Jiuping Xu and Xiaoyang Zhou

Fuzzy-Like Multiple Objective Decision Making

Studies in Fuzziness and Soft Computing, Volume 263

Editor-in-Chief

Prof. Janusz Kacprzyk
Systems Research Institute
Polish Academy of Sciences
ul. Newelska 6
01-447 Warsaw
Poland
E-mail: kacprzyk@ibspan.waw.pl

Further volumes of this series can be found on our homepage: springer.com

Vol. 248. Bing-Yuan Cao
Optimal Models and Methods with Fuzzy Quantities, 2010
ISBN 978-3-642-10710-8

Vol. 249. Bernadette Bouchon-Meunier, Luis Magdalena, Manuel Ojeda-Aciego, José-Luis Verdegay, Ronald R. Yager (Eds.)
Foundations of Reasoning under Uncertainty, 2010
ISBN 978-3-642-10726-9

Vol. 250. Xiaoxia Huang
Portfolio Analysis, 2010
ISBN 978-3-642-11213-3

Vol. 251. George A. Anastassiou
Fuzzy Mathematics: Approximation Theory, 2010
ISBN 978-3-642-11219-5

Vol. 252. Cengiz Kahraman, Mesut Yavuz (Eds.)
Production Engineering and Management under Fuzziness, 2010
ISBN 978-3-642-12051-0

Vol. 253. Badredine Arfi
Linguistic Fuzzy Logic Methods in Social Sciences, 2010
ISBN 978-3-642-13342-8

Vol. 254. Weldon A. Lodwick, Janusz Kacprzyk (Eds.)
Fuzzy Optimization, 2010
ISBN 978-3-642-13934-5

Vol. 255. Zongmin Ma, Li Yan (Eds.)
Soft Computing in XML Data Management, 2010
ISBN 978-3-642-14009-9

Vol. 256. Robert Jeansoulin, Odile Papini, Henri Prade, and Steven Schockaert (Eds.)
Methods for Handling Imperfect Spatial Information, 2010
ISBN 978-3-642-14754-8

Vol. 257. Salvatore Greco, Ricardo Alberto Marques Pereira, Massimo Squillante, Ronald R. Yager, and Janusz Kacprzyk (Eds.)
*Preferences and Decisions,*2010
ISBN 978-3-642-15975-6

Vol. 258. Jorge Casillas and Francisco José Martínez López
Marketing Intelligent Systems Using Soft Computing, 2010
ISBN 978-3-642-15605-2

Vol. 259. Alexander Gegov
Fuzzy Networks for Complex Systems, 2010
ISBN 978-3-642-15599-4

Vol. 260. Jordi Recasens
Indistinguishability Operators, 2010
ISBN 978-3-642-16221-3

Vol. 261. Chris Cornelis, Glad Deschrijver, Mike Nachtegael, Steven Schockaert, and Yun Shi (Eds.)
35 Years of Fuzzy Set Theory, 2010
ISBN 978-3-642-16628-0

Vol. 262. Zsófia Lendek, Thierry Marie Guerra, Robert Babuška, and Bart De Schutter
Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models, 2010
ISBN 978-3-642-16775-1

Vol. 263. Jiuping Xu and Xiaoyang Zhou
Fuzzy-Like Multiple Objective Decision Making, 2010
ISBN 978-3-642-16894-9

Jiuping Xu and Xiaoyang Zhou

Fuzzy-Like Multiple Objective Decision Making

Springer

Authors

Prof. Dr. Dr. Jiuping Xu
Uncertainty Decision-Making Laboratory
Sichuan University
610064 Chengdu
P. R. China

Assis. Prof. Xiaoyang Zhou
Uncertainty Decision-Making Laboratory
Sichuan University
610064 Chengdu
P. R. China

ISBN 978-3-642-16894-9 e-ISBN 978-3-642-16895-6

DOI 10.1007/978-3-642-16895-6

Studies in Fuzziness and Soft Computing ISSN 1434-9922

Library of Congress Control Number: 2010938617

© 2011 Springer-Verlag Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typeset & Cover Design: Scientific Publishing Services Pvt. Ltd., Chennai, India.

