

Lecture Notes Electrical Engineering

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Stanisław Rosłonec

Fundamental Numerical Methods for Electrical Engineering

Prof. Dr. Hab. Ing. Stanisław Rosłonec
Institute of Radioelectronics
Warsaw University of Technology
Nowowiejska 15/19
00-665 Warsaw
Poland
s.rosłonec@ire.pw.edu.pl

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About the Author



Stanisław Rosłonec received his M.Sc. degree in electronic engineering from the Warsaw University of Technology, Warsaw, in 1972. After graduation he joined the Department of Electronics, (Institute of Radioelectronics), Warsaw University of Technology where in 1976 he was granted with distinction his doctor's degree (Ph.D). The thesis has been devoted to nonlinear phenomena occurring in microwave oscillators with avalanche and Gunn diodes. In 1991, he received Doctorate in Science degree in electronic engineering from the Warsaw University of Technology for a habilitation thesis on new methods of designing linear microwave circuits. Finally, he received in 2001 the degree of professor of technical science. In 1996, he was appointed as associate professor in the Warsaw University of Technology, where he lectured on "*Fundamentals of radar and radionavigation techniques*", "*UHF and microwave antennas*", "*Numerical methods*" and "*Methods for analysis*

and synthesis of microwave circuits". His main research interest is computer-aided design of different microwave circuits, and especially planar multi-element array antennas. He is the author of more than 80 scientific papers, 30 technical reports and 6 books, viz. "*Algorithms for design of selected linear microwave circuits*" (in Polish), WkŁ, Warsaw 1987, "*Mathematical methods for designing electronic circuits with distributed parameters*" (in Polish), WNT, Warsaw 1988, "*Algorithms for computer-aided design of linear microwave circuits*", Artech House, Inc. Boston–London 1990, "*Linear microwave circuits – methods for analysis and synthesis*" (in Polish), WkŁ, Warsaw 1999 and "*Fundamentals of the antenna technique*" (in Polish), Publishing House of the Warsaw University of Technology, Warsaw 2006. The last of them is the present book "*Fundamental Numerical Methods for Electrical Engineering*". Since 1992, Prof. Rosloniec has been tightly cooperating with the Telecommunications Research Institute (PIT) in Warsaw. The main subject of his professional activity in PIT is designing the planar, in-phase array antennas intended for operation in long-range three-dimensional (3D) surveillance radar stations. A few of two-dimensional (planar) array antennas designed by him operate in radars of type TRD-12, RST-12M, CAR 1100 and TRS-15. These modern radar stations have been fabricated by PIT for the Polish Army and foreign contractors.

Introduction

Stormy development of electronic computation techniques (computer systems and software), observed during the last decades, has made possible automation of data processing in many important human activity areas, such as science, technology, economics and labor organization. In a broadly understood technology area, this development led to separation of specialized forms of using computers for the design and manufacturing processes, that is:

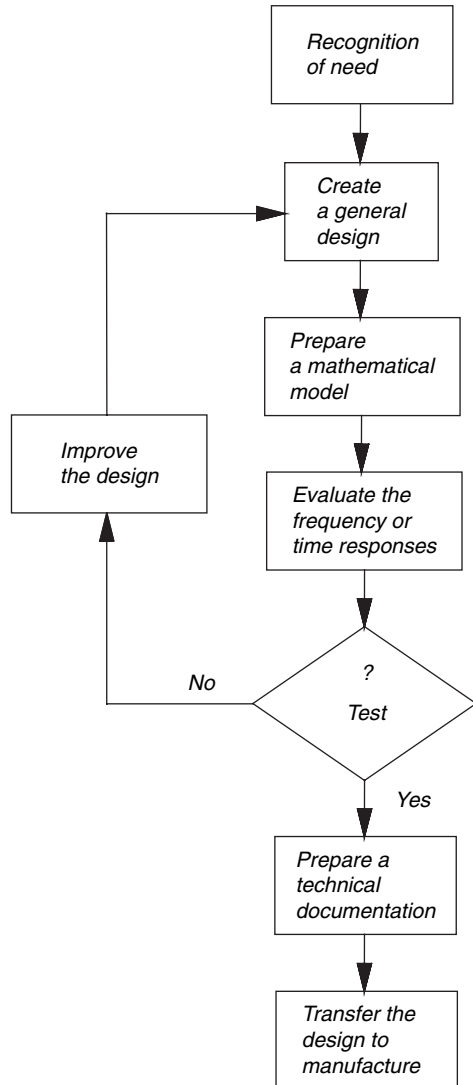
- computer-aided design (CAD)
- computer-aided manufacture (CAM)

