A Pseudo-logarithmic Image Processing Framework for Edge Detection*

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Abstract. The paper presents a new [pseudo-] Logarithmic Model for Image Processing (LIP), which allows the computation of gray-level addition, substraction and multiplication with scalars within a fixed gray-level range $[0; D]$ without the use of clipping. The implementation of Laplacian edge detection techniques under the proposed model yields superior performance in biomedical applications as compared with the classical operations (performed either as real axis operations, either as classical LIP models).

1 Introduction

Traditionally, edge identification is performed by derivative measures [1] (gradient or Laplacian) and thus implemented by linear filtering structures. The derivative is computed along a set of fixed orientation (as in the case of the compass operators) or along the orientation of the strongest intensity variation. Basically, all edge detection operators rely on the computation of an edge intensity map (that provides important values for the pixels that are on the boundaries of uniform regions) which is further thresholded in order to obtain a binary edge map.

The underlying physical properties of the imaging system used in biomedical applications are, most of the time, naturally multiplicative. (For instance, in the case of an X-ray image, the image values represent the transparency/ opacity of the real objects imaged by any given pixel.) The key to the logarithmic image processing (LIP) approaches is a homomorphism which transforms the product into a sum (by logarithm), allowing the use of the classical linear filtering in the presence of additive components. Also, it should be clear that the functions we use are bounded (say, they take values in a bounded interval $[0, D)$). During the image processing, the following problem may appear: the mathematical operations on real valued functions use implicitly the algebra of the real numbers i.e. on the whole real axis and we are faced with results that do not belong anymore to the interval $[0, D]$ – the only ones with physical meaning. Such an approach was discussed for instance in [2] for Xray enhancement or in [3] for the creation of high dynamic range images by bracketing.

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The remainder of the paper is organized as follows: first, section 2 briefly presents the existing logarithmic image processing models; section 3 presents the proposed pseudo-logarithmic image processing model that preserves, for all operations, the bounded gray-level value range. Finally, sections 4 and 5 present some experimental results and conclusions.

2 The Logarithmic Model of Image Processing

In this section we briefly present the key points of the only two existing logarithmic image processing models: the classical model introduced by Jourlin and Pinoli [4], [5], and the model introduced by Pătraşcu [6].

2.1 The Classical LIP Model

In the classical LIP model [4], [5], the intensity of an image is completely modelled by its gray tone function v, with $v \in [0, D)$. In the LIP model, the addition of two gray tone functions v_1 and v_2 and multiplication of v by a real number λ are defined in terms of usual $\mathbb{R}$ operations as:

$$v_1 \oplus v_2 = v_1 + v_2 - \frac{v_1 v_2}{D} \tag{1}$$

and respectively:

$$\lambda \otimes v = D - D\left(1 - \frac{v}{D}\right)^{\lambda} \quad . \tag{2}$$

The difference between two gray-levels v_1 and v_2, with $v_2 \geq v_1$ is given by:

$$v_2 \ominus v_1 = \frac{v_2 - v_1}{1 - v_1}. \tag{3}$$

All the right-hand operations in the equations above are the usual real scalar operations.

The vector space of gray-levels $(\mathbf{E}, \oplus, \otimes)$ is isomorphic to the space of real numbers $(\mathbf{R}, +, \cdot)$ by the function $T : \mathbf{E} \rightarrow \mathbf{R}$, defined as:

$$T(v) = -\ln(1 - v) \tag{4}$$

The particular nature of this isomorphism induces the logarithmic character of the mathematical model.

2.2 The Homomorphic LIP Model

The logarithmic model introduced in [6] does work with bounded real sets: the gray–tone values of the involved images, defined in $[0, D)$, is linearly applied onto the standard set $(-1, 1)$:

$$v = \frac{2}{D}\left(u - \frac{D}{2}\right) \tag{5}$$

where $u \in [0, D)$ and $v \in (-1, 1)$.

The $(-1, 1)$ interval plays the central role in the model: it is endowed with the structure of a linear (moreover: Euclidean) space over the scalar field of real numbers, $\mathbb{R}$. In this space, the addition between two gray-levels, z_1 and z_2 is defined as:

$$v_1 \oplus v_2 = \frac{v_1 + v_2}{1 + v_1 v_2} \tag{6}$$

The multiplication of a gray level, v with a real scalar, $\lambda \in \mathbb{R}$ is:

$$\lambda \otimes v = \frac{(1+v)^\lambda - (1-v)^\lambda}{(1+v)^\lambda + (1-v)^\lambda} \quad . \tag{7}$$

The difference between two gray-levels v_1 and v_2, with $v_2 \geq v_1$ is given by:

$$v_2 \ominus v_1 = \frac{v_2 - v_1}{1 - v_1 v_2}. \tag{8}$$

All the right-hand operations in the equations above are the usual real scalar operations.

