

Friends Forever: Social Relationships with a Fuzzy Agent-Based Model

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Abstract. Sociological research shows that friendship and partner choice tend to reveal a bias toward social similarity. These relations are ruled by the so called “proximity principle” which states that the more similar two individuals are, the more likely they will become friends. However, proximity, similarity or friendship are concepts with blurred edges and grades of membership (acquaintances, friends, couples). Therefore, in order to model the friendship dynamics we work on an Agent-Based Model that already manages the social relationships, together with demographics and evolutionary crossover. To introduce these theoretical concepts we decided to fuzzify the system, explaining the process in detail. Thus, we end up with fuzzy sets and operations, a fuzzy friendship relationship, and a logistic function for its evolution.

Keywords: agent-based modelling, friendship, fuzzy agent, fuzzy logic, social simulation.

1 Introduction

The dynamics of social relationships is a highly complex field to study. Even though it can be found many literature regarding friendship networks, weak links / acquaintances, relationship evolution and so on, we are still far from understanding all the processes involved. Social research has shown that people use numerous criteria when they consider the possibility of turning an acquaintance into a friend. But this paper considers only one: socio-demographical characteristics of people (i.e. ideology, age) that determine the emergence and evolution of friendship. After studying the theory available, we have decided to use the “proximity principle” in order to model the friendship dynamics. This principle assesses that the more similar two individuals are, the stronger their chances of becoming friends. Thus, we attempt to model the processes in which strangers turn to be acquaintances, those turn into friends, and some friends into couples.

In order to do that, we will work with an Agent-Based Model (ABM) which already handles social relationships: Mentat [1], which is deeply described. However, the application of the friendship modelling in the ABM has been accomplished using

fuzzy logic. Therefore, we expose how the theory has been guiding the fuzzification process step by step, resulting in a new ABM called Fezztat. The whole process consisted in the fuzzification of the agent characteristics, the similarity process, the fuzzification of the friendship relationship together with the introduction of an evolution function, and a new couples matchmaking calculation. Comparing the results of the different ABM we might assess that the fuzzy version deals with the problem in a more accurate way.

The section 2 explains some theoretical concepts of friendship dynamics. Section 3 resumes the needed basics of fuzzy logic, while the next one describes the Mentat ABM. Section 5 analyzes the fuzzification process, and the last two parts finish with a discussion about the results obtained and conclusions.

2 Friendship Dynamics

2.1 Understanding Friendship

Selecting a friend is among the most personal of human choices, and thus it is not surprising that friendship groups tend toward social homogeneity. Members of the working class usually associate with other workers, and middle-class individuals generally choose friends who are middle class.

A preliminary step to constructing a friendship modelling is an examination of the way that the social context structures friendship choice. Contextual explanations for individual behaviour argue that (i) individual preferences and actions are influenced through social interaction, and (ii) social interaction is structured by the individual's social characteristics [2]. This is consistent with the important homophily principle in social networks of [3]. Principles of meeting and “mating” by which strangers are converted to acquaintances, acquaintances to friends, and even maybe friends into partner, follow the same rules. Meeting depends on opportunities alone (that is, to be in the same place at the same time); instead, mating depends on both opportunities and attraction. How readily an acquaintance is converted to close friendship depends on how attractive two people find each other and how easily they can get together.

The “proximity principle” indicates that the more similar people are, the more likely they will meet and become friends [4]. Therefore, features like social status, attitudes, beliefs and demographic characteristics (that is, degree of “mutual similarity”) channel individual preferences and they tend to show more bias toward homogeneous friendship choices.

2.2 A Friendship Evolution Function

Similarity, proximity or friendship are vague or blurry categories, because they do not have clear edges. For this reason, we have developed a formal model of friendship dyads using the general framework presented above, but considering similarity and friendship as continuous variables. Besides, because friendship occurs through time, we have considered our model in dynamic terms.

We conceive the friendship process as a search for compatible associates, in terms of the proximity principle, and where strangers are transformed to acquaintances and acquaintances to friends as a continuous process over time.

