

Studies in Computational Intelligence, Volume 300

Editor-in-Chief

Prof. Janusz Kacprzyk
Systems Research Institute
Polish Academy of Sciences
ul. Newelska 6
01-447 Warsaw
Poland
E-mail: kacprzyk@ibspan.waw.pl

Further volumes of this series can be found on our homepage: springer.com

Vol. 278. Radomir S. Stankovic and Jaakko Astola
From Boolean Logic to Switching Circuits and Automata, 2010
ISBN 978-3-642-11681-0

Vol. 279. Manolis Wallace, Ioannis E. Anagnostopoulos, Phivos Mylonas, and Maria Bielikova (Eds.)
Semantics in Adaptive and Personalized Services, 2010
ISBN 978-3-642-11683-4

Vol. 280. Chang Wen Chen, Zhu Li, and Shiguo Lian (Eds.)
Intelligent Multimedia Communication: Techniques and Applications, 2010
ISBN 978-3-642-11685-8

Vol. 281. Robert Babuska and Frans C.A. Groen (Eds.)
Interactive Collaborative Information Systems, 2010
ISBN 978-3-642-11687-2

Vol. 282. Husrev Taha Sencar, Sergio Velastin, Nikolaos Nikolaidis, and Shiguo Lian (Eds.)
Intelligent Multimedia Analysis for Security Applications, 2010
ISBN 978-3-642-11754-1

Vol. 283. Ngoc Thanh Nguyen, Radoslaw Katarzyniak, and Shyi-Ming Chen (Eds.)
Advances in Intelligent Information and Database Systems, 2010
ISBN 978-3-642-12089-3

Vol. 284. Juan R. González, David Alejandro Pelta, Carlos Cruz, Germán Terrazas, and Natalio Krasnogor (Eds.)
Nature Inspired Cooperative Strategies for Optimization (NICSO 2010), 2010
ISBN 978-3-642-12537-9

Vol. 285. Roberto Cipolla, Sebastiano Battiato, and Giovanni Maria Farinella (Eds.)
Computer Vision, 2010
ISBN 978-3-642-12847-9

Vol. 286. Zeev Volkovich, Alexander Bolshoy, Valery Kirzhner, and Zeev Barzily
Genome Clustering, 2010
ISBN 978-3-642-12951-3

Vol. 287. Dan Schonfeld, Caifeng Shan, Dacheng Tao, and Liang Wang (Eds.)
Video Search and Mining, 2010
ISBN 978-3-642-12899-8

Vol. 288. I-Hsien Ting, Hui-Ju Wu, Tien-Hwa Ho (Eds.)
Mining and Analyzing Social Networks, 2010
ISBN 978-3-642-13421-0

Vol. 289. Anne Hökansson, Ronald Hartung, and Ngoc Thanh Nguyen (Eds.)
Agent and Multi-agent Technology for Internet and Enterprise Systems, 2010
ISBN 978-3-642-13525-5

Vol. 290. Weiliang Xu and John Bronlund
Mastication Robots, 2010
ISBN 978-3-540-93902-3

Vol. 291. Shimon Whiteson
Adaptive Representations for Reinforcement Learning, 2010
ISBN 978-3-642-13931-4

Vol. 292. Fabrice Guillet, Gilbert Ritschard, Henri Briand, Djamel A. Zighed (Eds.)
Advances in Knowledge Discovery and Management, 2010
ISBN 978-3-642-00579-4

Vol. 293. Anthony Brabazon, Michael O'Neill, and Dietmar Maringer (Eds.)
Natural Computing in Computational Finance, 2010
ISBN 978-3-642-13949-9

Vol. 294. Manuel F.M. Barros, Jorge M.C. Guilherme, and Nuno C.G. Horta
Analog Circuits and Systems Optimization based on Evolutionary Computation Techniques, 2010
ISBN 978-3-642-12345-0

Vol. 295. Roger Lee (Ed.)
Software Engineering, Artificial Intelligence, Networking and Parallel/Distributed Computing, 2010
ISBN 978-3-642-13264-3