The vector space of gray-levels $(\mathbf{E}, \oplus, \otimes)$ is isomorphic to the space of real numbers $(\mathbf{R}, +, \cdot)$ by the function $T : \mathbf{E} \to \mathbf{R}$, defined as:

$$T(v) = \frac{1}{2} \ln \left(\frac{1+v}{1-v} \right) \tag{9}$$

The particular nature of this isomorphism induces the logarithmic character of the mathematical model.

3 The Proposed Pseudo-logarithmic Image Processing Framework

We shall define the addition of two gray-levels $v_1, v_2 \in [0; 1]$ by:

$$v_1 \oplus v_2 = \frac{v_1 + v_2 - 2v_1 v_2}{1 - v_1 v_2}, \tag{10}$$

while the multiplication of a gray level, v with a real scalar, $\lambda \in \mathbb{R}$ is:

$$\lambda \otimes v = \frac{\lambda v}{1 + (\lambda - 1)v} \quad . \tag{11}$$

It can be shown that the interval $[0; 1)$ is isomorphic with $\mathbb{R}^+$ via the transform T defined as:

$$T(v) = \frac{v}{1-v} \tag{12}$$

The difference between two gray-levels v_1 and v_2, with $v_2 \geq v_1$ is given by:

$$v_2 \ominus v_1 = \frac{v_2 - v_1}{1 + v_1 v_2 - 2v_1}. \tag{13}$$

This is obtained by:

$$v_1 \ominus v_2 = T^{-1} \left(T(v_2) - T(v_1) \right). \tag{14}$$

All the right-hand operations in the equations above are the usual real scalar operations.

As opposed to the logarithmic nature of the transforms from (4) and (9), the isomorphic transform of the proposed model resembles a logarithmic function (as illustrated in figure 1), which justifies its given pseudo-logarithmic name.

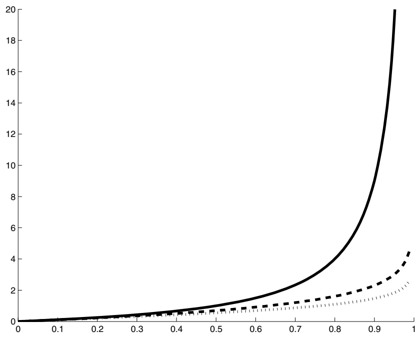


Fig. 1. Plot of the isomorphic transforms of the discussed LIP models: Jourlin and Pinoli's classical model (4)(dashed line), Pătraşcu's model (9)(dotted line) and proposed model (12) (continuous line)

