Vector Optimization

Series Editor:

Johannes Jahn
University of Erlangen-Nürnberg
Department of Mathematics
Martensstr. 3
91058 Erlangen
Germany
jahn@am.uni-erlangen.de

Vector Optimization

The series in Vector Optimization contains publications in various fields of optimization with vector-valued objective functions, such as multiobjective optimization, multi criteria decision making, set optimization, vector-valued game theory and border areas to financial mathematics, biosystems, semidefinite programming and multiobjective control theory. Studies of continuous, discrete, combinatorial and stochastic multiobjective models in interesting fields of operations research are also included. The series covers mathematical theory, methods and applications in economics and engineering. These publications being written in English are primarily monographs and multiple author works containing current advances in these fields.

Radu Ioan Boţ · Sorin-Mihai Grad · Gert Wanka

Duality in Vector Optimization

Springer

Dr. Radu Ioan Boţ
Faculty of Mathematics
Chemnitz University of Technology
Reichenhainer Str. 39
09126 Chemnitz
Germany
radu.bot@mathematik.tu-chemnitz.de

Dr. Sorin-Mihai Grad
Faculty of Mathematics
Chemnitz University of Technology
Reichenhainer Str. 39
09126 Chemnitz
Germany
sorin-mihai.grad@mathematik.tu-chemnitz.de

Professor Dr. Gert Wanka
Faculty of Mathematics
Chemnitz University of Technology
Reichenhainer Str. 39
09126 Chemnitz
Germany
gert.wanka@mathematik.tu-chemnitz.de

ISSN 1867-8971 e-ISSN 1867-898X
ISBN 978-3-642-02885-4 e-ISBN 978-3-642-02886-1
DOI 10.1007/978-3-642-02886-1
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2009932674

Cover design: WMXDesign GmbH

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Radu Ioan Boţ dedicates this book to Cassandra and Nina
Sorin-Mihai Grad dedicates this book to Carmen Lucia
Gert Wanka dedicates this book to Johanna

Preface

The continuous and increasing interest concerning vector optimization perceptible in the research community, where contributions dealing with the theory of duality abound lately, constitutes the main motivation that led to writing this book. Decisive was also the research experience of the authors in this field, materialized in a number of works published within the last decade. The need for a book on duality in vector optimization comes from the fact that despite the large amount of papers in journals and proceedings volumes, no book mainly concentrated on this topic was available so far in the scientific landscape. There is a considerable presence of books, not all recent releases, on vector optimization in the literature. We mention here the ones due to Chen, Huang and Yang (cf. [49]), Ehrgott and Gandibleux (cf. [65]), Eichfelder (cf. [66]), Goh and Yang (cf. [77]), Göpfert and Nehse (cf. [80]), Göpfert, Riahi, Tammer and Zălinescu (cf. [81]), Jahn (cf. [104]), Kaliszewski (cf. [108]), Luc (cf. [125]), Miettinen (cf. [130]), Mishra, Wang and Lai (cf. [131, 132]) and Sawaragi, Nakayama and Tanino (cf. [163]), where vector duality is at most tangentially treated. We hope that from our efforts will benefit not only researchers interested in vector optimization, but also graduate and undergraduate students.

The framework we consider is taken as general as possible, namely we work in (locally convex) topological vector spaces, going to the usual finite dimensional setting when this brings additional insights or relevant connections to the existing literature. We tried to add a certain order in the not always correct or rigorous results one can meet in the different segments of the vast literature addressed here. The investigations we perform in the book are always accompanied by the well-developed apparatus of conjugate duality for scalar convex optimization problems. Actually, a whole chapter is dedicated to classical results, but also to new achievements in this field. An additional motivation for this, as well as for displaying a consistent preliminary chapter on convex analysis and vector optimization, was our intention to keep the book as self-contained as possible. Four chapters remained for the vector duality itself, two of them directly extending the conjugate duality from the scalar case, another one focusing on the Wolfe and Mond-Weir duality concepts, while the last one deals with the broader class of set-valued optimization problems.