In order to show the role of computer in the first of the two applications mentioned above, let us consider basic stages of the design process for a standard piece of electronic system, or equipment:

- formulation of requirements concerning user properties (characteristics, parameters) of the designed equipment,
- elaboration of the initial, possibly general electric structure,
- determination of mathematical model of the system on the basis of the adopted electric structure,
- determination of basic responses (frequency- or time-domain) of the system, on the base of previously established mathematical model,
- repeated modification of the adopted diagram (changing its structure or element values) in case, when it does not satisfy the adopted requirements,
- preparation of design and technological documentation,
- manufacturing of model (prototype) series, according to the prepared documentation,
- testing the prototype under the aspect of its electric properties, mechanical durability and sensitivity to environment conditions,
- modification of prototype documentation, if necessary, and handing over the documentation to series production.

The most important stages of the process under discussion are illustrated in Fig. I.1.

Fig. I.1



According to the diagram presented above, the design process begins with the formulation of user requirements, which should be satisfied by the designed system in presence of the given construction and technological limitations. Next, among various possible solutions (electrical structures represented by corresponding structures), the ones, which best satisfy the requirements adopted at the start are chosen. During this stage, experience (knowledge and intuition) of the designer has decisive influence on the design process. For general solution chosen in this manner (values of system elements can be changed), mathematical model, in the form of transfer function, insertion losses function or state equations, is next determined. On the

base of the adopted mathematical model, frequency- or time-domain responses of the designed system are then calculated. These characteristics are analyzed during the next design stage. In case when the system fully satisfies the requirements taken at the start, it is accepted and its electric structure elaborated in this manner can be considered as the base for preparation of the construction and technological documentation. In the opposite case, the whole design cycle is repeated for changed values of elements of the adopted electrical structure. When modification of the designed system is performed with participation of the designer (manual control), the process organized in this way is called interactive design. It is also possible to modify automatically the parameters of the designed system, according to appropriate improvement criterions (goal function), which should take usually minimal or maximal values. Design process is then called optimization. During the stage of constructing mathematical model of the designed system, as well as during the stage of analysis, there is a constant need for repeated performing of basic mathematical procedures, such as:

- solving systems of linear algebraic equations,
- solving systems of nonlinear algebraic equations,
- approximation or interpolation of one or many variable functions,
- integration of one or many variable functions,
- integration of ordinary differential equations,
- integration of partial differential equations,
- solving optimization problems, the minimax problem included.

The second process mentioned above, namely the CAM, can be considered in a similar way. The author is convinced that efficient use of computer in both processes considered, requires extensive knowledge of mathematical methods for solving the problems mentioned above, known commonly under the name of numerical methods. This is, among other things the reason, why numerical methods became one of the basic courses, held in technical universities and other various kinds of schools with technical profile. Considerable cognitive virtues and specific beauty of this modern area of mathematics is the fact, which should also be emphasized here.

This book was worked out as education aid for the course “Numerical Methods in Radio Electronics” lead by the author on the Faculty of Electronics and Information Technology of Warsaw University of Technology. During its elaboration, considerable emphasis was placed on the transparency and completeness of discussed issues, and presented contents constitute sufficient base for writing calculation programs in arbitrary programming language, as for example in Turbo Pascal. Each time, when it was justified for editorial reasons, vector notation of the equation systems and vector operations were deliberately abandoned, the fact that facilitates undoubtedly the understanding of methods and numerical algorithms explained in this book. Numerous examples of engineering problems taken from electronics and high-frequency technology area serve for the same purpose.