

Applied Wave Mathematics

Ewald Quak • Tarmo Soomere
Editors

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Selected Topics in Solids, Fluids,
and Mathematical Methods



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*Dedicated to Jüri Engelbrecht,
the Founder and Leader of CENS,
on the Occasion of his 70th Birthday*

Preface

This edited volume consists of twelve contributions related to the EU Marie Curie Transfer of Knowledge Project *Cooperation of Estonian and Norwegian Scientific Centres within Mathematics and its Applications*, CENS-CMA (2005-2009), under contract MTKD-CT-2004-013909, which financed exchange visits to and from CENS, the *Centre for Nonlinear Studies* at the Institute of Cybernetics of Tallinn University of Technology in Estonia.

Seven contributions describe research highlights of CENS members, two the work of members of CMA, the *Centre of Mathematics for Applications*, University of Oslo, Norway, as the partner institution of CENS in the Marie Curie project, and three the field of work of foreign research fellows, who visited CENS as part of the project. The structure of the book reflects the distribution of the topics addressed:

Part I Waves in Solids

Part II Mesoscopic Theory

Part III Exploiting the Dissipation Inequality

Part IV Waves in Fluids

Part V Mathematical Methods

The papers are written in a tutorial style, intended for non-specialist researchers and students, where the authors communicate their own experiences in tackling a problem that is currently of interest in the scientific community. The goal was to produce a book, which highlights the importance of applied mathematics and which can be used for educational purposes, such as material for a course or a seminar.

To ensure the scientific quality of the contributions, each paper was carefully reviewed by two international experts. Special thanks go to all authors and referees, without whom making this book would not have been possible. We also thank Heiko Herrmann for his technical and TeXnical help. The friendly and effective collaboration with Springer Verlag through Martin Peters is kindly appreciated.

Tallinn,
April 2009

*Ewald Quak
Tarmo Soomere*

Contents

CENS, CMA and the CENS-CMA Project	1
Jüri Engelbrecht, Ragnar Winther & Ewald Quak	
 Part I Waves in Solids	
 Overview	9
Arkadi Berezovski	
 Deformation Waves in Solids	13
Jüri Engelbrecht	
 The Perturbation Technique for Wave Interaction in Prestressed Material	31
Arvi Ravasoo	
 Waves in Inhomogeneous Solids	55
Arkadi Berezovski, Mihhail Berezovski and Jüri Engelbrecht	
 Part II Mesoscopic Theory	
 Overview	85
Wolfgang Muschik	
 Dynamics of Internal Variables from the Mesoscopic Background for the Example of Liquid Crystals and Ferrofluids	89
Christina Papenfuss	
 Towards a Description of Twist Waves in Mesoscopic Continuum Physics	127
Heiko Herrmann	
 Part III Exploiting the Dissipation Inequality	
 Overview	149
Wolfgang Muschik	

Weakly Nonlocal Non-equilibrium Thermodynamics – Variational Principles and Second Law	153
Péter Ván	
 Part IV Waves in Fluids	
Overview	189
Tarmo Soomere	
Long Ship Waves in Shallow Water Bodies	193
Tarmo Soomere	
Modelling of Ship Waves from High-speed Vessels	229
Tomas Torsvik	
New Trends in the Analytical Theory of Long Sea Wave Runup	265
Ira Didenkulova	
 Part V Mathematical Methods	
Overview	299
Ewald Quak	
The Pseudospectral Method and Discrete Spectral Analysis	301
Andrus Salupere	
Foundations of Finite Element Methods for Wave Equations of Maxwell Type	335
Snorre H. Christiansen	
An Introduction to the Theory of Scalar Conservation Laws with Spatially Discontinuous Flux Functions	395
Nils Henrik Risebro	
Index	465

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