S.-M. Grad and G. Wanka are grateful to R. I. Boţ for the improvements he brought during the correction process. The authors want to express their sincere thanks to Ernö Robert Csetnek for reading a preliminary version of this book and for providing useful comments and suggestions that enhanced its quality. Thanks are also due to André Heinrich, Ioan Bogdan Hodrea and Catrin Schönyan for typewriting parts of the manuscript. We would like to thank our families for their unconditioned support and patience during writing this book. Without this background the authors would not have found the time and energy to bring this work to an end.

For updates and errata we refer the reader to

`http://www.tu-chemnitz.de/mathematik/approximation/dvo`

Chemnitz, Germany,
April 2009

Radu Ioan Boţ
Sorin-Mihai Grad
Gert Wanka

Contents

List of symbols and notations

Sets and elements

A_i	the i-th row of the matrix $A \in \mathbb{R}^{n \times m}$, $i = 1, \dots, n$
e^i	the i-th unit vector of $\mathbb{R}^n$, $i = 1, \dots, n$
e	the vector $(1, \dots, 1)^T$
Δ_{X^m}	the set $\{(x, \dots, x) \in X^m : x \in X\}$
$\mathrm{Pr}_X(U)$	the projection of the set $U \subseteq X \times Y$ on X
$l(K)$	linearity space of the cone K
$N(U, x)$	normal cone to the set U at x
$T(U, x)$	Bouligand tangent cone to the set U at x
$\mathrm{lin}(U)$	linear hull of the set U
$\mathrm{aff}(U)$	affine hull of the set U
$\mathrm{co}(U)$	convex hull of the set U
$\mathrm{cone}(U)$	conical hull of the set U
$\mathrm{coneco}(U)$	convex conical hull of the set U
$\mathrm{core}(U)$	algebraic interior of the set U
$\mathrm{icr}(U)$	intrinsic core of the set U
$\mathrm{int}(U)$	interior of the set U
$\mathrm{cl}(U)$	closure of the set U
$\overline{\mathrm{co}}(U)$	closed convex hull of the set U
$\mathrm{qri}(U)$	quasi relative interior of the set U
$\mathrm{qi}(U)$	quasi interior of the set U
$\mathrm{sqri}(U)$	strong quasi relative interior of the set U
$\mathrm{ri}(U)$	relative interior of the set U
X^*	topological dual space of X
$w(X, X^*)$	weak topology on X induced by X^*
$w(X^*, X)$	weak* topology on X^* induced by X
K^*	topological dual cone K^* of the cone K
K^{*0}	quasi interior of the dual cone of K
$\widehat{K}$	the cone $\mathrm{core}(K) \cup \{0\}$, where K is a convex cone