Vol. 296. Roger Lee (Ed.)
Software Engineering Research, Management and Applications, 2010
ISBN 978-3-642-13272-8

Vol. 297. Tania Tronco (Ed.)
New Network Architectures, 2010
ISBN 978-3-642-13246-9

Vol. 298. Adam Wierzbicki
Trust and Fairness in Open, Distributed Systems, 2010
ISBN 978-3-642-13450-0

Vol. 299. Vassil Sgurev, Mincho Hadjisiki, and Janusz Kacprzyk (Eds.)
Intelligent Systems: From Theory to Practice, 2010
ISBN 978-3-642-13427-2

Vol. 300. Baoding Liu
Uncertainty Theory, 2010
ISBN 978-3-642-13958-1

Baoding Liu

Uncertainty Theory

A Branch of Mathematics for Modeling Human
Uncertainty

Baoding Liu
Department of Mathematical Sciences
Tsinghua University
Beijing 100084
China
E-mail: liu@tsinghua.edu.cn

ISBN 978-3-642-13958-1

e-ISBN 978-3-642-13959-8

DOI 10.1007/978-3-642-13959-8

Studies in Computational Intelligence

ISSN 1860-949X

Library of Congress Control Number: 2010929355

© 2010 Springer-Verlag Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typeset & Cover Design: Scientific Publishing Services Pvt. Ltd., Chennai, India.

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Contents

Preface	IX
1 Uncertainty Theory	1
1.1 Uncertain Measure	1
1.2 Uncertain Variable	11
1.3 Uncertainty Distribution	14
1.4 Independence	21
1.5 Operational Law	23
1.6 Expected Value	44
1.7 Variance	52
1.8 Moments	54
1.9 Critical Values	56
1.10 Entropy	60
1.11 Distance	66
1.12 Inequalities	67
1.13 Convergence Concepts	69
1.14 Conditional Uncertainty	74
2 Uncertain Programming	81
2.1 Ranking Criteria	81
2.2 Expected Value Model	82
2.3 Chance-Constrained Programming	83
2.4 Dependent-Chance Programming	88
2.5 Uncertain Dynamic Programming	93
2.6 Uncertain Multilevel Programming	94
2.7 Hybrid Intelligent Algorithm	98
2.8 Ψ Graph	99
2.9 Project Scheduling Problem	100
2.10 Vehicle Routing Problem	103
2.11 Machine Scheduling Problem	108
2.12 Exercises	112
3 Uncertain Risk Analysis	115
3.1 Risk Index	115
3.2 Hazard Distribution	118
3.3 Boolean System	120
3.4 Risk Index Calculator	122

4	Uncertain Reliability Analysis	125
4.1	Reliability Index	125
4.2	Conditional Reliability	127
4.3	Boolean System	128
4.4	Reliability Index Calculator	130
5	Uncertain Process	131
5.1	Uncertain Process	131
5.2	Renewal Process	132
5.3	Martingale	138
5.4	Markov Process	138
5.5	Stationary Process	138
6	Uncertain Calculus	139
6.1	Canonical Process	139
6.2	Uncertain Integral	141
6.3	Chain Rule	143
6.4	Integration by Parts	144
7	Uncertain Differential Equation	147
7.1	Uncertain Differential Equation	147
7.2	Existence and Uniqueness Theorem	149
7.3	Stability Theorem	151
7.4	Numerical Method	152
7.5	Uncertain Differential Equation with Jumps	156
7.6	Uncertain Finance	158
8	Uncertain Logic	163
8.1	Uncertain Proposition	163
8.2	Connective Symbols	164
8.3	Uncertain Formula	164
8.4	Truth Function	164
8.5	Truth Value	165
8.6	Truth Value Theorem	166
8.7	Truth Value Solver	173
8.8	Uncertain Predicate Logic	174
9	Uncertain Entailment	177
9.1	Entailment Model	177
9.2	Modus Ponens	182
9.3	Modus Tollens	184
9.4	Hypothetical Syllogism	185
9.5	Automatic Entailment Machine	186