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Preface

Everyday people need to make decisions, and the decision makers usually face multiple, conflicting objectives and uncertain environments. The research about uncertain multi-objective decision making problems has been profolic. It mainly provides decision makers with the help to find optimal solutions for many objectives under uncertain environments, and has been a permanent focus for many years.

To trace the origins of the multi-objective decision making with certain parameters, we have to go back over the eighteenth century. B. Franklin introduced how to coordinate multiple objectives in 1772. A. A. Cournot proposed the multi-objective decision making model from the standpoint of the economics in 1836. V. Pareto firstly presented the optimal solution to the multi-objective decision making model from the standpoint of the mathematics in 1896 and then K. J. Arrow et al. proposed the concept of efficient points. Traditional multi-objective decision making is only aimed at problems with certain parameters, but as we know, it is usual that many decision making problems have uncertain factors. As people know more and more about the uncertain event, the research about random multi-objective decision making, fuzzy multi-objective decision making, rough multi-objective decision making and two-fold uncertain multi-objective decision making problems were gradually developed.

In the last 25 years, fuzzy set theory has been applied to many disciplines such as operations research, management science, control theory, artificial intelligent/expert systems, human behavior, etc. Growth of the applications of fuzzy set theory have been accumulating. In 1978, H. Kwakernaak combined randomness with fuzziness and initialized the concept of the fuzzy random variable, then introduced its basic definition and properties. This viewpoint which combined two different uncertain variables to describe complicated events received approval from many scholars and move forward a further step to uncertain events. Then many papers and books about the two-fold uncertain theory presented more and more, and therefore promoted the development of two-fold uncertain theory. This monograph presents systematically state-of-art of fuzzy-like multiple objective mathematical programming in both techniques and applications.

In real life, input data is usually imprecise or uncertain because of incomplete or non-obtainable information. So if we want to describe the uncertainty, especially subjective uncertainty, fuzzy variables will be used. For instance, If we ask people about the unit transport cost between two places, they say that the cost will be around a value, and they cannot give an absolute certain value. Since the information is described in linguistic terms, and not by the chance concept, conventional probability may not be a correct way to model these imprecise nature, so it can be derived that the unit transport cost could be a fuzzy variable rather than a constant. It is appropriate for us to use a LR fuzzy variables $(a,\theta,b)_{LR}$ to describe, where a,b are the left and right widths of the fuzzy variable, and θ is the middle value. Further more, we consider about the middle value θ, due to the conditions of road, traffic and weather, etc., this value may not be the constant either, but here may be a random variable or fuzzy variable or rough variable, thus is regarded as a fuzzy-like two-fold uncertain variable. This book mainly concentrates on one type of two-fold uncertain theory, that is, the fuzzy-like two-fold uncertain variable including fuzzy random variables, bifuzzy variables and fuzzy rough variables, and then deduces their properties and the application to the real world.

In the classical multi-objective decision making model, all data and information are assumed to be absolutely accurate, and the objectives and constraints are all assumed to be well expressed by mathematical formations. However, it is difficult to clearly describe the objective functions and constraints by mathematical equations for many realistic problems, thus, the multi-objective decision making model with certain parameters cannot deal with all real-life problems. So uncertain multi-objective decision making models are proposed. In 1965, after L.A. Zadeh proposed the fuzzy set theory, it was rapidly and widely applied in the filed of operations, management science, control theory and so on. In 1970, R. E. Bellman and L. A. Zadeh collaborated to propose the fuzzy decision making model based on multi-objective programming. That's the original work which is the basis of this monograph.

The multi-objective decision making model with fuzzy-like variables can be summarized as follows:

$$\begin{cases} \max\ f(x,\xi) = [f_1(x,\xi), f_2(x,\xi), \cdots, f_m(x,\xi)] \\ \text{s.t.} \begin{cases} g_r(x,\eta) \le 0, r = 1,2,\cdots,p \\ x \in X, \end{cases} \end{cases}$$

where x is an n-dimensional decision variable, f is the objective function, ξ and η are both fuzzy-like variables.