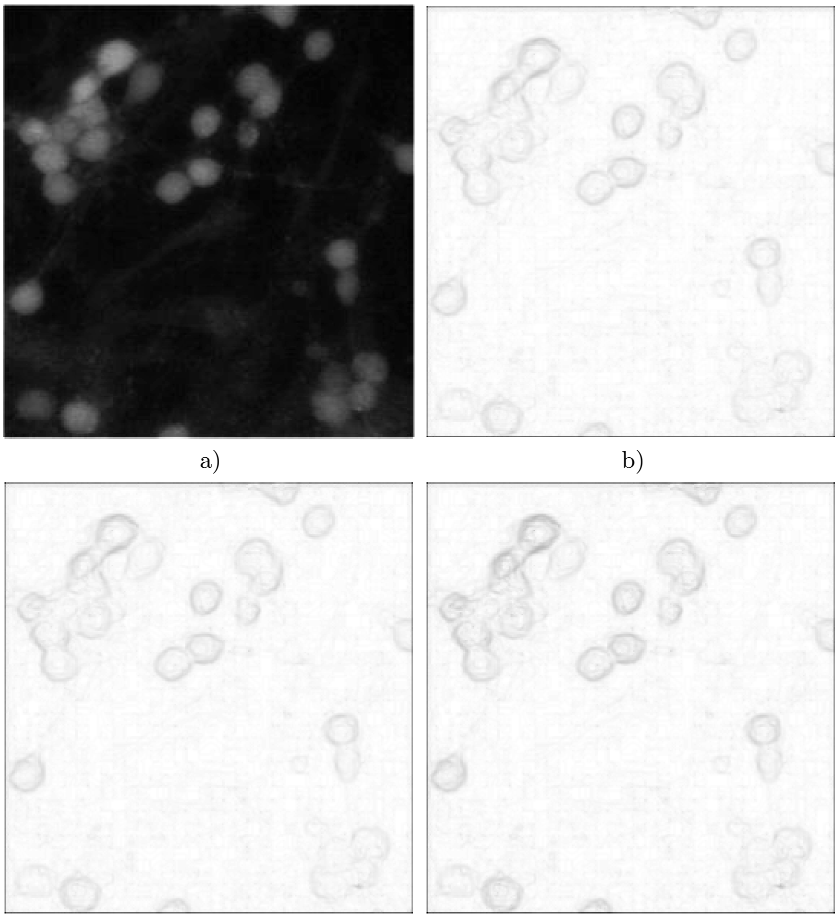
4 Implementation and Tests

We shall use the luminance-invariant inter-color distance introduced in (2) for the implementation of a color Laplacian operator L. The proposed operator is a modification of the classical (derivative-type) V4-neighborhood Laplacian operator; the proposed Laplacian is the average difference within the value (gray-scale or color) at the current processed location (i, j) and its immediate neighboring values from the image f. Mathematically we can express the proposed Laplacian at location (i, j) as:

$$\varDelta(i, j = \sum_{(k,l) \in V_4} \left(f(i+k, j+l) \ominus f(i,j) \right)/4. \tag{15}$$

Such a Laplacian operator was tested on images taken from various biomedical applications (microscopy images under visible or fluorescent lighting, Xrays, etc...) under the four implementation scenarios for the $\ominus$ operation in (15): standard $\mathbb{R}$ substraction or logarithmic substraction (as defined in (3), (8), (13)). Some results (for both gray-level and color images) are presented in figures 2 - 4 below.

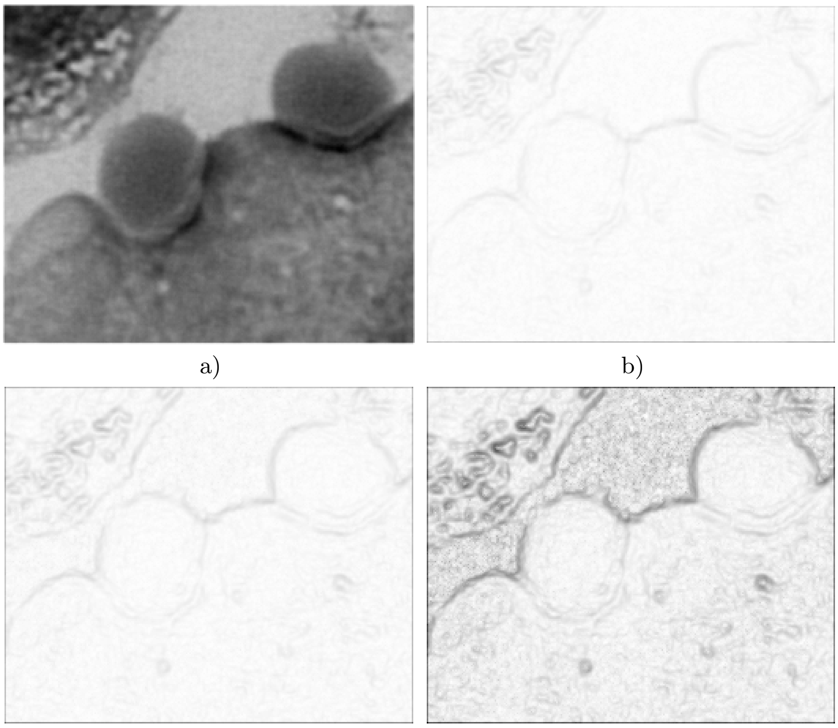
It is well-known that linear derivative filtering (such as the Laplacian) amplifies the noise present within the image. In the case of the classical derivative



a) b) c) d)

Fig. 2. a) Original gray-level image detail from a biomedical application (neuron nuclei under fluorescent light); b) classical Laplacian of the original image; c) P'atra'scu LIP type Laplacian; d) proposed LIP type Laplacian. The transition intensity images from figures b) - d) are inversely mapped to gray-levels (dark corresponds to more intense contours).

filtering, there is a linear dependency between the noise power in the original image and the noise in the filtered image. In the case of derivative operators based on the logarithmic image processing operations, the power of the noise is significantly reduced in the output image. A simple experimental setup consists in measuring the noise parameters (standard deviation) for an uniform image corrupted by Gaussian, zero-mean, additive noise before and after Laplacian filtering. The plots in figure 5 show the good behavior of the proposed logarithmic model based edge detector.



a) b) c) d)

Fig. 3. a) Original gray-level image detail from a biomedical application (blood cells in color visible microscopy); b) classical Laplacian of the original image; c) P'atra'scu LIP type Laplacian; d) proposed LIP type Laplacian. The transition intensity images from figures b) - d) are inversely mapped to gray-levels (dark corresponds to more intense contours).