10 Uncertain Set Theory	187
10.1 Uncertain Set	187
10.2 Membership Degree	190
10.3 Membership Function	192
10.4 Uncertainty Distribution	199
10.5 Independence	201
10.6 Operational Law	203
10.7 Expected Value	206
10.8 Critical Values	210
10.9 Hausdorff Distance	212
10.10 Conditional Uncertainty	213
11 Uncertain Inference	215
11.1 Inference Rule	215
11.2 Uncertain System	219
11.3 Inference Control	222
11.4 Inverted Pendulum	223
A Supplements	225
A.1 Law of Truth Conservation	225
A.2 Maximum Uncertainty Principle	225
A.3 How to Determine Distribution?	226
A.4 Evolution of Measures	229
A.5 Uncertainty vs. Randomness	232
A.6 Uncertainty + Randomness	233
A.7 Uncertainty vs. Fuzziness	236
A.8 What Is Uncertainty?	237
B Probability Theory	239
B.1 Probability Space	239
B.2 Random Variable	242
B.3 Probability Distribution	244
B.4 Independence	247
B.5 Expected Value	248
B.6 Variance	254
B.7 Moments	255
B.8 Critical Values	256
B.9 Entropy	257
B.10 Conditional Probability	261
B.11 Random Set	263
C Credibility Theory	267
C.1 Credibility Space	267
C.2 Fuzzy Variable	277
C.3 Membership Function	288
C.4 Credibility Distribution	282

C.5	Independence	284
C.6	Extension Principle of Zadeh	286
C.7	Expected Value	288
C.8	Variance	293
C.9	Moments	295
C.10	Critical Values	295
C.11	Entropy	297
C.12	Conditional Credibility	302
C.13	Fuzzy Set	306
D	Chance Theory	309
D.1	Chance Space	309
D.2	Hybrid Variable	317
D.3	Chance Distribution	323
D.4	Expected Value	324
D.5	Variance	326
D.6	Critical Values	326
D.7	Conditional Chance	328
	Bibliography	333
	List of Frequently Used Symbols	346
	Five Plus One	347
	Index	349

Preface

Some information and knowledge are usually represented by human language like “about 100km”, “approximately 39 °C”, “roughly 80kg”, “low speed”, “middle age”, and “big size”. Perhaps some people think that they are subjective probability or they are fuzziness. However, a lot of surveys showed that those imprecise quantities behave neither like randomness nor like fuzziness. How do we understand them? How do we model them? These questions provide a motivation to invent another mathematical tool to model those imprecise quantities. In order to do so, an uncertainty theory was founded and became a branch of axiomatic mathematics. Since then, uncertainty theory has been developed steadily and applied widely.

Chapter 1 is devoted to the uncertainty theory. The first fundamental concept in uncertainty theory is uncertain measure that is used to measure the truth degree of an uncertain event. The second one is uncertain variable that is used to represent imprecise quantities. The third one is uncertainty distribution that is used to describe uncertain variables in an incomplete but easy-to-use way. Uncertainty theory is thus deduced from those three foundation stones, and plays the role of mathematical model to deal with uncertain phenomena.

Uncertain programming is a type of mathematical programming involving uncertain variables. A key problem in uncertain programming is how to rank uncertain variables, and different ranking criteria produce different classes of uncertain programming. Chapter 2 will introduce four ranking criteria and then provide a spectrum of uncertain programming with applications to project scheduling problem, vehicle routing problem and machine scheduling problem.

The term risk has been used in different ways in literature. In this book the risk is defined as the accidental loss plus the uncertain measure of such loss, and a risk index is defined as the uncertain measure that some specified loss occurs. Chapter 3 will introduce uncertain risk analysis that is a tool to quantify risk via uncertainty theory.

Reliability index is defined as the uncertain measure that some system is working. Thus reliability and risk have the same root in mathematics. They are separately treated for application convenience in practice rather than theoretical demand. Chapter 4 will introduce uncertain reliability analysis that is a tool to deal with system reliability via uncertainty theory.

An uncertain process is essentially a sequence of uncertain variables indexed by time or space. Thus an uncertain process is usually used to model

uncertain phenomena that vary with time or space. Some basic concepts of uncertain process will be presented in Chapter 5.