In this book, we take real-life problems as background and guidance, and present the application of the fuzzy-like multi-objective decision making (FLMODM) model to the real world, and finally construct a relative model. After that, basic theories, models and algorithms are proposed and applied to solve realistic problems. This book consists of 6 chapters. Chapter 1 reviews some relative preliminary knowledge such as fuzzy set, fuzzy numbers, fuzzy arithmetic and membership functions. Chapter 2 mainly introduces multi-objective decision making with fuzzy parameters and its application to farm structure optimization problem. Three

sections introduce the fuzzy expected value model (Fuzzy EVM), the fuzzy chance-constrained model (Fuzzy CCM) and the fuzzy dependent-chance model (Fuzzy DCM). In each section, the equivalent model of those problems which have some special fuzzy variables are deduced and the traditional solution methods for the crisp equivalent models are introduced the technique of fuzzy simulations-based hybrid algorithms to deal with the general decision making problems under fuzzy environments. Finally, in the next section, the proposed models and algorithms have been applied to solve the realistic problems introduced in the first section. Chapters 3, 4 and 5 have the same structure as Chapter 2. Chapter 3 proposes a multi-objective decision making model with fuzzy random parameters and introduces its application to portfolio selection problems under a fuzzy random environment. The equivalent models of the fuzzy random expected value model (Fu-Ra EVM), fuzzy random chance-constrained model (Fu-Ra CCM) and fuzzy random dependent-chance model (Fu-Ra DCM) are deduced. Chapter 4 proposes the multi-objective decision making model with a bifuzzy parameters and introduces its application to material purchasing problems under a bifuzzy environment. The equivalent models of the bifuzzy expected value model (Fu-Fu EVM), bifuzzy chance-constrained model (Fu-Fu CCM) and bifuzzy dependent-chance model (Fu-Fu DCM) are deduced. Chapter 5 proposes a multi-objective decision making model with fuzzy rough parameters and introduces its application to logistics problems under a fuzzy rough environment. The equivalence models of the fuzzy rough expected value model (Fu-Ro EVM), fuzzy rough chance-constrained model (Fu-Ro CCM) and fuzzy rough dependent-chance model (Fu-Ro DCM) are deduced. Finally in Chapter 6, the main problems are concluded, models, methods and algorithms to obtain the organic systems are discussed, and a methodological system for the FLMODM is proposed.

The authors wish to thank the support from the National Science Foundation for Distinguished Young Scholars, P. R. China (Grant No. 70425005) and the National Natural Science Foundation of P. R. China (Grant No. 79760060, and No. 70171021). The authors also wish to acknowledge the assistance of Prof. Janusz Kacprzyk for his helpful comments and recommendations to publish this book in this series. This book benefited many literatures, and the authors also wish to take this opportunity to thank these researchers here. For discussion and advice, the authors are grateful to researchers from the Uncertainty Decision-Making Laboratory of Sichuan University, J. Li, Y. Liu, Z. Tao, L. Yao and others, who have done a lot of work in this field and have made a number of corrections. Finally the authors express their deepest gratitude to the staff of Springer, especially, Dr. Thomas Ditzinger and Renuka Devi for the wonderful cooperation.

Sichuan University, Jiuping Xu
June 2009 Xiaoyang Zhou

Contents

1 Fuzzy Set Theory 1
1.1 Fuzzy Sets 2
1.2 Basic Concepts of Fuzzy Sets 7
1.3 Fuzzy Number 14
1.4 Fuzzy Arithmetic 17
1.5 Membership Function 37
1.6 Possibility Distribution 51

2 Fuzzy Multiple Objective Decision Making 57
2.1 Farm Structure Optimization Problem under Fuzzy Environment 57
2.2 Fuzzy Variable 59
2.2.1 Definition of Fuzzy Variable 59
2.2.2 Possibility, Necessity and General Fuzzy Measure 64
2.2.3 Expected Value Operator of Fuzzy Variables 69
2.3 Fuzzy EVM 77
2.3.1 General Model for Fuzzy EVM 77
2.3.2 Linear Fuzzy EVM and Two-Stage Method 79
2.3.3 Non-linear Fuzzy EVM and Fuzzy Simulation-Based PSO 84
2.4 Fuzzy CCM 92
2.4.1 General Model for Fuzzy CCM 92
2.4.2 Linear Fuzzy CCM and Goal Programming Method 95
2.4.3 Non-linear Fuzzy CCM and Fuzzy Simulation-Based PSO with Preference Order 107
2.5 Fuzzy DCM 111
2.5.1 General Model for Fuzzy DCM 111
2.5.2 Linear Fuzzy DCM and Ideal Point Method 114

