

Zsófia Lendek, Thierry Marie Guerra, Robert Babuška, and Bart De Schutter

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Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy  
Models

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# Preface

Many problems in decision making, monitoring, fault detection, and control rely on the knowledge of state variables and time-varying parameters that are not directly measured by sensors. In such situations, observers, or estimators, can be employed that use the measured input and output signals along with a dynamic model of the system in order to estimate the unknown states or parameters. An essential requirement in designing an observer is to guarantee the convergence of the estimates to the true values or at least to a small neighborhood of the true values. For linear models, a wide array of estimation techniques are available, such as the Kalman filter and its variants. However, no general method exists for the design of estimators for nonlinear systems. The design and tuning of a nonlinear observer is generally complicated and involves large computational costs.

This book provides a range of methods and tools to design observers for nonlinear systems represented by a special type of a dynamic nonlinear model – the Takagi–Sugeno (TS) fuzzy model. The TS model is a convex combination of affine linear models. This structure facilitates stability analysis and observer design by using effective algorithms based on Lyapunov functions and linear matrix inequalities. TS models are known to be universal approximators and, in addition, a broad class of nonlinear systems can be exactly represented as a TS system.

In the fuzzy systems literature, observer design is typically considered as a dual problem to control design, and as such it has not received much attention yet. This book aims at filling this gap by addressing observer design for TS systems in its own right, with a special attention to large-scale, decentralized systems. To this end, three particular structures of large-scale TS models are considered: cascaded systems, distributed systems, and systems affected by unknown disturbances. The reader will find in-depth theoretical analysis accompanied by illustrative examples and simulations of real-world systems. Stability analysis of TS models is also addressed in detail.

The intended audience are graduate students and researchers both from academia and industry. For newcomers to the field, the book provides a concise introduction dynamic TS fuzzy models along with two methods to construct TS models for a given nonlinear system.

While this monograph focusses mainly on the theory and methodology of state and parameter estimation in nonlinear distributed dynamic systems, the methods presented can readily be used in applications such as industrial processes, traffic systems, environmental systems, energy and water distribution networks, and so on.

Supplementary information relevant to this book is available at the website:

<http://www.dsc.tudelft.nl/fuzzybook/>

Comments, suggestions, or questions concerning the book or the website are welcome. Interested readers are encouraged to get in touch with the authors using the contact information on the website.

We thank Janusz Kacprzyk, the series editor, for giving us the opportunity to publish our book with Springer, and the editorial and production team at Springer, especially Thomas Ditzinger, for their valuable help. We gratefully acknowledge the financial support of the BSIK-ICIS project Interactive Collaborative Information Systems (grant no. BSIK03024 of Senter, Ministry of Economic Affairs of the Netherlands), as well as of the International Campus on Safety and Intermodality in Transportation, the Nord-Pas-de-Calais Region, the European Community, the Regional Delegation for Research and Technology, the French Ministry of Higher Education and Research.

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Zsófia Lendek  
Thierry Marie Guerra  
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# Acronyms

|      |  |
|------|--|
| TS   | Takagi-Sugeno                            |
| LMI  | Linear Matrix Inequality                 |
| ISS  | Input-to-State Stable                    |
| GAS  | Globally Asymptotically Stable           |
| UGAS | Uniformly Globally Asymptotically Stable |
| PDC  | Parallel Distributed Compensation        |
| SISO | Single-Input Single-Output               |
| MIMO | Multi-Input Multi-Output                 |
| FP   | Feasibility Problem                      |
| EVP  | Eigenvalue Problem                       |
| GEVP | Generalized Eigenvalue Problem           |