &= FSR(t_{i-1}^k, M) + CA(t_i) + TA(t_i), \end{aligned} \tag{10}$$

where

$$CA(t_i) = E_0^{t_{i-1}^k}(SR(t_{i-1}^k, M)) - FSR(t_{i-1}^k, M), \tag{11}$$

$$TA(t_i) = \frac{P(0, t_{i-1}^k)}{P(0, t_i)}\ CoVar_0^{t_{i-1}^k}(P(t_{i-1}^k, t_i), SR(t_{i-1}^k, M)). \tag{12}$$

[3] The first line of (9) is a special case of the change of numéraire theorem, see [3].

Equations (6), (10), (11), and (12) clarify that the forward swap rate has to be adjusted by two terms: first, by the difference $CA(t_i)$ between the t_{i-1}^k-forward measure expectation of the swap rate and the corresponding forward swap rate. Second, by the scaled covariance $TA(t_i)$ between the discount factor from t_i to t_{i-1}^k and the swap rate under the t_{i-1}^k-forward measure. The two adjustments sum up to the total adjustment $AD(t_i) = CA(t_i) + TA(t_i)$.

2.3 Convexity and Timing Adjustment

We first analyze the so called 'convexity adjustment' $CA(t_i)$. Based on the definition of the covariance and of the swap rate (1) we obtain

$$\begin{aligned} & E_0^{t_{i-1}^k}(SR(t_{i-1}^k, M)) \\ &= E_0^{t_{i-1}^k}\left(\frac{1}{\sum_{j=1}^M P(t_{i-1}^k, t_{i-1}^k + j)}\right) E_0^{t_{i-1}^k}(1 - P(t_{i-1}^k, t_{i-1}^k + M)) \\ &\quad + CoVar_0^{t_{i-1}^k}\left(\frac{1}{\sum_{j=1}^M P(t_{i-1}^k, t_{i-1}^k + j)}, 1 - P(t_{i-1}^k, t_{i-1}^k + M)\right). \end{aligned} \tag{13}$$

As the function $x \to 1/x$ is convex Jensen's inequality, (3) and (2) imply

$$\begin{aligned} & E_0^{t_{i-1}^k}\left(\frac{1}{\sum_{j=1}^M P(t_{i-1}^k, t_{i-1}^k + j)}\right) E_0^{t_{i-1}^k}(1 - P(t_{i-1}^k, t_{i-1}^k + M)) \\ &\geq \frac{1}{\sum_{j=1}^M E_0^{t_{i-1}^k}(P(t_{i-1}^k, t_{i-1}^k + j))} E_0^{t_{i-1}^k}(1 - P(t_{i-1}^k, t_{i-1}^k + M)) \\ &= \frac{1}{\sum_{j=1}^M \frac{P(0, t_{i-1}^k + j)}{P(0, t_{i-1}^k)}} \frac{P(0, t_{i-1}^k) - P(0, t_{i-1}^k + M)}{P(0, t_{i-1}^k)} \\ &= FSR(t_{i-1}^k, M). \end{aligned} \tag{14}$$

As the covariance term in (13) is positive in general, we obtain the following inequality for the expected swap rate:

$$E_0^{t_{i-1}^k}(SR(t_{i-1}^k, M)) \geq FSR(t_{i-1}^k, M). \tag{15}$$

This implies that the convexity adjustment $CA(t_i)$ is positive in general. Note that it only depends on the fixing date t_{i-1}^k and not on the payment date t_i of the deposit rate. Therefore, it is also independent of the difference between these two dates, i.e. the time lag. The convexity adjustment would also be necessary if the deposit rate fixed in t_{i-1}^k were paid in t_{i-1}^k rather than in t_i.

In contrast, the second adjustment $TA(t_i)$ depends only on the difference between the fixing date and the payment date, i.e. the time lag. Therefore, it is called 'timing adjustment'.[4] If the time lag is zero, i.e. if $k = 1$, we have $P(t_{t-1}^k, t_i) = 1$ in (12). Therefore, the covariance term equals zero so that the timing adjustment vanishes, i.e. $TA(t_i) = 0$. Since generally all interest rates are positively correlated the covariance in (12) and, hence, the timing adjustment is negative for $k < 1$. Note that the timing adjustment is even necessary if we have a natural time lag, i.e. if $k = 0$.

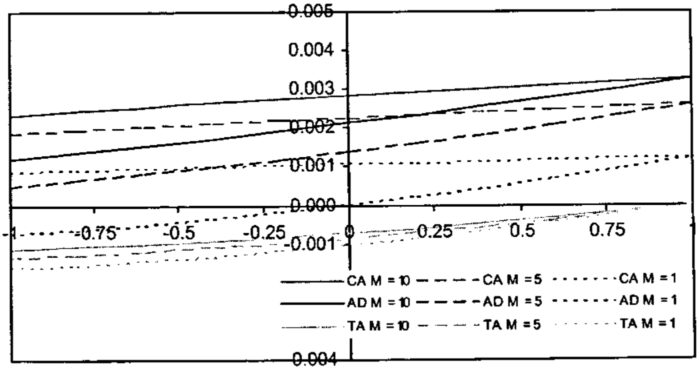

Fig. 1. Convexity and timing adjustments

This figure shows numerical results for the convexity adjustment CA, the timing adjustment TA and the total adjustment AD for different maturities M of the swap rate in dependence of the time lag k. The adjustments are calculated for $t_i = 5$. Calculations are based on the short rate model of Hull and White (see [5]) with the following input parameters: today's spot rate structure = flat at 5%; mean reversion speed = 0.1; short rate volatility = 0.02

Of course, the concrete size of the convexity adjustment, the timing adjustment, and the total adjustment depends on the term structure model. Figure 1 shows some numerical results for the model by Hull and White (see [5]). The timing adjustment equals zero for $k = 1$ and

[4] See also [4].

is becoming negative and smaller for smaller k. The convexity adjustment is always positive and is becoming larger for larger k. The total adjustment is increasing in k and is positive in general. For $k < 0$ and small maturities M of the reference swap rate it can become negative. Obviously, most effects are more pronounced for longer maturities of the swap rate.

3 Concluding Remarks

In this paper, we analyzed the valuation of deposits when the deposit rates are linearly linked to long-term swap rates. We allowed for natural and unnatural time lags and provided term structure model-free results on the valuation of these deposits. We especially focused on the structure of necessary adjustments on the forward swap rates: the convexity and the timing adjustment. Our analysis can easily be transferred to the case of other capital market yields such as spot rates or yields of fixed-coupon bonds (see [2]).

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