2.5.3 Non-linear Fuzzy DCM and Fuzzy Simulation-Based PSO with Shrinkage Factor . . . 119
2.6 Application of Farm Structure Optimization Problem . . . 122
2.7 Multi-objective Decision Making with Fuzzy Relations . . . 128
2.7.1 Multi-objective Decision Making with Fuzzy Constraints . . . 128
2.7.2 Multi-objective Decision Making with Fuzzy Objectives . . . 131

3 Fuzzy Random Multiple Objective Decision Making . . . 135
3.1 Portfolio Selection Problem under a Fuzzy Random Environment . . . 136
3.2 Fu-Ra Variable . . . 138
3.2.1 Definition of Fu-Ra Variable . . . 138
3.2.2 Expected Value Operator of Fu-Ra Variables . . . 140
3.2.3 Chance Operator of Fu-Ra Variables . . . 142
3.3 Fu-Ra EVM . . . 144
3.3.1 General Model for Fu-Ra EVM . . . 144
3.3.2 Linear Fu-Ra EVM and Fuzzy Satisfied Method . . . 147
3.3.3 Non-linear Fu-Ra EVM and Fu-Ra Simulation-Based GA . . . 153
3.4 Fu-Ra CCM . . . 168
3.4.1 General Model for Fu-Ra CCM . . . 168
3.4.2 Linear Fu-Ra CCM and Surrogate Worth Trade-Off Method . . . 170
3.4.3 Non-linear Fu-Ra CCM and Fu-Ra Simulation-Based Adaptive Weight GA . . . 180
3.5 Fu-Ra DCM . . . 186
3.5.1 General Model for Fu-Ra DCM . . . 187
3.5.2 Linear Fu-Ra DCM and Satisfying Trade-Off Method . . . 189
3.5.3 Non-linear Fu-Ra DCM and Fu-Ra Simulation-Based Compromise GA . . . 198
3.6 Application to the Portfolio Selection Problem . . . 202
3.6.1 Assumptions . . . 202
3.6.2 Chance Constrained Portfolio Selection Model . . . 204
3.6.3 λmean-Variance Portfolio Selection Model . . . 207
3.7 Another Way to Deal with Fuzzy Random Multi-objective Decision Making Model . . . 215
3.7.1 (α,β)-Satisfied Solution for Fu-Ra Multi-objective Decision Making Model . . . 216
3.7.2 Application to Inventory Problems . . . 220

4 Bifuzzy Multiple Objective Decision Making . . . 227
4.1 Raw Material Purchasing Problem under Bifuzzy Environment . . . 227
4.2 Fu-Fu Variable . . . 229

4.2.1 Level-2 Fuzzy Set, Type-2 Fuzzy Sets and Fu-Fu Variable . . . 229
4.2.2 Expected Value Operator of Fu-Fu Variables . . . 231
4.2.3 Chance Operator of Fu-Fu Variables . . . 232
4.3 Fu-Fu EVM . . . 233
4.3.1 General Model for Fu-Fu EVM . . . 233
4.3.2 Linear Fu-Fu EVM and Step Method . . . 234
4.3.3 Nonlinear Fu-Fu EVM and Fu-Fu Simulation Based SA . . . 244
4.4 Fu-Fu CCM . . . 249
4.4.1 General Model for Fu-Fu CCM . . . 249
4.4.2 Linear Fu-Fu CCM and Lexicographic Method . . . 251
4.4.3 Nonlinear Fu-Fu CCM and Fu-Fu Simulation Based Parallel SA . . . 260
4.5 Fu-Fu DCM . . . 265
4.5.1 General Model for Fu-Fu DCM . . . 265
4.5.2 Linear Fu-Fu DCM and Weight Sum Method . . . 266
4.5.3 Nonlinear Fu-Fu DCM and Fu-Fu Simulation Based Adaptive SA . . . 271
4.6 Application to Purchasing Problem in a Large-Scale Integrated Steel Plant . . . 277
4.6.1 Modelling . . . 278
4.6.2 Data Collection . . . 281
4.6.3 Model Processing Specification . . . 281
4.7 Another Way to Deal with Fu-Fu Multi-objective Decision Making Model . . . 286