Uncertain calculus is a branch of mathematics that deals with differentiation and integration of function of uncertain processes. As the very core of uncertain calculus, canonical process is a Lipschitz continuous uncertain process that has stationary and independent increments and every increment is a normal uncertain variable. Chapter 6 will introduce the uncertain calculus including canonical process, uncertain integral and chain rule.

Uncertain differential equation is a type of differential equation driven by canonical process. Chapter 7 will discuss the existence, uniqueness and stability of solutions of uncertain differential equations, and will design a numerical method for solving monotone uncertain differential equations. This chapter will also present an application of uncertain differential equation in finance.

Uncertain logic is a generalization of mathematical logic for dealing with uncertain knowledge via uncertainty theory. A key point in uncertain logic is that the truth value of an uncertain proposition is defined as the uncertain measure that the proposition is true. One advantage of uncertain logic is the well consistency with classical logic. In other words, uncertain logic obeys the law of truth conservation, and is consistent with the law of excluded middle and the law of contradiction. Chapter 8 will discuss uncertain propositional logic and uncertain predicate logic.

Uncertain entailment is a methodology for calculating the truth value of an uncertain formula via the maximum uncertainty principle when the truth values of other uncertain formulas are given. That is, we will assign an uncertain formula a truth value as close to 0.5 as possible. Chapter 9 will introduce an entailment model from which modus ponens, modus tollens and hypothetical syllogism are deduced.

Uncertain set is a measurable function from an uncertainty space to a collection of sets. In other words, uncertain set is a set-valued function on an uncertainty space. Thus the main difference between uncertain set and uncertain variable is that the former takes values of set and the latter takes values of point. The concepts of membership function and uncertainty distribution are two basic tools to describe uncertain sets, where membership function is intuitionistic for us but frangible for arithmetic operations, and uncertainty distribution is hard-to-understand for us but easy-to-use for arithmetic operations. Fortunately, an uncertainty distribution may be uniquely determined by a membership function. In practice, we first determine membership functions for uncertain sets, and convert membership functions to uncertainty distributions. Then we perform arithmetic operations on uncertain sets via uncertainty distributions rather than membership functions. Chapter 10 will provide an uncertain set theory that is a generalization of uncertainty theory to the domain of uncertain sets.

Some knowledge and evidence in human brain are actually uncertain sets rather than fuzzy sets or random sets. This fact encourages us to propose

a theory of uncertain inference that is a process of deriving consequences from uncertain knowledge or evidence via the tool of conditional uncertain set. Chapter 11 will present an inference rule with applications to uncertain system and inference control.

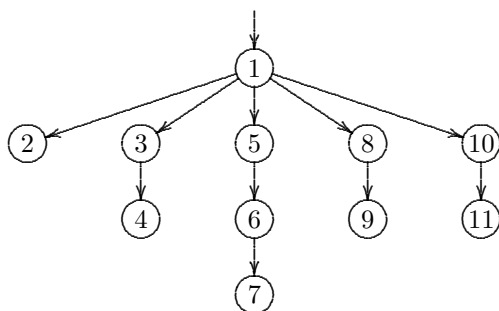
The book is suitable for mathematicians, researchers, engineers, designers, and students in the field of mathematics, information science, operations research, system science, industrial engineering, computer science, artificial intelligence, finance, control, and management science. The readers will learn the axiomatic approach of uncertainty theory, and find this work a stimulating and useful reference.

Lecture Slides

If you need lecture slides for uncertainty theory, please download them from the website at <http://orsc.edu.cn/liu/resources.htm>.

A Guide for the Reader

The readers are not required to read the book from cover to cover. The logic dependence of chapters is illustrated by the figure below.



Acknowledgment

I am indebted to a series of grants from from National Natural Science Foundation, Ministry of Education, and Ministry of Science and Technology of China. I also express my deep gratitude to Professor Janusz Kacprzyk for the invitation to publish this book in his series, and Dr. Thomas Ditzinger of Springer for his wonderful cooperation and helpful comments.

February 2010

Baoding Liu
Tsinghua University
<http://orsc.edu.cn/liu>

To My Wife Jinlan