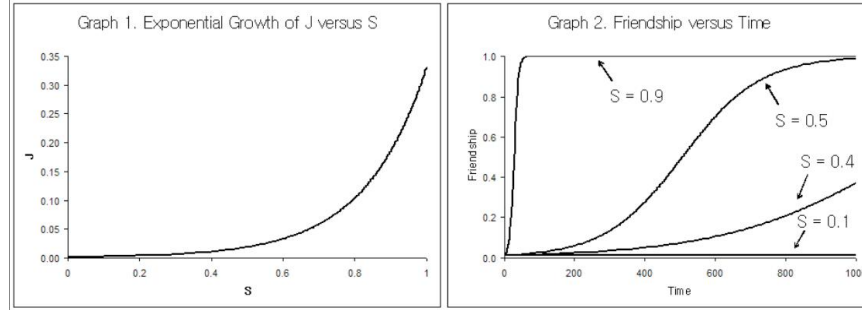


Fig. 1. Graphs showing exponential growth of J and the logistic friendship function

We propose the hypothesis that a logistic function [5] can describe formally the “friendship relation” or degree of friendship for every couple of individuals. The logistic function is one of the most useful (and heavily exploited) modelling strategies used in the social sciences. In order to model the evolution of friendship, we have specified it as in Equation 1:

$$W(t) = \frac{dF}{dt} = F(t) \times K(t) \times r \quad (1)$$

The equation expresses the hypothesis that friendship increases over time; thus, at each point of time, $F(t)$ defines the minimum degree of friendship that is given as an initial condition ($0 < F(t) < K$); K is the maximum degree of friendship that agents can reach (K can be understood as the level of “close friends”), and finally r value defines the growth rate of friendship. However, this equation does not include the “proximity principle” described above. We can include this principle in equation (1) by modifying the growth rate r and stating it as follows: the more similar in social characteristics two individuals are, the higher the growth rate of their friendship is (we need to make r sensitive to the similarity value). Thus, we can express the following equation:

$$r = S \times J \quad (2)$$

Where S is a measure of similarity and J defines a multiplicative factor that increases the magnitude of S within r . The objective of J is turning r more sensitive to S values, and specially sensitive to high S values. For this reason, J describes an exponential growth depending on S values. We can formalize J as follows:

$$J(s) = J_0 \times e^{ps} \quad (3)$$

Where J_0 is the initial value of J , P defines the constant of proportionality and S is the similarity value between the individuals. In the following graphs we can see how the friendship will develop over time given different initial conditions¹.

¹ In the Graph 1 of Figure 1 it is assumed that P is equal to 5.8 and J_0 is equal to 0.001. In Graph 2 it is assumed that K is equal to 1 and F_0 is equal to 0.01; r value is equal to $S \times J(s)$. The constants were generated by experimental procedures.

3 Be Fuzzy, My Friend

3.1 Why, What and When Fuzzy Logic

Individuals are often vague about their beliefs, desires and intentions. They use linguistic categories with blurred edges and gradations of membership, for instance: “acquainted or friend”. Fuzzy logic is oriented at modelling the imprecise modes of reasoning in environment of uncertainty and vagueness [6]. Thus, because vagueness is such a common thing in the social realm, fuzzy logic provides us with a useful way to handle this vagueness systematically and constructively [7].

Fuzzy logic shows that if A is a fuzzy set in a universe of discourse U , then every member of U has a grade of membership in A between 0 and 1 (instead of the classical two-valued-logic). This membership function defines A as a fuzzy subset of U [8]. According to this framework, the mapping of the function is denoted by $m_a: A \rightarrow [0,1]$.

3.2 The Importance of Fuzzy Logic in ABM

There is an increasing interest among social scientists for adding fuzzy logic to the social science toolbox [9]. Likewise, even though it is still incipient, there are numerous examples of researches linking fuzzy logic with social simulation. For instance, in some ABM, agents decide according to fuzzy logic rules; “fuzzy controls” or “fuzzy agents” are expert systems based in “If \rightarrow Then” rules where the premises and conclusions are unclear. Unlike traditional multi-agent models, where these completely determined agents are an over-simplification of real individuals, fuzzy agents take into account the stochastic component of the human behaviour.