5 Fuzzy Rough Multiple Objective Decision Making . . . 295
5.1 Integrated Logistics Problem under Fuzzy Rough Environment . . . 295
5.2 Fu-Ro Variable . . . 298
5.2.1 Definition of Fu-Ro Variable . . . 298
5.2.2 Expected Value Operator of Fu-Ro Variables . . . 308
5.2.3 Chance Operator of Fu-Ro Variables . . . 311
5.3 Fu-Ro EVM . . . 312
5.3.1 General Model for Fu-Ro EVM . . . 312
5.3.2 Linear Fu-Ro EVM and Minimax Point Method . . . 314
5.3.3 Non-linear Fu-Ro EVM and Fu-Ro Simulation-Based TS . . . 320
5.4 Fu-Ro CCM . . . 327
5.4.1 General Model for Fu-Ro CCM . . . 328
5.4.2 Linear Fu-Ro CCM and Fuzzy Goal Method . . . 330
5.4.3 Non-linear Fu-Ro CCM and Fu-Ro Simulation-Based Parametric TS . . . 341
5.5 Fu-Ro DCM . . . 347
5.5.1 General Model for Fu-Ro DCM . . . 347
5.5.2 Linear Fu-Ro DCM and ε-Constraint Method . . . 349

5.5.3 Non-linear Fu-Ro DCM and Fu-Ro Simulation Based Parallel TS . . . 358
5.6 Application to Integrated Logistics Network Design Problem . . . 364
5.6.1 Modelling for Integrated Reuse Logistics Network under Fuzzy Rough Environment . . . 365
5.6.2 Uncertain Linear Multi-objective Model . . . 368
5.6.3 Application to Beer Company . . . 370

6 Methodological System for FLMODM . . . 375
6.1 Motivation of Researching FLMODM . . . 376
6.2 Physics-Based Model System . . . 378
6.3 Mathematical Model System . . . 379
6.4 Model Analysis System . . . 383
6.5 Algorithm System . . . 388
6.6 Research Ideas and Paradigm: 5MRP . . . 392

References . . . 397

A Procedures . . . 417
A.1 Procedures for Example 2.6 . . . 417
A.2 Procedures for Fu-Ra Portfolio Selection Problem . . . 423

Index . . . 439

List of Figures

1.1 Graphic explanation for the fuzzy set of good shooter 3
1.2 Derived degree of membership for the fuzzy set A 4
1.3 The membership(possibility) function of the interest rate 5
1.4 The fuzzy set "real numbers close to 10" 6
1.5 An α-set 10
1.6 α-cut of LR fuzzy number 11
1.7 α-cut of triangular fuzzy number 11
1.8 A convex fuzzy set 12
1.9 A discrete fuzzy number M 15
1.10 A continuous fuzzy number M 15
1.11 LR type fuzzy numbers. 16
1.12 Triangular and trapezoidal fuzzy numbers. 17
1.13 Extension principle 20
1.14 Extension principle with $r = 1$ 21
1.15 The fuzzy numbers M, N, $M(+)N$, and $M(-)N$ 24
1.16 Fuzzy number $M(\cdot)N$ 30
1.17 Example of the fuzzy max 32
1.18 Example of the fuzzy min 32
1.19 Example of the fuzzy min 34
1.20 Calibration of the interval for measurement 42
1.21 Monotonic membership functions of "very young man," "young man," "old man" and "very old man" 44
1.22 Unimodal membership functions of "very young man" and "young man" 44
1.23 Payoff function of the assertion "Yao is tall" 50

2.1 The membership function curve of trapezoidal fuzzy variable (r_1, r_2, r_3, r_4) 62
2.2 The membership function curve of triangular fuzzy variable (r_1, r_2, r_3) 62
2.3 The membership function curve of centeral triangular fuzzy variable $(a, \frac{a+b}{2}, b)(a < b)$ 63

