

Towards Compatible and Interderivable Semantic Specifications for the Scheme Programming Language, Part I: Denotational Semantics, Natural Semantics, and Abstract Machines

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Abstract

We derive two big-step abstract machines, a natural semantics, and the valuation function of a denotational semantics based on the small-step abstract machine for Core Scheme presented by Clinger at PLDI'98. Starting from a functional implementation of this small-step abstract machine, (1) we fuse its transition function with its driver loop, obtaining the functional implementation of a big-step abstract machine; (2) we adjust this big-step abstract machine so that it is in defunctionalized form, obtaining the functional implementation of a second big-step abstract machine; (3) we refunctionalize this adjusted abstract machine, obtaining the functional implementation of a natural semantics in continuation style; and (4) we closure-unconvert this natural semantics, obtaining a compositional continuation-passing evaluation function which we identify as the functional implementation of a denotational semantics in continuation style. We then compare this valuation function with that of Clinger's original denotational semantics of Scheme.

1. Introduction

Motivation: Somewhat facetiously, in an earlier work [7], Biernacka and the author concluded:

Call/cc was introduced in Scheme [11] as a Church encoding of Reynolds's escape operator [48]. A typed version of it is available in Standard ML of New Jersey [29] and Griffin has identified its logical con-

tent [28]. It is endowed with a variety of specifications: a CPS transformation [18], a CPS interpreter [30, 48], a denotational semantics [33], a computational monad [55], a big-step operational semantics [29], the CEK machine [26], calculi in the form of reduction semantics [25], and a number of implementation techniques [12, 15, 31]—not to mention its call-by-name variant in the archival version of Krivine's machine [34].

Question: How do we know that all the artifacts¹ in this semantic jungle define the same call/cc?

In some sense, the same can be said of the Scheme programming language today: it is independently specified with a denotational semantics [46], a structural operational semantics [45] and a reduction semantics [37], and its tail-recursion property is accounted for with a small-step abstract machine [14]. Which one specifies Scheme? The newest one? Or does one supersede a previous one of the same kind, just as the revisedⁿ⁺¹ report supersedes the revisedⁿ report [13, 33, 46, 50, 52, 53]? Or perhaps each semantics specifies one particular facet, however large that facet may be, of Scheme, irrespective of others? In that case, is the reference semantics the most complete one at any point of time?

Background: It is the author's thesis² that functional representations of small-step operational semantics (i.e., structured operational semantics), reduction semantics (i.e., small-step operational semantics with an explicit representation of the reduction context), small-step abstract machines, big-step abstract machines, big-step operational semantics (i.e., natural semantics), and denotational semantics are inter-derivable using elementary program transformations such as CPS transformation [18, 44, 47, 51], defunctionalization [21, 47], fixed-point fusion [19, 43], and refocusing [22].

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¹ "Artifact" means "man-made construct."

² A real one, actually [17], together with several others [1, 5, 9, 32, 38, 39, 41, 42], as a matter of fact.

This work: As a proof of concept, we derive the functional implementation of the denotational semantics corresponding to Clinger’s abstract machine as presented at PLDI’98 to specify the meaning of proper tail recursion [14]. We then compare it with Clinger’s denotational semantics in the R^3RS [46]. Each of these semantics is significant: the denotational one was profoundly influential in that it revealed formal semantics in action to a whole generation of computer scientists, and the operational one was instrumental to substantiate the precise meaning of what it means for an implementation to be properly tail-recursive. They are also unique because of their use of permutation/unpermutation functions to account for the undetermined sequencing order of sub-expressions in an application.

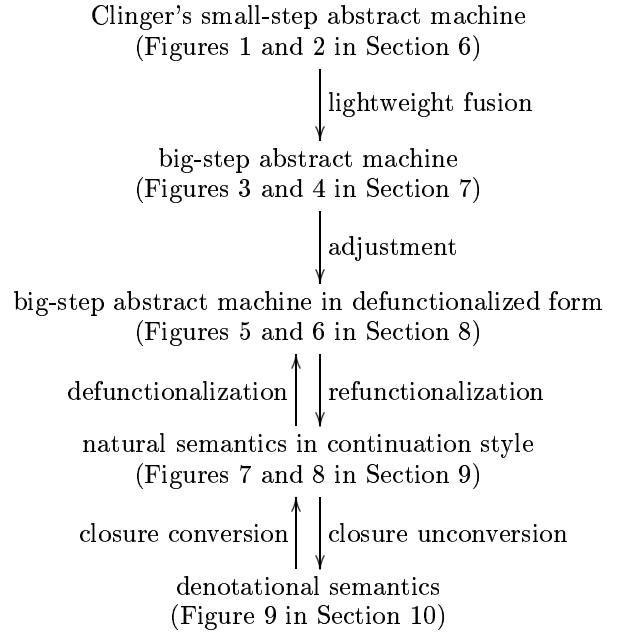
Our starting point is that, as pioneered by Reynolds in “Definitional Interpreters” [47] and pursued by the author and his students [2–4, 6, 16], closure-converting and defunctionalizing a continuation-passing evaluation function yields a big-step abstract machine. Here, we restate Clinger’s machine to operate with big steps, we adjust it so that it is in defunctionalized form, and we refunctionalize and then closure-unconvert the functional implementation of this adjusted machine, obtaining an evaluation function which is compositional and in continuation-passing style.

