Design and Analysis of Learning Classifier Systems

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Vol. 139. Jan Drugowitsch Design and Analysis of Learning Classifier Systems, 2008 ISBN 978-3-540-79865-1 Jan Drugowitsch

# Design and Analysis of Learning Classifier Systems

A Probabilistic Approach



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#### **Foreword**

This book is probably best summarized as providing a principled foundation for Learning Classifier Systems. Something is happening in LCS, and particularly XCS and its variants that clearly often produces good results. Jan Drugowitsch wishes to understand this from a broader machine learning perspective and thereby perhaps to improve the systems. His approach centers on choosing a statistical definition – derived from machine learning – of "a good set of classifiers", based on a model according to which such a set represents the data. For an illustration of this approach, he designs the model to be close to XCS, and tests it by evolving a set of classifiers using that definition as a fitness criterion, seeing if the set provides a good solution to two different function approximation problems. It appears to, meaning that in some sense his definition of "good set of classifiers" (also, in his terms, a good model structure) captures the essence, in machine learning terms, of what XCS is doing.

In the process of designing the model, the author describes its components and their training in clear detail and links it to currently used LCS, giving rise to recommendations for how those LCS can directly gain from the design of the model and its probabilistic formulation. The seeming complexity of evaluating the quality of a set of classifiers is alleviated by giving an algorithmic description of how to do it, which is carried out via a simple Pittsburgh-style LCS. A final chapter on sequential decision tasks round out the model-centered formulation that has until then focused on function approximation and classification, by providing criteria for method stability and insights into new developments.

The link provided between LCS on a theoretical level and machine learning work in general is important, especially since the latter has a more developed theory and may in part stand off from LCS because of LCS's relative lack thereof (I stress "relative"). Also the problem addressed is important because out of greater theoretical understanding may result better classifier systems, as already demonstrated in this work by the improvements suggested for current LCS.

A particularly appealing feature of Drugowitsch's novel approach is its universal applicability to any kind of LCS that seeks to perform function approximation, classification, or handle sequential decision tasks by means of dynamic

programming or reinforcement learning. Its close relation to XCS in this book results from the authors commitment to an LCS model structure that relates to XCS, but nothing speaks against applying the same approach to greatly different model types, resulting in different, potentially novel, LCS.

While its connection to Pittsburgh-style LCS is straightforward and clearly established in this work, using the same approach for the design of Michigan-style LCS remains a significant future challenge. Also, it will be interesting to see how the author's theoretical basis for reinforcement learning is built upon in future LCS, and how LCS designed by the model-based approach perform in comparison to currently existing LCS.

Overall, the work is elegant and approaches LCS from a refreshingly different perspective. It's also stylistically pretty novel for LCS work - but that's surely healthy!

Concord, MA, USA March, 2008 Stewart W. Wilson

## **Preface**

I entered the world of Learning Classifier Systems (LCS) through their introduction by Will Browne as part of a lecture series on "Advanced Artificial Intelligence" at the University of Reading, UK. Their immediate appeal as as a flexible architecture that combines the power of evolutionary computation with machine learning by splitting larger problems into tractable sub-problems made me decide to pursue them further, for which I got the opportunity during my Ph.D., supervised by Alwyn Barry, at the University of Bath.

Modest dissatisfaction followed my initial euphoria when I had to discover that their theoretical basis that I planned to rest my work upon did not live up to my initial expectation. Indeed, despite being generally referred to as Genetic-based Machine Learning, their formal development had little in common with machine learning itself. Their loose definition, ad-hoc design, complex structure of interwoven sub-components, and yet surprisingly competitive performance made me comprehend why David Goldberg referred to them as "a glorious, wondrous, and inventing quagmire, but a quagmire nonetheless."

The work presented in this book is an attempt to "clean up" on LCS and lay the foundations for a principled approach to their design by pragmatically following the road of machine learning, in order to bridge the gap between LCS and machine learning. Their design is approached from first principles, based on the question "What is a classifier system supposed to learn?". As presented here, the work is intended for researchers in LCS, genetic-based machine learning, and machine learning, but also for anyone else who is interested in LCS. The content is in most parts based on work performed during my Ph.D., but also includes extensions to it, most notably a complete formulation for classification tasks rather than only regression tasks. The content of this book is not to be understood as the development of a new LCS, but rather as the groundwork for a new approach to their design that I and hopefully others will build upon.

Numerous people have supported me in performing this work, and I am grateful for their constant encouragement. Most notably, I would not have been able to fully focus on my work without the generous financial support of my parents, Elsbeth and Knut Drugowitsch, during my Ph.D. time. Also, my Ph.D.

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supervisor, Alwyn Barry, helped me to stay focused on the main questions, and his guidance, his constructive comments, and his initiative were essential to the completion of this work. Many people in an around Bath, UK, have helped me with comments, discussions, or equally valuable moral support: Dan Richardson, Marelee Hurn, Hagen Lehmann, Tristan Caulfield, Mark Price, Jonty Needham, Joanna Bryson, and especially Will Lowe for emphasising the model behind each method. Various researchers in LCS and machine learning have offered their support thought constructive discussions at conferences or per e-mail: Pier Luca Lanzi, Daniele Loiacono, Martin Butz, Stewart Wilson, Will Browne, Tim Kovacs, Gavin Brown, James Marshall, Lashon Booker, Xavier Llorà, Gavin Brown, Christopher Bishop, Markus Svensén, Matthew Beal, Tommi Jaakkola, Lei Xu, Peter Grünwald, Arta Doci, and Michael Littman. Special thanks go to Larry Bull for not giving me a too hard time at my Ph.D. viva, and for encouraging me to publish my work as a book, therefore taking full responsibility for it. Last, but certainly not least, I am deeply grateful for the moral support and patience of Odali Sanhueza throughout the years that I was working on what resulted in this book.

Rochester, NY, USA March, 2008 Jan Drugowitsch

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