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Baoding Liu

Uncertainty Theory

A Branch of Mathematics for Modeling Human Uncertainty

Springer

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Contents

Preface

Some information and knowledge are usually represented by human language like "about 100km", "approximately 39 °C", "roughly 80kg", "low speed", "middle age", and "big size". Perhaps some people think that they are subjective probability or they are fuzziness. However, a lot of surveys showed that those imprecise quantities behave neither like randomness nor like fuzziness. How do we understand them? How do we model them? These questions provide a motivation to invent another mathematical tool to model those imprecise quantities. In order to do so, an uncertainty theory was founded and became a branch of axiomatic mathematics. Since then, uncertainty theory has been developed steadily and applied widely.

Chapter 1 is devoted to the uncertainty theory. The first fundamental concept in uncertainty theory is uncertain measure that is used to measure the truth degree of an uncertain event. The second one is uncertain variable that is used to represent imprecise quantities. The third one is uncertainty distribution that is used to describe uncertain variables in an incomplete but easy-to-use way. Uncertainty theory is thus deduced from those three foundation stones, and plays the role of mathematical model to deal with uncertain phenomena.

Uncertain programming is a type of mathematical programming involving uncertain variables. A key problem in uncertain programming is how to rank uncertain variables, and different ranking criteria produce different classes of uncertain programming. Chapter 2 will introduce four ranking criteria and then provide a spectrum of uncertain programming with applications to project scheduling problem, vehicle routing problem and machine scheduling problem.

The term risk has been used in different ways in literature. In this book the risk is defined as the accidental loss plus the uncertain measure of such loss, and a risk index is defined as the uncertain measure that some specified loss occurs. Chapter 3 will introduce uncertain risk analysis that is a tool to quantify risk via uncertainty theory.

Reliability index is defined as the uncertain measure that some system is working. Thus reliability and risk have the same root in mathematics. They are separately treated for application convenience in practice rather than theoretical demand. Chapter 4 will introduce uncertain reliability analysis that is a tool to deal with system reliability via uncertainty theory.

An uncertain process is essentially a sequence of uncertain variables indexed by time or space. Thus an uncertain process is usually used to model

uncertain phenomena that vary with time or space. Some basic concepts of uncertain process will be presented in Chapter 5.

Uncertain calculus is a branch of mathematics that deals with differentiation and integration of function of uncertain processes. As the very core of uncertain calculus, canonical process is a Lipschitz continuous uncertain process that has stationary and independent increments and every increment is a normal uncertain variable. Chapter 6 will introduce the uncertain calculus including canonical process, uncertain integral and chain rule.

Uncertain differential equation is a type of differential equation driven by canonical process. Chapter 7 will discuss the existence, uniqueness and stability of solutions of uncertain differential equations, and will design a numerical method for solving monotone uncertain differential equations. This chapter will also present an application of uncertain differential equation in finance.

Uncertain logic is a generalization of mathematical logic for dealing with uncertain knowledge via uncertainty theory. A key point in uncertain logic is that the truth value of an uncertain proposition is defined as the uncertain measure that the proposition is true. One advantage of uncertain logic is the well consistency with classical logic. In other words, uncertain logic obeys the law of truth conservation, and is consistent with the law of excluded middle and the law of contradiction. Chapter 8 will discuss uncertain propositional logic and uncertain predicate logic.

Uncertain entailment is a methodology for calculating the truth value of an uncertain formula via the maximum uncertainty principle when the truth values of other uncertain formulas are given. That is, we will assign an uncertain formula a truth value as close to 0.5 as possible. Chapter 9 will introduce an entailment model from which modus ponens, modus tollens and hypothetical syllogism are deduced.

Uncertain set is a measurable function from an uncertainty space to a collection of sets. In other words, uncertain set is a set-valued function on an uncertainty space. Thus the main difference between uncertain set and uncertain variable is that the former takes values of set and the latter takes values of point. The concepts of membership function and uncertainty distribution are two basic tools to describe uncertain sets, where membership function is intuitionistic for us but frangible for arithmetic operations, and uncertainty distribution is hard-to-understand for us but easy-to-use for arithmetic operations. Fortunately, an uncertainty distribution may be uniquely determined by a membership function. In practice, we first determine membership functions for uncertain sets, and convert membership functions to uncertainty distributions. Then we perform arithmetic operations on uncertain sets via uncertainty distributions rather than membership functions. Chapter 10 will provide an uncertain set theory that is a generalization of uncertainty theory to the domain of uncertain sets.

Some knowledge and evidence in human brain are actually uncertain sets rather than fuzzy sets or random sets. This fact encourages us to propose

a theory of uncertain inference that is a process of deriving consequences from uncertain knowledge or evidence via the tool of conditional uncertain set. Chapter 11 will present an inference rule with applications to uncertain system and inference control.

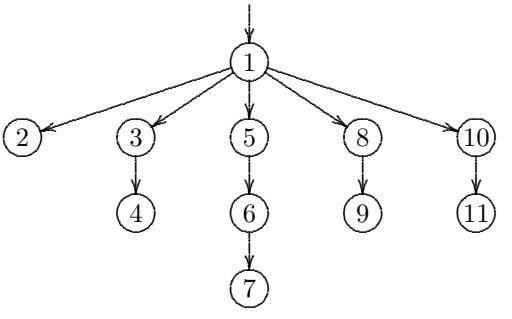
The book is suitable for mathematicians, researchers, engineers, designers, and students in the field of mathematics, information science, operations research, system science, industrial engineering, computer science, artificial intelligence, finance, control, and management science. The readers will learn the axiomatic approach of uncertainty theory, and find this work a stimulating and useful reference.

Lecture Slides

If you need lecture slides for uncertainty theory, please download them from the website at http://orsc.edu.cn/liu/resources.htm.

A Guide for the Reader

The readers are not required to read the book from cover to cover. The logic dependence of chapters is illustrated by the figure below.



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February 2010

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To My Wife Jinlan