Prerequisites: Naturally, we assume the reader to be aware of Clinger’s denotational semantics of Scheme in the R^3RS [46] and of his abstract machine for Core Scheme as presented at PLDI’98 [14]. In addition, we also expect a basic knowledge of Standard ML [40], a pure subset of which we use as a functional meta-language,³ and a passing familiarity with defunctionalization [21, 47] and its left inverse, refunctionalization [19] as well as with closure conversion [35] and its left inverse, closure unconversion. As for the technique of fusing the transition function of a small-step abstract machine with its driver loop, it is described in a recent note by Millikin and the author [20].

Overview: We first review the domain of discourse (Section 2): stores, environments, and the permutation/unpermutation functions that are idiosyncratic to Clinger’s semantic specifications of Scheme. We then specify the syntax (Section 3) and the semantics (Section 4) of Core Scheme, and a garbage-collection rule (Section 5). Thus equipped, we present Clinger’s small-step abstract machine (Section 6) and then its big-step counterpart (Section 7). We put this big-step counterpart in defunctionalized form (Section 8), we present its refunctionalized counterpart (Section 9), and we closure-unconvert it (Section 10). We then compare the resulting compositional evaluation function in continuation-passing style to the valuation function of Clinger’s denotational semantics (Section 11) and then conclude (Section 12).

³In that we keep in mind that the full name of the Scheme workshop is “Scheme and Functional Programming.” (The emphasis is ours.)

Pictorially:



2. Domain of discourse

In the interest of brevity and abstractness, we only present ML signatures for the store (Section 2.1) and the environment (Section 2.2). We also explicitly treat Clinger’s permutations and unpermutations (Section 2.3).

2.1 Store

A store is a mapping from locations to storable values. We specify it as a polymorphic abstract data type with the usual algebraic operators to allocate fresh locations and initialize them with given storable values, dereference a given location in a given store, and update a given store at a given location with a given storable value.

```

signature STO = sig
  type 'a sto
  type loc

  val empty : 'a sto

  val new : 'a sto * 'a -> loc * 'a sto
  val news : 'a sto * 'a list -> loc list * 'a sto

  val fetch : loc * 'a sto -> 'a option
  val update : loc * 'a * 'a sto -> 'a sto
end

structure Sto : STO = struct
  (* deliberately omitted *)
end
  
```

At the price of modularity, we could define the allocation operators to also be given an environment and a context so that the fresh location(s) they return can be more precisely characterized as not occurring in the environment and in the context. This less modular definition would let us implement Clinger’s small-step abstract machine even more faithfully.

2.2 Environment

An environment is a mapping from identifiers to denotable values. We specify it as a polymorphic abstract data type with the usual algebraic operators to extend a given environment with new bindings and to look up identifiers in a given environment. We also need a predicate testing whether a given environment is empty.

```
signature ENV = sig
  type 'a env

  val empty : 'a env
  val empty? : 'a env -> bool

  val extend : ide * 'a * 'a env -> 'a env
  val extends : ide list * 'a list * 'a env -> 'a env

  val lookup : ide * 'a env -> 'a option
end
```

```
structure Env : ENV = struct
  (* deliberately omitted *)
end
```

In the present study, the denotable values are locations in a store.

2.3 Permutations and unpermutations

Both in his denotational semantics and his small-step abstract machine, Clinger non-deterministically uses a pair of permutation functions: one over the sub-terms in an application, and the inverse one over the resulting values (see Note #1 in Section 6.1). We implement this non-determinism by threading a stream of pairs of permutations through Clinger's semantic specifications. We materialize this stream with the following polymorphic abstract data type.

```
signature PERMUTATION = sig
  type 'a permutation = 'a * 'a list -> 'a * 'a list
  type ('v, 't) permutations

  val new : ('v, 't) permutations
    -> ('v permutation * 't permutation)
    * ('v, 't) permutations
  val init : ('v, 't) permutations
end

structure Permutation : PERMUTATION = struct
  (* deliberately omitted *)
end
```

