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Sparse Grid Quadrature in High Dimensions with Applications in Finance and Insurance

Springer

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Preface

This text deals with the numerical analysis and efficient numerical treatment of high-dimensional integrals using sparse grids and related dimension-wise integration techniques and discusses applications to finance and insurance. The focus on sparse grid quadrature methods on the one hand is thereby complemented by larger parts of this text which are devoted to dimension-wise decompositions of high-dimensional functions, such as the ANOVA or the anchored-ANOVA, on the other hand. The main intention is to cover these two areas of research, which have attracted independently of each other considerable attention in recent years, in a self-contained, easily accessible and unified way. In particular the interplay between the convergence behaviour of sparse grid methods with effective dimensions (which are a measure for the decay of the dimension-wise decompositions) and with coordinate transformations (which aim to improve the decay of the dimension-wise decompositions) is studied. The text moreover aims to investigate potential benefits but also limitations of these techniques with respect to applications from mathematical finance and insurance and to give some recommendations based on the theoretical and numerical results presented in the manuscript.

This manuscript mainly originated during my time from July 2004 to January 2009 at the Institute for Numerical Simulation at the University of Bonn where I worked in the area of sparse grid methods and high-dimensional integration problems. I had the chance to participate in several research projects, in which we investigated, partly in close collaboration with financial institutions, the use of these methods for applications from financial engineering and insurance. In addition I was involved in the years 2004–2008 in the teaching of the laboratory "Computational Finance" that was offered for master students with special focus on the computational aspects of option pricing problems from mathematical finance. This manuscript summarizes material and results which have been developed during these different research projects and teaching activities in this period of time.

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