Dynamical Systems

Werner Krabs • Stefan Pickl

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Stability, Controllability and Chaotic Behavior

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Foreword

Reflecting modelling dynamical systems by mathematical methods can be enriched by philosophical categories. The following introduction might catch the reader's interest concerning some interdisciplinary dimensions and completes the holistic approach.

"It has been said– and I was among those saying it– that any theory of explanation worth its salt should be able to make good predictions. If good predictions could not be made, the explanation could hardly count as serious. (...) I now want to move on to my main point. There is, I claim, no major conceptual difference between the problems of explaining the unpredictable in human affairs and in non-human affairs. There are, it is true, many remarkable successes of prediction in the physical sciences of which we are all aware, but these few successes of principled science making principled predictions are, in many ways, misleading. (...) The point I want to emphasize is that instability is as present in purely physical systems as it is in those we think of as characteristically human. Our ability to explain but not predict human behavior is in the same general category as our ability to explain but not predict many physical phenomena. The underlying reasons for the inability to predict are the same. (...) The concept of instability which accounts for many of these failures is one of the most neglected concepts in philosophy. We philosophers have as a matter of practice put too much emphasis on the contrast between deterministic and probabilistic phenomena. We have not emphasized enough the salient differences between stable and unstable phenomena. One can argue that the main sources of probabilistic or random behavior lie in instability. We might even want to hold the speculative thesis that the random behavior of quantum systems will itself in turn be explained by unstable behavior of classical dynamical systems. (...)

Chaos, the original confusion in which all the elements were mixed together, was personified by the Greeks as the most ancient of the gods. Now in the twentieth century, chaos has returned in force to attack that citadel of order and harmony, classical mechanics. We have come to recognize how rare and special are those physical systems whose behavior can be predicted in detail. The naivet and hopes of earlier years will not return. For many phenomena in many domains there are principled reasons to believe that we shall never be able to move from good explanations to good predictions."

PATRICK SUPPES "EXPLAINING THE UNPREDICTABLE"
Lucie Stern Professor of Philosophy, Stanford University
Director and Faculty Advisor, Education Program for Gifted Youth, Stanford University

Preface

At the end of the nineteenth century Lyapunov and Poincaré developed the so called qualitative theory of differential equations and introduced geometric-topological considerations which have led to the concept of dynamical systems. In its present abstract form this concept goes back to G.D. Birkhoff.

This is also the starting point of Chapter 1 of this book in which uncontrolled and controlled time-continuous and time-discrete systems are investigated under the aspect of stability and controllability. Chapter 1 starts with time-continuous dynamical systems. After the description of elementary properties of such systems it focusses on stability in the sense of Lyapunov and gives applications to systems in the plane such as the mathematical pendulum, to general predator-prey models, and to evolution matrix games.

The time-discrete case is divided into the autonomous and the non-autonomous part where the latter is no more a dynamical system in the strong sense. It is the counter part of the time-continuous case where the right-hand side of the system of differential equations which describes the dynamics of the system depends explicitly on the time.

Controlled dynamical systems could be considered as dynamical systems in the strong sense, if the controls were incorporated into the state space. We, however, adopt the conventional treatment of controlled systems as in control theory. We are mainly interested in the question of controllability of dynamical systems into equilibrium states. In the non-autonomous time-discrete case we also consider the problem of stabilization.

Chapter 3 is concerned with chaotic behaviour of autonomous time discrete systems. We consider three different types of chaos: chaos in the sense of Devaney, disorder chaos and chaos in the sense of Li and Yorke. The chapter ends with two examples of strange (or chaotic) attractors.

The Appendix A is concerned with a dynamical method for the calculation of Nash equilibria in non-cooperative n-person games. The method is based on the fact that Nash equilibria are fixed points of certain continuous mappings of the Cartesian product of the strategy sets of the players into itself. This gives rise to an iteration method for the calculation of Nash equilibria the set of which can be considered as the Ω-limit set of a time-discrete dynamical system.

In Appendix B we consider two optimal control problems in chemotherapeutic treatment of cancer. These two problems are somehow dual to each other and are shown to have solutions of the same type.

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Werner Krabs, Stefan Pickl, Munich, May 2010

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