Some authors have proposed to improve the agents' strategy choices within the iterated prisoner's dilemma using fuzzy logic decision-rules [10]. Others researchers have claimed that simulation based on two-player games can use fuzzy strategies when analytic solutions do not exist or are computationally very difficult to obtain (because agents use fuzzy strategies; i.e., “If I think my opponent will choose action x , I will choose action y ”) [11]. Examples of multi-agent based models are the fate of spatial dilemmas [12], an extension of sugar space model with fuzzy agents [13] and computational modelling of “fuzzy love and romance” [14].

4 The Case Study: Mentat, the Original ABM

Therefore, to study the friendship dynamics commented, an ABM has been chosen for its fuzzification. The aim of the Mentat model [15, 1] is to understand the evolution of the moral values in Spain from 1980 to 2000. It carries out an analysis of the evolution of multiple factors in the period, trying to determine to which extent the demographic dynamics explains the magnitude of mentality change in Spain. Due to its broad spectrum, it needs to cope with: gender, age, education, economy, political ideology, religiosity, family, friend relationships, matchmaking and reproduction patterns, demographic dynamics, life cycles and others. The aggregated statistics of these variables evolve over time, together with the agent network.

The agents in Mentat are initialised using data from the Spanish census, research studies and sample surveys [16]. Thus, each agent has different values of their attributes, with a behaviour deeply influenced by distributions representing demographic rules (life expectancy, fertility rate, etc).

The simulation has been configured with a population of 3000 static agents, randomly distributed in a space 100x100 (thus, around one agent each 3.3 cells), and simulated for a period of 20 years (1000 agent steps). The agents are able to communicate, establishing friendship and couple relationships, and reproduce. The communication is always local, with a Moore neighbourhood of distance 6 (168 cells). This means that an agent will be able to communicate with around 50 other “possible friends” along its life. From these, each agent will be able to choose, which ones will be its friends. This election will be determined by a probability directly proportional to a similarity measure between the two agents.

It should be mentioned that agents are randomly distributed in the space because the context of an individual can be initially considered random: in which neighbourhood you grew, in which university did you study. Only with the common context already chosen, the individuals can relate with each other taking into account non-random factors: once the student is in the university, they will decide whom will be their friends.

An agent will choose a couple among their friends, if certain conditions are given: reproduction probability taken into account the age and the demographic distributions; election of the “candidates” based on boolean conditions as “not child”, “not married”, “same sex”, etc; candidate choice determined by the similarity rate. These rules are consistent with the friendship dynamics explained in the section 2. Again according to the demography implemented, the couples will give birth a certain number of children that will inherit their characteristics and values.

Thus, the agents form a network where the nodes are the individuals and the links can be of type “friend” or “family” (couple, parents, children). The more friendships exist, the more couples and families will be formed. However, it is not only the quantity which is important: the matchmaking process should return similar couples, minimizing the exceptional cases where two very different people are married.

5 Fuzzification of Mentat

5.1 The Baby's First Steps: Attributes and Similarity

Applying the concepts of section 3 into Mentat, it has been modified, step by step, fuzzifying a collection of aspects: agent characteristics, similarity measure, friendship relationship, and the matchmaking process. Besides, we will introduce the friendship evolution function previously mentioned in section 2. The result ABM has been called Fezztat.

Mentat uses a similarity function for several purposes, as it has already been explained. It is built with a gratification method based on the comparison of the agent attributes. However, the technique is not very sophisticated and could be improved. The use of fuzzy logic would significantly increase its accuracy. But if we want to use

fuzzy operators, first we have to fuzzify the variables where they are applied or, formally, define fuzzy sets over these variables.

Thus, the agent attributes, very different from each other, were normalized in the real interval $[0, 1]$. For example, we would have the fuzzy set $\mu_{\text{economy}}: U \rightarrow [0, 1]$, and an individual with a $\mu_{\text{economy}}(\text{ind}) = 0.7$ would be a person quite wealthy.