3. Syntax

The following module implements the internal syntax of Core Scheme [14, Figure 1].

```
structure Syn = struct
  datatype quotation = QBOOL of bool
    | QNUMB of int
    | QSymb of ide
    | QPAIR of Sto.loc * Sto.loc
    (* QVECT of ... *)
    (* ... *)
end
```

```
datatype term = QUOTATION of quotation
  | VAR of ide
  | LAM of ide list * term
  | APP of term * term list
  | CND of term * term * term
  | SET of ide * term
```

end

4. Semantics

The following module implements the expressible values and evaluation contexts [14, Figure 4]. Like Clinger, we focus on Core Scheme and do not consider primitive procedures and first-class continuations here. (These are unproblematic to add [4, 7].)

```
structure Sem = struct
  (*datatype primop = ...*)

  datatype value = QUOTED of Syn.quotation
    | UNSPECIFIED
    | UNDEFINED
    (* PRIMOP of primop *)
    (* ESCAPE of Sto.loc * cont *)
    | CLOSURE of Sto.loc *
      (ide list * Syn.term) *
      Sto.loc Env.env

  and cont = HALT
    | SELECT of Syn.term *
      Syn.term *
      Sto.loc Env.env *
      cont
    | ASSIGN of ide *
      Sto.loc Env.env *
      cont
    | PUSH of Syn.term list *
      value list *
      value Permutation.permutation *
      Sto.loc Env.env *
      cont
    | CALL of value list * cont
end
```

Note: Compared to the original [14, Figure 4], we fixed one typo in the declaration of the evaluation-context constructor for calls, which holds a list of semantic values, not a list of syntactic terms.

5. Garbage collection

The following module is used to implement the garbage-collection rule [14, Figure 5].

```
signature GC = sig
  val gc : Sem.value *
    Sto.loc Env.env *
    Sem.cont *
    Sem.value Sto.sto
    -> Sem.value Sto.sto
end

structure Gc : GC = struct
  (* deliberately omitted *)
end
```

```

structure M_small_step = struct
  val r_init = Env.empty

  datatype term_or_value = TERM of Syn.term | VALUE of Sem.value

  datatype halting_state = RESULT of Sem.value * Sem.value Sto.sto | STUCK of string

  datatype state = FINAL of halting_state | INTER of configuration
  withtype configuration = term_or_value * Sto.loc Env.env * Sem.cont * Sem.value Sto.sto
    * (Sem.value, Syn.term) Permutation.permutations

  fun move ...
    = ...

  fun drive (FINAL answer)
    = answer
  | drive (INTER configuration)
    = drive (move configuration)

  fun evaluate t
    = drive (INTER (TERM t, r_init, Sem.HALT, Sto.empty, Permutation.init))
end

```

Figure 1. Clinger’s small-step abstract machine without the GC rule, part 1/2: configurations, driver loop and initialization

6. Clinger’s small-step abstract machine

We first present the machine without the GC rule (Section 6.1) and then with a GC rule (Section 6.2).

6.1 Without the GC rule

The abstract machine is displayed in Figures 1 and 2. It operates on a quintuple: a term or a value, an environment mapping variables to store locations, a context, a store mapping locations to values, and a stream of permutations. When the first component is a term, this term is dispatched upon. When the first component is a value, the third component (i.e., the context) is dispatched upon.

Note #1: A new pair of permutations is explicitly allocated every time an application is evaluated. The second one operates over terms and is used immediately over the sub-terms of the application. The first one operates over values and it will be subsequently used over the results of evaluating each of these sub-terms.

Clinger’s formulation uses an infinite set of rules generated by a rule schema that is parameterized by a permutation and its inverse. We model his non-deterministic choice of permutation with an oracle that picks in the current stream of permutations.

Note #2: Compared to the original [14, Figure 5] and to Clinger’s denotational semantics, this semantics embodies the official specification of Scheme about assignments:

When evaluating an assignment, the expression is evaluated, and the resulting value is stored in the location to which the variable is bound. This vari-

able must be bound either in some region enclosing the assignment or at the top level.

Accordingly, in Figure 2, the expression is evaluated, and then the variable is checked to be bound.

For the rest, Figures 1 and 2 display a scrupulously faithful implementation of Clinger’s small-step abstract machine.