2.4 Conclusions of Theorem 2.2 66
2.5 A trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, r_4)$ 69
2.6 Flow chart of PSO 89
2.7 Flow chart of Fu-Ra simulation-based GA 91
2.8 The relationship of fuzzy sets C, F and D 128

3.1 Fuzzy random portfolio selection problem 138
3.2 Primitive chance curve $Ch\{f_j(\xi) \leq 0, j = 1, 2, \cdots, m\}$ 143
3.3 Search in a fixed direction in criterion space 161
3.4 Search in multiple directions in criterion space 162
3.5 Flow chart of Fu-Ra simulation-based GA 166
3.6 Convergency curve 168
3.7 Adaptive weights and adaptive hyperplane 184
3.8 LR-type fuzzy random return $\tilde{\bar{r}}_j$ 203
3.9 The relationship of different λ-mean variance efficient portfolios 212
3.10 λ efficient frontier and mean-variance efficient frontier (I) 213
3.11 λ efficient frontier and mean-variance efficient frontier (II) 214
3.12 λ efficient frontier and mean-variance efficient frontier (III) 214
3.13 λ efficient frontiers 215
3.14 The probability density function of fuzzy random variable ρ 218
3.15 The membership function of $\tilde{\omega}_{\tilde{\xi}}$ 219
3.16 A multi-item inventory system 221

4.1 Raw materials supply system 228
4.2 Triangular Fu-Fu variable 231
4.3 Basic steps for simulated annealing 247
4.4 Detailed partial figure 247
4.5 Time line of the MMC parallel SA 262
4.6 Flow chart of the MMC parallel SA 263
4.7 Flow chart of the adaptive simulated annealing. 273
4.8 Transformation process from the Fu-Fu variable to the (α_1, α_2)-level fuzzy variable 290

5.1 Integrated logistics system 297
5.2 Conceptual model of reuse integrated logistics network 298
5.3 Apartment 304
5.4 Tangent 305
5.5 Intersection 305
5.6 Inclusion 305
5.7 Rough approximation 307
5.8 Layout of tabu search 322
5.9 Partition of current solution neighborhood 322
5.10 A standard ”backtracking”(depth first) branch-and-bound approach 325

5.11 TTS approach 325
5.12 Hierarchical classification of parallel TS strategies. 360
5.13 Architecture of the parallel adaptive TS. 362

6.1 Development of FLMODM 376
6.2 Problem under fuzzy-like environment 378
6.3 Fuzzy-like model system 381
6.4 Transformation to crisp equivalent models 384
6.5 Fuzzy-like hybrid algorithm system 389
6.6 Algorithm design 391
6.7 Ideal route 392
6.8 Problem system 393
6.9 Model system 393
6.10 Research system 394
6.11 Methodological system for FLMODM 395

List of Tables

1.1 Solutions of example 1.9 9
1.2 Fuzzy set A in Example 1.17 13
1.3 Second power of A 13
1.4 Fuzzy set A and B in Example 1.18 18
1.5 Fuzzy set A_1 and A_2 in Example 1.19 19
1.6 Composition of A_1 and A_2 in Example 1.19 20
1.7 Possible (x_1, x_2) pairs in Example 1.19 20
1.8 $\mu_{A_1}(x_1)$ and $\mu_{A_2}(x_2)$ in Example 1.19 20
1.9 Fuzzy numbers M and N 23
1.10 Addition of fuzzy numbers M and N 23
1.11 Membership values, and $\mu_M(x) \wedge \mu_N(y)$ 24
1.12 α-cut values 25
1.13 Result of $M(-)N$ 26
1.14 Result of $(\mu_M(x) \wedge \mu_N(y))$ 27
1.15 α-cut values 27
1.16 Membership value of z 29
1.17 Membership value $\mu_{M(\cdot)N}(14)$ 29
1.18 Fuzzy numbers M and N 33
1.19 Fuzzy max of M and N 33
1.20 (x, y) pairs that satisfy $3 = x \vee y$ 34
1.21 Minimum of M and N 34
1.22 Fuzzy min of M and N 34
1.23 Algebraic operation for $M = (m, \alpha, \beta)$, $N = (n, \gamma, \delta)$ 35
1.24 Algebraic operation for $M = (a, b, \alpha, \beta)$, $N = (c, d, \gamma, \delta)$ 35
1.25 Algebraic operation for $M = (l, m, n)$, $N = (a, b, c)$ 36
1.26 Algebraic operation for $M = (a_1, b_1, c_1, d_1)$, $N = (a_2, b_2, c_2, d_2)$ 36
1.27 Heights and true values of $A =$ "Yao is tall" and $B =$ "Yi is tall" 47
1.28 The possibility and probability distributions associated with X 54