Afterwards, the fuzzy similarity can be defined using a T-indistinguishability, which generalizes the classical equivalence relations. The mathematical explanation beneath it can be found in [17], but roughly the distance between the attributes of the two agents compared is “how far are they”, so its negation will point out “how similar are they”. The aggregation of each couple of attribute similarities will return the total similarity rate. The negation used is a fuzzy strong negation N [18] and the aggregation an OWA operator [19]. And so, the relation is:

$$R_{\text{similarity}}(\text{ind}, \text{ind}_2) = \text{OWA}(\forall \mu_i \in \text{ind}, N(d(\mu_i(\text{ind}), \mu_i(\text{ind}_2)))) \quad (4)$$

An OWA is a family of multicriteria combination (aggregation) procedures. By specifying suitable order weights (which sum will result always 1) it is possible to change the form of aggregation. The arithmetic average in the example OWA would need a value of $1/n$ to each weight (where “ n ” would be the number of attributes). Through these weights, it is possible to control the importance of each attribute in the global similarity.

5.2 Growing-Up: Friendship and Couples

Although the agents comparisons have been improved with the previous method, the potential of this new subtle similarity function would not be used if left like that. Supporting what it was pointed out in section 2, we will link the concepts of similarity and friendship in a fuzzy way. For doing so, the friendship is turned into a fuzzy relationship, completely different from its boolean nature in Mentat. This relationship is naturally “fuzzy”, and the previous use (to be or not to be a friend) was an oversimplification not found in reality: in real world, there is a continuous range of degrees of friendship. With the new $R_{\text{friend}}: U \times U \rightarrow [0, 1]$, each agent has a range from close friends to acquaintances.

Now that it is formally defined, it is needed to specify an evolution for it. Therefore, the friendship logistic function of (1) is used here. Every “step” of time, each agent will update its friendship with its linked agents. Depending on how similar they are, and how old is their friendship, they will end up being very close friends or just stay as acquaintances. Formally (where W is the already defined logistic function (1) and t_{friend} is the time of friendship):

$$R_{\text{friend}}(\text{ind}, \text{ind}_2) = W(t_{\text{friend}}(\text{ind}, \text{ind}_2), R_{\text{similarity}}(\text{ind}, \text{ind}_2)) \quad (5)$$

The final improvement through fuzzy logic is related to couples and the matchmaking process. The “has couple” relation is clearly boolean and cannot be fuzzified. But the process of choosing the couple, as it was described in the section 4, can take into account new information obtained with the modifications already made. Mentat has several boolean conditions for filtering the “candidates”, and these can not be changed (“same sex” or “not married” are impossible to fuzzify). The similarity function that

uses has already been improved. And now we propose to introduce the friendship degree in the selection.

Therefore, it has been defined a new relationship “compatible” that will measure the possibilities of a candidate to be selected as a couple:

$$R_{compatible}(ind, ind_2) = OWA(R_{friend}(ind, ind_2), R_{similarity}(ind, ind_2)) \quad (6)$$

The weights of the OWA can be redefined depending on the importance given to each term. After some experimentation, Fezztat uses equal weights. It could be argued that, as the friendship evolution already takes into account the similarity, this one would not be necessary, as its information is already included. However, the friendship relationship can achieve its maximum with several candidates of the same agent, and it is specially in those cases where the similarity is very useful.

6 Results and Discussion

Here we present a comparison between several versions of the ABM. Four different implementations, each one with two configurations have been analyzed, focusing in three measures. The fuzzy modifications have been grouped in two main ones: “*Fuzz-Sim*”, when the attributes are normalized and the similarity operator fuzzified, as stands the subsection 5.1; and “*Fuzz-Fri*”, when the friendship turns to be fuzzy, evolving over time and affecting the partner choice (subsection 5.2). The four ABM represent all the possible combinations between these, represented in the table 1 in the pair (*Fuzz-Sim*, *Fuzz-Fri*). Thus, we have the classic version of *Mentat* explained in section 4, with no fuzzy properties; the *Mentat_{FuzzSim}*, simply the same ABM but with the “*Fuzz-Sim*”; the *Mentat_{FuzzFri}* with “*Fuzz-Fri*” but not “*Fuzz-Sim*”; and last *Fezztat*, with all the fuzzy modifications.

The two configurations deal with two possible ways of friendship emerging: one promoting random friends (and therefore an agent can be linked to a non-similar neighbor) and the other promoting similarity-based friends (and therefore an agent will rarely be linked to a non-similar neighbor, as it will give priority to the most similar ones). This is not a trivial decision, because the friendship evolution function already deals with similarity, and if a neighbor is not similar at all, it will never be more than an acquaintance. It is not evident if the closer way to real-world is giving double strength to similarity (in the second option) or letting randomness to decide who will be the friend (and thus maybe ignoring similar people). It has to be mentioned that none of the two configurations is so deterministic and both are based on probabilities.