6.2 With a GC rule

The following implementation deterministically applies the GC rule every time a value is returned to the evaluation context, i.e., at every reduction step. Compared to Figure 1, only the driver loop is changed.

```

structure M_small_step_with_gc_rule = struct
  ...
  fun drive (FINAL answer)
    = answer
  | drive (INTER (TERM t, r, c, s, p))
    = drive (move (TERM t, r, c, s, p))
  | drive (INTER (VALUE v, r, c, s, p))
    = let val s' = Gc.gc (v, r, c, s)
      in drive (move (VALUE v, r, c, s', p))
      end
  ...
end

```

In contrast, Clinger’s formulation of the GC rule is non-deterministic.

```

structure M_small_step = struct
...
fun move (TERM (Syn.QUOTATION q), r, c, s, p)
  = INTER (VALUE (Sem.QUOTED q), r, c, s, p)
| move (TERM (Syn.VAR i), r, c, s, p)
  = (case Env.lookup (i, r)
    of (SOME l)
      => (case Sto.fetch (l, s)
        of (SOME v)
          => (case v
            of Sem.UNDEFINED
              => FINAL (STUCK "attempt to reference undefined variable")
            | -
              => INTER (VALUE v, r, c, s, p))
        | NONE
          => FINAL (STUCK "attempt to read an invalid location"))
    | NONE
      => FINAL (STUCK "attempt to reference an undeclared variable"))
| move (TERM (Syn.LAM (is, t)), r, c, s, p)
  = let val (l, s') = Sto.new (s, Sem.UNSPECIFIED)
    in INTER (VALUE (Sem.CLOSURE (l, (is, t), r)), r, c, s', p) end
| move (TERM (Syn.CND (t0, t1, t2)), r, c, s, p)
  = INTER (TERM t0, r, Sem.SELECT (t1, t2, r, c), s, p)
| move (TERM (Syn.SET (i, t)), r, c, s, p)
  = INTER (TERM t, r, Sem.ASSIGN (i, r, c), s, p)
| move (TERM (Syn.APP (t0, ts)), r, c, s, p)
  = let val ((pi, rev_pi_inv), p') = Permutation.new p
    val (t0', ts') = rev_pi_inv (t0, ts)
    in INTER (TERM t0', r, Sem.PUSH (ts', nil, pi, r, c), s, p') end
| move (VALUE v, r', Sem.HALT, s, p)
  = if Env.empty r'
    then FINAL (RESULT (v, s))
    else INTER (VALUE v, Env.empty, Sem.HALT, s, p)
| move (VALUE (Sem.QUOTED (Syn.QBOOL false)), r', Sem.SELECT (t1, t2, r, c), s, p)
  = INTER (TERM t2, r, c, s, p)
| move (VALUE _, r', Sem.SELECT (t1, t2, r, c), s, p)
  = INTER (TERM t1, r, c, s, p)
| move (VALUE v, r', Sem.ASSIGN (i, r, c), s, p)
  = (case Env.lookup (i, r)
    of (SOME l)
      => (case Sto.fetch (l, s)
        of (SOME v)
          => (case v
            of Sem.UNDEFINED
              => FINAL (STUCK "attempt to assign undefined variable")
            | -
              => INTER (VALUE Sem.UNSPECIFIED, r, c, Sto.update (l, v, s), p))
        | NONE
          => FINAL (STUCK "attempt to write an invalid location"))
    | NONE
      => FINAL (STUCK "attempt to assign an undeclared variable"))
| move (VALUE v0', r', Sem.PUSH (nil, vs', pi, r, c), s, p)
  = let val (v0, vs) = pi (v0', vs')
    in INTER (VALUE v0, r, Sem.CALL (vs, c), s, p) end
| move (VALUE v0', r', Sem.PUSH (t1' :: ts', vs', pi, r, c), s, p)
  = INTER (TERM t1', r, Sem.PUSH (ts', v0' :: vs', pi, r, c), s, p)
| move (VALUE (Sem.CLOSURE (l, (is, t), r)), r', Sem.CALL (vs, c), s, p)
  = let val (ls, s') = Sto.news (s, vs)
    in INTER (TERM t, Env.extends (is, ls, r), c, s', p) end
| move (VALUE v, r', Sem.CALL (vs, c), s, p)
  = FINAL (STUCK "attempt to apply a non-procedure")
...
end

```

Figure 2. Clinger's small-step abstract machine without the GC rule, part 2/2: transition function

```

structure M_big_step = struct
  val r_init = Env.empty

  datatype term_or_value = TERM of Syn.term | VALUE of Sem.value

  datatype answer = RESULT of Sem.value * Sem.value Sto.sto | STUCK of string

  fun iterate ...
    = ...

  fun evaluate t
    = iterate (TERM t, r_init, Sem.HALT, Sto.empty, Permutation.init)
end

```

Figure 3. Big-step counterpart of Figure 1, part 1/2: configurations and initialization

7. Big-step version of Clinger’s abstract machine

We first present the big-step version of the machine without any GC rule (Section 7.1) and then with a GC rule (Section 7.2).