2.1 Results obtained from Interactive process 107
2.2 Constraints set 124

2.3 Fuzzy coefficients of constraints 125
2.4 Fuzzy cost coefficients 125
2.5 Payoff table 132
2.6 Payoff table of example 2.12 133

3.1 Results obtained from interactive process 152
3.2 Results obtained from Fu-Ra simulation based-random weight GA 168
3.3 Employ the interactive fuzzy satisfied method based on possibility($\gamma_i = 0.9, \delta_i = 0.9$) 179
3.4 Employ the interactive fuzzy satisfied method based on necessity($\gamma_i = 0.9, \delta_i = 0.9$) 180
3.5 The expected return rates, left and right spreads of 30 stocks (%) 206
3.6 Investment scale for optimistic investor ($\gamma = 0.6, \delta = 0.6$) 206
3.7 Investment scale for pessimistic investor ($\gamma = 0.6, \delta = 0.6$) 207
3.8 Investor's subjective optimistic degrees for the returns of 30 stocks (I) 213
3.9 Investor's subjective optimistic degrees for the returns of 30 stocks (II) 214
3.10 Investor's subjective optimistic degrees for the returns of 30 stocks (III) 214

4.1 Payoff table 241
4.2 Unit purchasing cost of the jth item from the ith vendor 282
4.3 Tardy-delivery ratio of the jth item from the ith vendor 283
4.4 Scrap fraction of the jth item from the ith vendor 284
4.5 Scrap fraction of the jth item from the ith vendor 285

5.1 Listing of the ECTS parameters 323
5.2 Another taxonomy dimension for parallel TS algorithms 327
5.3 The resource demand in producing process 339
5.4 The result computed by parallel TS algorithm at different weights 347
5.5 Another taxonomy dimension for parallel TS algorithms 361
5.6 The result computed by parametric TS algorithm 364
5.7 Amount of recycling and demand 370
5.8 Default processing cost and time limit (h) of every wholesaler 371
5.9 The expected value of transport cost, and time (h) from collectors to recyclers 371
5.10 The expected value of transport cost, and time (h) from collectors to distributors (recycled bottles) 371
5.11 The expected value of transport cost, and time (h) from distributors to wholesalers (products) 372

5.12 Transport cost and time from producers to distributors (recycled bottles) 372
5.13 Transport cost and time from distributors to producers (products) 372
5.14 The expected value of transport cost from recyclers to producers 372
5.15 Processing cost, processing time (s) and disposal cost of recyclers 373
5.16 The expected value of processing cost, time (s) and disposal cost of distributors 373
5.17 Location decision 373
5.18 The results of TS (Appr=0.8, Pos=0.8) 374

Acronyms

MODM Multiple objective decision making
FLMODM Fuzzy-like multiple objective decision making
EVM Expected value model
CCM Chance-constrained model
DCM Dependent-chance model
ECM Expectation model with chance constraints
CEM Chance-constrained model with expectation constraints
DEM Dependent-chance model with expectation constraints
Fu-Ra EVM Fuzzy random expected value model
Fu-Ra CCM Fuzzy random chance-constrained model
Fu-Ra DCM Fuzzy random dependent-chance model
Fu-Fu EVM Bifuzzy expected value model
Fu-Fu CCM Bifuzzy chance-constrained model
Fu-Fu DCM Bifuzzy dependent-chance model
Fu-Ro EVM Fuzzy rough expected value model
Fu-Ro CCM Fuzzy rough chance-constrained model
Fu-Ro DCM Fuzzy rough dependent-chance model
PSO Particle swarm optimization algorithm
GA Genetic algorithm
SA Simulated annealing algorithm
TS Tabu search algorithm