The parameters analyze the couples and how they are affected by the changes in the configuration and fuzzification. The $R_{similarity}$ shows the proximity taking into account all the characteristics of each partner in a couple. The R_{friend} focuses in the friendship link between them, which in a way (according to the logistic function) depends in their similarity too, but also in the time spent together. The $R_{compatibility}$ is taken as an average of the other two. The values have been obtained after averaging the output of several executions of each version. And in every execution, it is the mean of the property in every couple.

As the first two ABM has a boolean friendship, their compatibility is always the same as the similarity. In the first configuration, when the friendship is rarely involved in the neighbors linked, the similarity rates are very similar in all the versions. However, in the second one it's clear that the ones with fuzzy similarity slightly increase their success. But the bigger changes can be observed in the friendship: *Fezztat* beats the other versions with a greater R_{Friend} and $R_{\text{Compatibility}}$, specially in the second configuration. These results approach the theoretical qualitative assessments.

Finally, we compared the four different orders of fuzzification described above for both $R_{\text{Similarity}}$ and R_{Friend} of couples, by using the statistical test "One-Way Analysis of Variance", in order to detect evidence of difference among the population means. The Fisher's statistical significance test equals to 7.281, with a P -value $P < .0001$. This small P -value provides strong evidence against null hypothesis, namely, that the difference in the means among the four orders of fuzzification are by chance, both for $R_{\text{Similarity}}$ and R_{Friend} of couples. Therefore, the differences among the means analysed can be attributed to model's fuzzification.

Table 1. Comparison among the different ABM, in increasing order of fuzzification

	<i>Mentat</i> (0,0)	<i>Mentat</i> _{FuzzSim} (1,0)	<i>Mentat</i> _{FuzzFri} (0,1)	<i>Fezztat</i> (1,1)
Config. Random-friendship				
Mean $R_{\text{Similarity}}$ of couples	0.76*	0.77	0.76*	0.77
Mean R_{Friend} of couples	(**)	(**)	0.72*	0.80
Mean $R_{\text{Compatibility}}$ of couples	0.76*	0.77	0.54*	0.62
Config. Similar-friendship				
Mean $R_{\text{Similarity}}$ of couples	0.73*	0.77	0.73*	0.78
Mean R_{Friend} of couples	(**)	(**)	0.54*	0.76
Mean $R_{\text{Compatibility}}$ of couples	0.73*	0.77	0.39*	0.59

*: The original *Mentat*'s similarity has other range, but here they have been normalized in the interval [0,1] in order to be compared.

**: When the friendship is not fuzzified, all the couples are friends (as this is a boolean property).

7 Concluding Remarks

In this paper we have explained some concepts of social relationship dynamics, including an evolution function that was applied for the changing of friendship over time. After justifying the suitability of fuzzy logic in this context, we proceeded to apply it in an existing ABM. Therefore, we defined fuzzy sets over each agent attribute, and a new fuzzy similarity operator that would influence friendship emergence and partner choice. The friendship relationship nature and importance in the model was significantly modified, fuzzifying it, making it evolve using the defined logistic function, and letting it influence in the partner choice as much as the similarity rate. The results of these changes are clearly positive, as long as they improve the proximity to the qualitative assessments of the theory.

To sum up, we have exposed a theory, formalized it, searched where it can be applied, found the useful tools to do that, implemented the application and extracted a collection of results that are used to validate the model against the theory. This validated formalization of the theory could be useful for further study in the field.

Future research lines that could be followed could take into account other interesting friendship theories. There are deep studies in homophily in social networks [3] that could be implemented. An aspect that our model ignores but it is important enough to be considered is the stability of friendship [20]. Besides, Fezztat could be extended to analyze the importance of weak links along one's life, a new possibility that Mentat did not allow.

Acknowledgments. We acknowledge support from the project *Methods and tools for modelling multi-agent systems*, supported by Spanish Council for Science and Technology, with grant TIN2005-08501-C03-01, and the Ford Foundation International Fellowships Program together with the Chilean National Scholarship Program for Graduate Studies.

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