7.1 Without the GC rule

In Figure 1, the ‘drive’ function iteratively calls the ‘move’ function until a final answer is obtained, if any. As pointed out by Millikin and the author [20], such small-step abstract machines are candidates for lightweight fusion by fixed-point promotion [43]: the composition of ‘drive’ and ‘move’ can be fused into an ‘iterate’ function where the outer recursive call to ‘drive’ has been distributed to all the return points in the definition of ‘move.’ The result is the big-step abstract machine displayed in Figures 3 and 4. Since Ohori and Sasano’s fixed-point promotion is fully correct, this big-step abstract machine is also correct, by construction.

7.2 With a GC rule

The following implementation deterministically applies the GC rule every time a function is about to be applied. Compared to Figure 4, only one clause is changed.

```

structure M_big_step_with_gc_rule = struct
  (* ... *)
  | iterate (VALUE v0',
             r',
             Sem.PUSH (nil, vs', pi, r, c),
             s,
             p)
    = let val (v0, vs) = pi (v0', vs')
      val s' = Gc.gc (v0,
                    r,
                    Sem.CALL (vs, c),
                    s)
    in iterate (VALUE v0,
               r,
               Sem.CALL (vs, c),
               s',
               p) end
  (* ... *)
end

```

```

structure M.big_step = struct
  ...
  fun iterate (TERM (Syn.QUOTATION q), r, c, s, p)
    = iterate (VALUE (Sem.QUOTED q), r, c, s, p)
  | iterate (TERM (Syn.VAR i), r, c, s, p)
    = (case Env.lookup (i, r)
      of (SOME l)
        => (case Sto.fetch (l, s)
          of (SOME v)
            => (case v
              of Sem.UNDEFINED
                => STUCK "attempt to reference undefined variable"
              | -
                => iterate (VALUE v, r, c, s, p))
          | NONE
            => STUCK "attempt to read an invalid location")
        | NONE
          => STUCK "attempt to reference an undeclared variable")
  | iterate (TERM (Syn.LAM (is, t)), r, c, s, p)
    = let val (l, s') = Sto.new (s, Sem.UNSPECIFIED)
      in iterate (VALUE (Sem.CLOSURE (l, (is, t), r)), r, c, s', p) end
  | iterate (TERM (Syn.CND (t0, t1, t2)), r, c, s, p)
    = iterate (TERM t0, r, Sem.SELECT (t1, t2, r, c), s, p)
  | iterate (TERM (Syn.SET (i, t)), r, c, s, p)
    = iterate (TERM t, r, Sem.ASSIGN (i, r, c), s, p)
  | iterate (TERM (Syn.APP (t0, ts)), r, c, s, p)
    = let val ((pi, rev_pi_inv), p') = Permutation.new p
      val (t0', ts') = rev_pi_inv (t0, ts)
      in iterate (TERM t0', r, Sem.PUSH (ts', nil, pi, r, c), s, p') end
  | iterate (VALUE v, r', Sem.HALT, s, p)
    = if Env.emptytyp r'
      then RESULT (v, s)
      else iterate (VALUE v, Env.empty, Sem.HALT, s, p)
  | iterate (VALUE (Sem.QUOTED (Syn.QB00L false)), r', Sem.SELECT (t1, t2, r, c), s, p)
    = iterate (TERM t2, r, c, s, p)
  | iterate (VALUE _, r', Sem.SELECT (t1, t2, r, c), s, p)
    = iterate (TERM t1, r, c, s, p)
  | iterate (VALUE v, r', Sem.ASSIGN (i, r, c), s, p)
    = (case Env.lookup (i, r)
      of (SOME l)
        => (case Sto.fetch (l, s)
          of (SOME v)
            => (case v
              of Sem.UNDEFINED
                => STUCK "attempt to assign undefined variable"
              | -
                => iterate (VALUE Sem.UNSPECIFIED, r, c, Sto.update (l, v, s), p))
          | NONE
            => STUCK "attempt to write an invalid location")
        | NONE
          => STUCK "attempt to assign an undeclared variable")
  | iterate (VALUE v0', r', Sem.PUSH (nil, vs', pi, r, c), s, p)
    = let val (v0, vs) = pi (v0', vs')
      in iterate (VALUE v0, r, Sem.CALL (vs, c), s, p) end
  | iterate (VALUE v0', r', Sem.PUSH (t1' :: ts', vs', pi, r, c), s, p)
    = iterate (TERM t1', r, Sem.PUSH (ts', v0' :: vs', pi, r, c), s, p)
  | iterate (VALUE (Sem.CLOSURE (l, (is, t), r)), r', Sem.CALL (vs, c), s, p)
    = let val (ls, s') = Sto.news (s, vs)
      in iterate (TERM t, Env.extends (is, ls, r), c, s', p) end
  | iterate (VALUE v, r', Sem.CALL (vs, c), s, p)
    = STUCK "attempt to apply a non-procedure"
  ...
end