It should be noted that the presented implementation can be extended to any derivative-type edge detection technique (such as gradient, compass operators, etc.)

5 Conclusions

We presented a new [pseudo-] Logarithmic Model for Image Processing (LIP), which allows the computation of gray-level addition, substraction and multiplication with scalars within a fixed gray-level range $[0; D]$ without the use of clipping. The implementation of classical edge detection techniques under the proposed model yields significant superior performance as compared with the classical operations (performed either as real axis operations, either as classical LIP models). The tests performed on various images from biomedical applications show the good performance of the proposed approach.

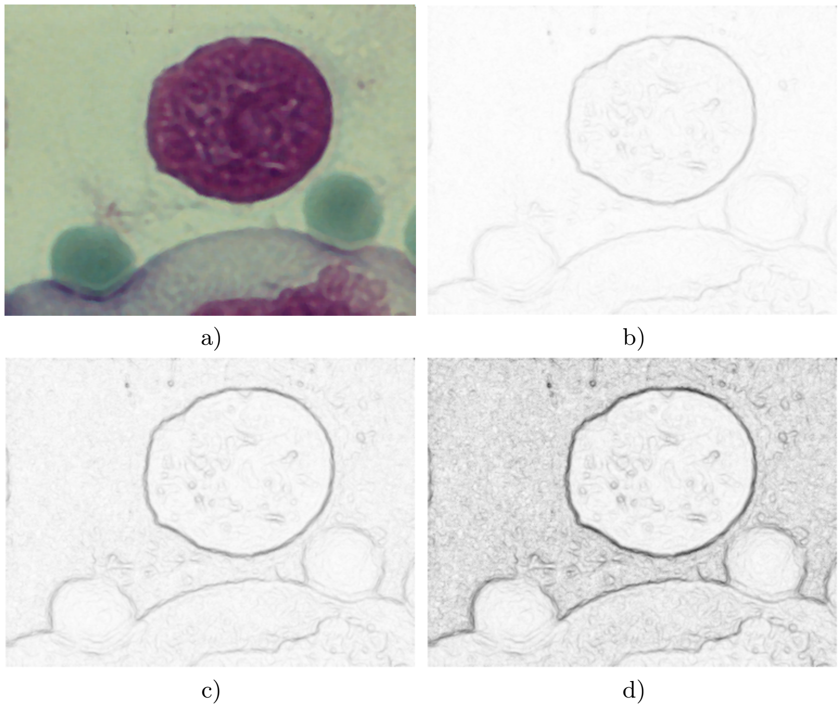


Fig. 4. a) Original image detail from a biomedical application (blood cells in color visible microscopy); b) classical Laplacian of the original image; c) classical LIP type Laplacian; d) proposed LIP type Laplacian. The transition intensity images from figures b) - d) are inversely mapped to gray-levels (dark corresponds to more intense contours).

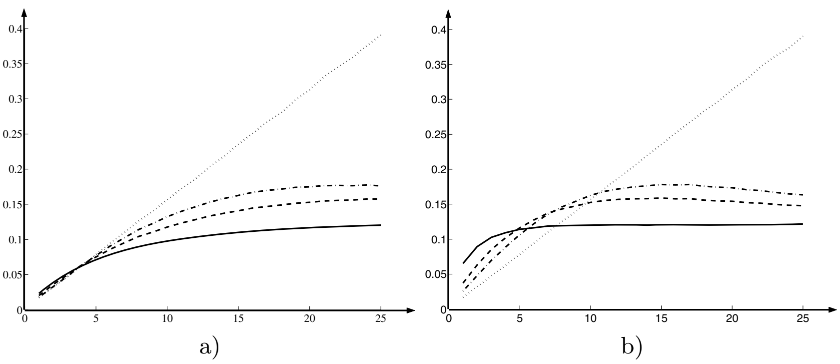


Fig. 5. Output noise dispersion vs. input noise dispersion for the classical Laplacian (upper dotted line), Laplacian under the P'atra'scu logarithmic model (dash-dotted line), Laplacian under the classical Jourlin-Pinoli logarithmic model (dashed line) and Laplacian under the proposed logarithmic-like model (lower continuous line) under two constant gray levels of a) 50 and b)150

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