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Mathematics of Fuzziness – Basic Issues

Springer

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Foreword

The word *fuzziness,* as introduced by our friend and mentor Professor Lotfi A. Zadeh in 1965, has evolved to characterize the information associated with the human language. The human language which carries a lot of information and is being used in our everyday decision making processes, in a mathematical sense, is not very precise. For example, in Saskatoon in the month of January day time temperature is usually around -15^0 C to -20^0C. A resident of Saskatoon will classify this temperature as *'not very cold'*. By a friend just arriving from India, his classification for this temperature would be *'very- very cold'*.

Professor Zadeh has given us a mathematical tool called the *fuzzy mathematics,* which gives precision to uncertainty inherent in our human language. The meaning of the wordings that we use in our everyday life are context dependent, subjective, and have a cognitive flavor. Thus, we can say that *the fuzzy set theory is a mathematical tool that provides a figure of certainty to a cognitive type of uncertain phenomenon inherent in our human language.*

Since the inception of notion of fuzzy sets in 1965, many research papers and books have appeared in the field of fuzzy uncertainty. In the present book entitled '*Mathematics of Fuzziness, Basic Issues'* the authors, Drs. Xuzhu Wang, Da Ruan and Etienne Kerre who are among the *pioneers* in the field of fuzzy mathematics, have provided a lucid mathematical characterization of 'fuzzy uncertainty'. In this book the authors introduce a basic notion of *'fuzziness' and* provide a conceptual mathematical framework to characterize such fuzzy phenomena.

In general, mathematical methods have evolved in order to characterize the phenomena we are surrounded with. This book which contains six chapters in 281 pages lays the basic mathematical basis of fuzzy phenomena, the phenomena which are inherent in our human perception and cognitive processes. The authors start with the preliminaries on sets and relations, in Chapter 1. Then in Chapter 2 they provide some conceptual basics of fuzzy sets. In Chapter 3 they provide a detailed description of the methodology of fuzzy relations with applications in several fields such as fuzzy clustering, information retrieval and multiple attribute decision making analysis. Following this basic introduction, the authors move to Chapter 4 to describe the extension principle and fuzzy numbers, in Chapter 5 some mathematical topics such as fuzzy measures and fuzzy integrals, and finally in Chapter 6 some applications oriented topics such as fuzzy inference and fuzzy control. Each chapter is very well appended with exercises and reference material.

In the years ahead, the notion of *fuzziness* is likely to be an integral part for dealing with cognitive uncertainty in a computationally effective way. Thus, this

book on mathematics of fuzziness which has evolved from the authors' lecture notes to both undergraduate and graduate students provides a basic mathematical exposition to this growing field of soft-computing. The authors and the publisher, Springer Verlag, have produced a treatise that addresses, with high authority and high level of expertise, the mathematics devoted in the studies of fuzziness and soft-computing. For this very informative and timely book the authors deserve our heartiest congratulations!

University of Saskatchewan Madan M. Gupta
January 28, 2009

Preface

This book is intended to serve as a basic textbook for a fuzzy set and system course at the undergraduate level and as a preliminary material for further research and applications on some special topics. Accordingly, the readers may contain undergraduate as well as graduate students in mathematics, computer science and engineering, engineers and researchers in the related fields. The seven aspects feature our book.

(1) The most fundamental subjects in fuzzy set theory such as fuzzy sets, fuzzy relations and fuzzy numbers are introduced in detail.
(2) The most important applications of fuzzy sets such as pattern recognition, clustering analysis and fuzzy control are explicitly highlighted.
(3) The well-developed pure mathematical branches such as fuzzy measures, fuzzy integrals, fuzzy algebra, and fuzzy topology are briefly discussed.
(4) While most parts of the book are devoted to the introduction to the basics of fuzzy set theory and its applications, some parts contain advanced materials based our research.
(5) Remarks are made to indicate further reading in the end of some sections particularly with important and theoretical sections.
(6) The whole book is basically self-contained. There is no difficulty for the reader with basic knowledge of mathematical analysis and linear algebra to get through all chapters except Chapter 5.
(7) Many exercises arranged as the last section of every chapter help the reader to understand the concepts, approaches and theories in the chapter and serve as a supplement to main results in the chapter.

The book is structured as follows. In Chapter 1, we recall basic concepts of set theory and abstract algebra including set, relation, isomorphism, lattice, Boolean algebra, and soft algebra. These concepts play an important role in the investigation of fuzzy sets and fuzzy relations. In Chapter 2, we introduce the basics of fuzzy set theory. Considering general fuzzy logic connective operations are visible everywhere in the current fuzzy literature, a lot of space is

assigned to deal with negations, t-norms, t-conorms, fuzzy implications and equivalencies besides the Zadeh's set-theoretic operations. Then we expose the links between fuzzy sets and crisp sets which include the decomposition theorems and mathematical representations of fuzzy sets in terms of a nest of sets. Afterwards, fuzzy sets are extended to L-fuzzy sets and a similar investigation is carried out. Finally, an important application of fuzzy set theory – fuzzy pattern recognition – is introduced. In Chapter 3, we deal with fuzzy relations. We begin with the investigation of various compositions of fuzzy relations and followed by the introduction of fuzzy equivalence relations and fuzzy tolerance relations. Then comes an exhaustive investigation into the main properties of fuzzy relations: negativity, semi-transitivity, Ferrers property, consistency, weak transitivity and acyclicity. After that, we discuss a type of composite fuzzy relation equations, and finally mention several applications of fuzzy relations. In Chapter 4, three tasks are fulfilled. The first one is the introduction of the Zadeh's extension principle. The second one is the discussion of fuzzy numbers and their algebraic operations. The last one is the detailed investigation of ranking methods for fuzzy numbers, which can find wide applications when fuzzy data are involved and processed. In Chapter 5, we introduce three well-developed fuzzified mathematical branches: (i) fuzzy measures and fuzzy integrals; (ii) fuzzy algebraic structures including fuzzy groups, fuzzy rings and fuzzy ideals; (iii) fuzzy topology. The chapter is especially ready for the reader with some crisp mathematical prerequisites and supplies them with primary fuzzification approaches of a mathematical theory, which will open a door to further theoretical research. The major subject of Chapter 6 is fuzzy control. We outline the principle of fuzzy control by a typical application case. Around fuzzy control, we briefly introduce some concepts related to fuzzy inference such as linguistic variables, hedges, fuzzy propositions, IF-THEN rules and special fuzzy inference approaches. Considering that fuzzification and defuzzification often play an important role in the design of fuzzy controllers, we also investigate them in this chapter.

This book has evolved from our lectures for both undergraduate and graduate students. We are thankful to our students and colleagues for their constructive suggestions. We are deeply indebted to the authors whose works are employed and not explicitly cited, but certainly listed in our bibliography. We gratefully acknowledge the research support by the China-Flanders Foundation and the Natural Science Foundation of Shanxi Province.

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