```

Figure 4. Big-step counterpart of Figure 2, part 2/2: transition function

```

structure M_big_step_defunct = struct
  val r_init = Env.empty

  datatype term_or_value = TERM of Syn.term | VALUE of Sem.value

  datatype answer = RESULT of Sem.value * Sem.value Sto.sto | STUCK of string

  fun eval ...
    = ...
  and continue ...
    = ...

  fun evaluate t
    = eval (TERM t, r_init, Sem.HALT, Sto.empty, Permutation.init)
end

```

Figure 5. Version of Figure 3 in defunctionalized form, part 1/2: configurations and initialization

8. Big-step version of Clinger’s abstract machine in defunctionalized form

8.1 Without the GC rule

Like the SECD machine [16, 35], the big-step version of Clinger’s abstract machine is not in defunctionalized form. Fortunately, it can easily be made to be so [19], by using the type isomorphism between the transition function

```
iterate : term_or_value * ... -> answer
```

and two mutually recursive transition functions

```

eval : term * ... -> answer
continue : value * ... -> answer.

```

The reformulated version is displayed in Figures 5 and 6, and can readily be recognized as an ‘eval/apply’ abstract machine [36]:⁴ the ‘eval’ transition function dispatches on terms and the ‘apply’ transition function (or more accurately, the ‘continue’ transition function) dispatches on (the top constructor of) the context. This abstract machine is in defunctionalized form in that the evaluation context and the second transition function are the defunctionalized counterpart of a function. As shown in the next section, this function is a continuation since the refunctionalized abstract machine is in CPS.⁵

8.2 With a GC rule

As in Section 7.2, it is simple to deterministically apply the GC rule, e.g., every time a function is about to be applied.

⁴ A more accurate term than ‘eval/apply’, though, would be ‘eval/continue.’

⁵ Hence the point about terminology in Footnote 4.

```

structure M.big_step_defunct = struct
...
fun eval (Syn.QUOTATION q, r, c, s, p)
  = continue (Sem.QUOTED q, r, c, s, p)
| eval (Syn.VAR i, r, c, s, p)
  = (case Env.lookup (i, r)
    of (SOME l)
      => (case Sto.fetch (l, s)
        of (SOME v)
          => (case v
            of Sem.UNDEFINED
              => STUCK "attempt to reference undefined variable"
            | -
              => continue (v, r, c, s, p))
        | NONE
          => STUCK "attempt to read an invalid location")
    | NONE
      => STUCK "attempt to reference an undeclared variable")
| eval (Syn.LAM (is, t), r, c, s, p)
  = let val (l, s') = Sto.new (s, Sem.UNSPECIFIED)
    in continue (Sem.CLOSURE (l, (is, t), r), r, c, s', p) end
| eval (Syn.CND (t0, t1, t2), r, c, s, p)
  = eval (t0, r, Sem.SELECT (t1, t2, r, c), s, p)
| eval (Syn.SET (i, t), r, c, s, p)
  = eval (t, r, Sem.ASSIGN (i, r, c), s, p)
| eval (Syn.APP (t0, ts), r, c, s, p)
  = let val ((pi, rev_pi_inv), p') = Permutation.new p
    val (t0', ts') = rev_pi_inv (t0, ts)
    in eval (t0', r, Sem.PUSH (ts', nil, pi, r, c), s, p') end
and continue (v, r', Sem.HALT, s, p)
  = if Env.empty r'
    then RESULT (v, s)
    else continue (v, Env.empty, Sem.HALT, s, p)
| continue (Sem.QUOTED (Syn.QB00L false), r', Sem.SELECT (t1, t2, r, c), s, p)
  = eval (t2, r, c, s, p)
| continue (_, r', Sem.SELECT (t1, t2, r, c), s, p)
  = eval (t1, r, c, s, p)
| continue (v, r', Sem.ASSIGN (i, r, c), s, p)
  = (case Env.lookup (i, r)
    of (SOME l)
      => (case Sto.fetch (l, s)
        of (SOME v)
          => (case v
            of Sem.UNDEFINED
              => STUCK "attempt to assign undefined variable"
            | -
              => continue (Sem.UNSPECIFIED, r, c, Sto.update (l, v, s), p))
        | NONE
          => STUCK "attempt to write an invalid location")
    | NONE
      => STUCK "attempt to assign an undeclared variable")
| continue (v0', r', Sem.PUSH (nil, vs', pi, r, c), s, p)
  = let val (v0, vs) = pi (v0', vs')
    in continue (v0, r, Sem.CALL (vs, c), s, p) end
| continue (v0', r', Sem.PUSH (t1' :: ts', vs', pi, r, c), s, p)
  = eval (t1', r, Sem.PUSH (ts', v0' :: vs', pi, r, c), s, p)
| continue (Sem.CLOSURE (l, (is, t), r), r', Sem.CALL (vs, c), s, p)
  = let val (ls, s') = Sto.news (s, vs)
    in eval (t, Env.extends (is, ls, r), c, s', p) end
| continue (v, r', Sem.CALL (vs, c), s, p)
  = STUCK "attempt to apply a non-procedure"
...
end