Functions and operators

id_X	identity function on X
$\mathcal{L}(X,Y)$	the set of linear continuous mappings from X to Y
$\mathcal{L}_+(X,Y)$	the set of positive mappings from X to Y
$\langle x^*, x\rangle$	the value of $x^* \in X^*$ at $x \in X$
A^*	adjoint mapping of $A \in \mathcal{L}(X,Y)$
$f_1 \Box \ldots \Box f_m$	infimal convolution of the functions f_i, $i = 1, \ldots, m$
δ_U	indicator function of the set U
σ_U	support function of the set U
δ_U^V	vector indicator function of the set U
γ_U	gauge of the set U
$\mathrm{dom}\, f$	(effective) domain of the (vector) function f
$\mathrm{epi}\, f$	epigraph of the function f
$\mathrm{epi}_K\, h$	K-epigraph of the vector function h
(v^*h)	the function $\langle v^*, h\rangle$, where h is a vector function and $v^* \in K^*$
Tf	infimal function of the function f through $T \in \mathcal{L}(X,Y)$
$h(\cdot)$	the scalar infimal value function
$\mathrm{co}\, f$	convex hull of the function f
$\bar{f}$	lower semicontinuous hull of the function f
$\overline{\mathrm{co}} f$	lower semicontinuous convex hull of the function f
f^*	conjugate function of the function f
f_S^*	conjugate function of the function f with respect to the set S
$\partial f(x)$	subdifferential of the function f at $x \in X$
$\nabla f(x)$	gradient of the function f at $x \in X$
F^*	conjugate map of the set-valued map F
$\mathrm{dom}\, F$	domain of the set-valued map F
$\mathrm{gph}\, F$	graph of the set-valued map F
$\mathrm{epi}_K\, F$	K-epigraph of the set-valued map F
$\partial F(x; v)$	subdifferential of the set-valued map F at $(x, v) \in \mathrm{gph}\, F$
$\partial F(x)$	subdifferential of the set-valued map F at $x \in X$
$H(\cdot)$	the set-valued minimal or infimal value map
F_k^*	k-conjugate map of the set-valued map F
$\partial_k F(x; v)$	k-subdifferential of the set-valued map F at $(x, v) \in \mathrm{gph} F$
$\partial_k F(x)$	k-subdifferential of the set-valued map F at $x \in X$

Partial orderings

$\leqq_K$	the partial ordering induced by the convex cone K
$x \leq_K y$	$x \leqq_K y$ and $x \neq y$
$x <_K y$	$y - x \in \mathrm{core}(K)$ (or $y - x \in \mathrm{int}(K)$), where K is a convex cone with $\mathrm{core}(K) \neq \emptyset$ ($\mathrm{int}(K) \neq \emptyset$)

$+\infty_K$	a greatest element with respect to the ordering cone K attached to a space
$-\infty_K$	a smallest element with respect to the ordering cone K attached to a space
$\overline{V}$	the space V to which the elements $\pm\infty_K$ are added
$A(M)$	the set of elements above the set M
$B(M)$	the set of elements below the set M

Minimality notions (with respect to the cone $K \subseteq V$)

$\mathrm{Min}(M, K)$	the set of minimal elements of the set M
$\mathrm{Max}(M, K)$	the set of maximal elements of the set M
$\mathrm{WMin}(M, K)$	the set of weakly minimal elements of the set M
$\mathrm{WMax}(M, K)$	the set of weakly maximal elements of the set M
$\mathrm{PMin}(M, K)$	generic notation for sets of properly minimal elements of the set M
$\mathrm{Min}\, M$	abbreviation for the minimal set of the set $M \subseteq \overline{V}$
$\mathrm{Max}\, M$	abbreviation for the maximal set of the set $M \subseteq \overline{V}$
$\mathrm{WMin}\, M$	abbreviation for the weak minimum of the set $M \subseteq \overline{V}$
$\mathrm{WMax}\, M$	abbreviation for the weak maximum of the set $M \subseteq \overline{V}$
$\mathrm{WInf}\, M$	abbreviation for the weak infimum of the set $M \subseteq \overline{V}$
$\mathrm{WSup}\, M$	abbreviation for the weak supremum of the set $M \subseteq \overline{V}$

Generic notations

$(P^{\cdots})$	primal optimization problem
$(PV^{\cdots})$	primal vector optimization problem
$(PSV^{\cdots})$	primal set-valued optimization problem
$v(P^{\cdots})$	the optimal objective value of the problem $(P^{\cdots})$
$(D^{\cdots})$	dual optimization problem
$(DV^{\cdots})$	dual vector optimization problem
$(DSV^{\cdots})$	dual set-valued optimization problem
$\mathcal{A}^{\cdots}$	feasible set of a primal vector problem
$\mathcal{B}^{\cdots}$	feasible set of a dual vector problem
$h^{\cdots}$	objective function of a dual vector problem
$\Phi^{\cdots}$	(vector) perturbation function or set-valued perturbation map
$(RC^{\cdots})$	regularity condition
$\gamma^{\cdots}$	gap function