```

Figure 6. Version of Figure 4 in defunctionalized form, part 2/2: transition functions

```

structure Sem = struct
  datatype value = QUOTED of Syn.quotation
                  | UNSPECIFIED
                  | UNDEFINED
                  | CLOSURE of Sto.loc * (ide list * Syn.term) * Sto.loc Env.env
  and answer = RESULT of value * value Sto.sto
             | STUCK of string
  withtype cont = value * Sto.loc Env.env * value Sto.sto * value Permutation.permutation -> answer
end

structure M_big_step_refunct = struct
  val r_init = Env.empty

  datatype term_or_value = TERM of Syn.term | VALUE of Sem.value

  fun eval ...
    = ...

  fun evaluate t
    = eval (t, r_init, fn (v, -, s, p) => Sem.RESULT (v, s), Sto.empty, Permutation.init)
end

```

Figure 7. Refunctionalized version of Figure 5, part 1/2: configurations and initialization

9. Big-step version of Clinger’s abstract machine, refunctionalized

9.1 Without the GC rule

The refunctionalized version is displayed in Figures 7 and 8: defunctionalizing it yields back Figures 5 and 6. It is the evaluation function in continuation-passing style of a natural semantics [23, 29]. As exploited in Section 10, it is also in the range of closure conversion.

9.2 With a GC rule

Refunctionalization has made us cross a line: continuations are now higher-order, which prevents us to implement the GC rule as directly as in Section 7.2.

```

structure M.big_step_refunct = struct
  ...
  fun eval (Syn.QUOTATION q, r, c, s, p)
    = c (Sem.QUOTED q, r, s, p)
  | eval (Syn.VAR i, r, c, s, p)
    = (case Env.lookup (i, r)
      of (SOME l)
        => (case Sto.fetch (l, s)
          of (SOME v)
            => (case v
              of Sem.UNDEFINED
                => Sem.STUCK "attempt to reference undefined variable"
              | -
                => c (v, r, s, p))
          | NONE
            => Sem.STUCK "attempt to read an invalid location")
      | NONE
        => Sem.STUCK "attempt to reference an undeclared variable")
  | eval (Syn.LAM (is, t), r, c, s, p)
    = let val (l, s') = Sto.new (s, Sem.UNSPECIFIED)
      in c (Sem.CLOSURE (l, (is, t), r), r, s', p) end
  | eval (Syn.CND (t0, t1, t2), r, c, s, p)
    = eval (t0, r, fn (Sem.QUOTED (Syn.QB00L false), r', s, p)
      => eval (t2, r, c, s, p)
      | (_, r', s, p)
        => eval (t1, r, c, s, p),
      s, p)
  | eval (Syn.SET (i, t), r, c, s, p)
    = eval (t, r, fn (v, r', s, p)
      => (case Env.lookup (i, r)
        of (SOME l)
          => (case Sto.fetch (l, s)
            of (SOME v)
              => (case v
                of Sem.UNDEFINED
                  => Sem.STUCK "attempt to assign undefined variable"
                | -
                  => c (Sem.UNSPECIFIED, r, Sto.update (l, v, s), p))
            | NONE
              => Sem.STUCK "attempt to write an invalid location")
          | NONE
            => Sem.STUCK "attempt to assign an undeclared variable"),
        s, p)
  | eval (Syn.APP (t0, ts), r, c, s, p)
    = let val ((pi, rev.pi_inv), p') = Permutation.new p
      val (t0', ts') = rev.pi_inv (t0, ts)
      in eval (t0', r, fn (v0', r', s, p) => evlis (ts', v0', nil, pi, r, c, s, p), s, p') end
and evlis (nil, v0', vs', pi, r, c, s, p)
    = let val (v0, vs) = pi (v0', vs')
      in case v0
        of (Sem.CLOSURE (l, (is, t), r))
          => let val (ls, s') = Sto.news (s, vs)
            in eval (t, Env.extends (is, ls, r), c, s', p)
            end
        | -
          => Sem.STUCK "attempt to apply a non-procedure" end
  | evlis (t1' :: ts', v0', vs', pi, r, c, s, p)
    = eval (t1', r, fn (v1', r', s, p) => evlis (ts', v1', v0' :: vs', pi, r', c, s, p), s, p)
  ...
end

```

Figure 8. Refunctionalized version of Figure 6, part 2/2: evaluation functions

```

structure Sem = struct
  datatype value = ...
    | CLOSURE of Sto.loc * (value list * cont * value Sto.sto *
                           (value, Syn.term) Permutation.permutations -> answer) (* << *)
  and answer = RESULT of value * value Sto.sto
    | STUCK of string
  withtype cont = value * Sto.loc Env.env * value Sto.sto * (value, Syn.term) Permutation.permutations
    -> answer
end

structure M_big_step_refunct_higher_order = struct
  (* ... *)
  | eval (Syn.LAM (is, t), r, c, s, p)
    = let val (l, s') = Sto.new (s, Sem.UNSPECIFIED)
      in c (Sem.CLOSURE (l, fn (vs, c, s, p)
                           => let val (ls, s') = Sto.news (s, vs)
                              in eval (t, Env.extends (is, ls, r), c, s', p)
                              end),
          r, s', p) end
      (* << *)
      (* << *)
      (* << *)
      (* << *)
  (* ... *)
  and evlis (nil, v0', vs', pi, r, c, s, p)
    = let val (v0, vs) = pi (v0', vs')
      in case v0
        of (Sem.CLOSURE (l, f))
          => f (vs, c, s, p)
          | -
          => Sem.STUCK "attempt to apply a non-procedure" end
      (* << *)
  (* ... *)
end

```

Figure 9. Closure-unconverted version of Figures 7 and 8 (the modified parts are marked on the right)

10. Big-step version of Clinger's abstract machine, refunctionalized and closure-unconverted

10.1 Without the GC rule

Figure 9 displays the higher-order counterpart of Figures 7 and 8: closure-converting it yields back these two figures. It is a compositional evaluation function in continuation-passing style.

10.2 With a GC rule

Closure unconversion has made us cross another line: the higher-order functions in the domain of values further prevent us to implement the GC rule as directly as in Section 7.2.

11. Analysis

11.1 From abstract machine to denotational semantics

The evaluation function of Figure 9 differs from the one in Clinger's denotational semantics in two ways:

- the continuation domains are not the same: the operational continuations are passed an environment whereas the denotational ones are not;
- sub-terms in an application are not evaluated in the same fashion: in the denotational semantics, there is an explicit `evlis` function whereas in the operational semantics, the sub-terms are reversed after being permuted and the corresponding values are iteratively accumulated.

11.2 From denotational semantics to abstract machine

Conversely, defunctionalizing Clinger's denotational semantics yields a big-step abstract machine that also differs from the one presented at PLDI'98: it is a traditional eval/continue abstract machine where the 'continue' transition function is not passed any environment. In particular, the permuted sub-terms are not reversed prior to be evaluated and the corresponding list of values is not accumulated as in Figure 2; instead, there is an additional 'evlis' transition function.

11.3 Related work

Earlier on, Biernacki and the author carried out the same experiment for Propositional Prolog with cut [10]. Comparing de Bruin and de Vink's operational and denotational semantics [24], we found mismatches that are similar to the ones reported in this section.

For the rest, the literature is rich with connections and derivations between semantic artifacts. To the best of our knowledge, none are as simple and as effective as the ones pioneered by Reynolds and used here.

12. Conclusion and perspectives

We have presented new semantic specifications for Core Scheme as specified by Clinger in his PLDI'98 article. These semantic specifications are compatible and inter-derived. It is our analysis that structurally, they differ from similar semantics that have independently been published.

There are two next logical steps to this preliminary work, an analytical one and a constructive one:

- Redo this experiment on a larger scale with the other semantic specifications of Scheme. We will then be in position to compare all the small-step semantics, all the abstract machines, and all the big-step semantics of Scheme with respect to each other, and verify whether Scheme is uniformly and uniquely specified.
- Specify Scheme in part or in toto with inter-derivable specifications so that the compatibility of these specifications is a corollary of the correctness of the derivations. This corollary would let us flesh out, for example, Gasbichler, Knauel, and Sperber's conjecture of equivalence for their operational and denotational semantics of Scheme with multiple threads [27, Section 6.4].

The author does not have any opinion about the particular goodness of one or another semantic specification of Scheme. He however feels strongly that Scheme's semantic artifacts should be compatible with each other. PhD students are thus invited to apply for six-months visits to the BRICS PhD School at the University of Aarhus to help carrying out the experiments above together with the author. Any other input or collaboration is also cordially welcome.

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