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Hak-Keung Lam and Frank Hung-Fat Leung

# Stability Analysis of Fuzzy-Model-Based Control Systems

## Linear-Matrix-Inequality Approach

Springer

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*To my family and my wife, Esther Wing See Chan, for standing by me.*

*H.K. Lam*

*To Hypatia of Alexandria*
*F.H.F. Leung*

# Preface

Fuzzy logic control has been studied and developed for decades. It is known to be an effective control approach to some ill-defined and complex control processes. Thanks to fuzzy logic, expert knowledge about the control processes can be employed to heuristically design fuzzy controllers with some linguistic IF-THEN rules. Practically, human knowledge can be represented as linguistic statements and incorporated into the fuzzy logic controller. As a result, the design of the control action can be understood easily, and the fuzzy logic controller can operate with intelligence.

With years of investigation, the analysis and design of fuzzy logic control systems have been developing into a broad paradigm of active research. Instead of following the conventional heuristic approach, the design of fuzzy logic controllers can be realized by adopting a fuzzy-model-based (FMB) approach with engineering concerns such as the system stability and performance. The FMB control approach has received great attention of the researchers in the fuzzy control community owing to the systematic mathematical platform it offers to carry out rigorous analysis on general nonlinear control systems. The Takagi-Sugeno/Takagi-Sugeno-Kang (TS/TSK) fuzzy model is generally accepted as a powerful mathematical tool to represent/model/identify nonlinear plants in a systematic way to facilitate the stability analysis and controller synthesis. Based on the Lyapunov stability theory, the stability and performance of the FMB control systems can be investigated. Stability and performance conditions in terms of linear matrix inequalities (LMI) have been reached to aid the design of stable and well-performed FMB control systems. Convex programming techniques can be applied to find the solution of the LMI-based conditions numerically.

In this book, the state-of-the-art FMB based control approaches are covered. A comprehensive review about the stability analysis of type-1 and type-2 FMB control systems using the Lyapunov-based approach is given, presenting a clear picture to researchers who would like to work on this field. A wide variety of continuous-time nonlinear control systems such as state-feedback, switching, time-delay and sampled-data FMB control systems, are covered.

In short, this book summarizes the recent contributions of the authors on the stability analysis of the FMB control systems. It discusses advanced stability analysis techniques for various FMB control systems, and founds a concrete theoretical basis to support the investigation of FMB control systems at the research level. The analysis results of this book offer various mathematical approaches to designing stable and well-performed FMB control systems. Furthermore, the results widen the applicability of the FMB control approach and help put the fuzzy controller in practice. A wide range of advanced analytical and mathematical analysis techniques will be employed to investigate the system stability and performance of FMB-based control systems in a rigorous manner. Detailed analysis and derivation steps are given to enhance the readability, enabling the readers who are unfamiliar with the FMB control systems to follow the materials easily. Simulation examples, with figures and plots of system responses, are given to demonstrate the effectiveness of the proposed FMB control approaches.

This book is organized in 10 chapters. Chapter 1 offers a review on the state-of-the-art stability analysis techniques of FMB control systems. Various techniques under the categories of adaptive, sampled-data, state-feedback, switching, and time-delay fuzzy control schemes, are presented. The stability of the FMB control systems under the different categories of fuzzy controllers are discussed.

In in the first half of Chapter 2, the basic concept and mathematical background of the FMB control systems are given. The technical details of the TS fuzzy model that represents the nonlinear plant and facilitates the stability analysis and controller synthesis are given. A state-feedback fuzzy controller is used to control the nonlinear plant represented by the TS fuzzy model, forming an FMB control system. Some published membership-function-shape-independent (MFSI) stability conditions and performance conditions in terms of LMIs are given in the second half of the chapter.

In chapter 3, the membership-function-shape-dependent (MFSD)-based stability of FMB control systems with perfectly/imperfectly matched membership functions is investigated based on quadratic Lyapunov function. The first approach employs the membership function boundary (MFB) information of both the TS fuzzy model and the fuzzy controller to facilitate the stability analysis. The MFB information is carried by some slack matrices in the stability analysis for relaxing the stability conditions. LMI-based stability and performance conditions are derived to realize a stable and well-performed FMB control system. The second approach employs the staircase membership functions (SMF) to approximate the membership functions of the TS fuzzy model and fuzzy controller for conducting the stability analysis. Using this approach, the membership functions of the TS fuzzy model and fuzzy controller are able to help reaching the stability conditions. Consequently, the LMI-based stability conditions are dedicated to the FMB control system under consideration, which makes the approach to be more suitable for dealing with practical control problems.

In Chapter 4, a state-feedback fuzzy controller with time-varying feedback gains are proposed to strengthen the feedback compensation capability. Based on a quadratic Lyapunov function, stability conditions in terms of bilinear matrix inequalities (BMIs) are derived. The BMI conditions demonstrate a potential to further relax the stability conditions thanks to their nonlinear nature. However, efficient numerical methods are lacked to look for the solution. In this chapter, in order to make the proposed fuzzy control scheme feasible, a genetic-algorithm (GA)-based convex programming technique is developed to search for the solution to the BMI stability conditions. The proposed fuzzy control scheme offers a mathematical analysis platform that extends the LMI-based analysis approach in Chapter 2 to BMI-based ones for the further relaxation of stability conditions.

In Chapter 5, a parameter-dependent Lyapunov function (PDLF) is employed to investigate the system stability of the FMB control systems. Compared with the quadratic Lyapunov function, which is a kind of parameter-independent Lyapunov function (PILF), the PDLF is a nonlinear function that facilitates the stability analysis in different operating regions governed by the membership functions. The results show that the PDLF offers a nice property for the relaxation of stability conditions in some cases.

In Chapter 6, a regional switching fuzzy controller is proposed. The fuzzy controllers described in Chapter 2 to Chapter 5 are for nonlinear plants working in full operating region. As the nonlinear plant demonstrates stronger nonlinearity when it works at the full operating region than the sub-operating region, the idea of regional switching control is motivated. The full operating region is divided into a number of sub-operating regions. Corresponding to each sub-operating region, a regional fuzzy controller is designed. As the nonlinearity of the sub-operating regions is relatively weak, it is more likely to come up with stable regional fuzzy controllers than a fuzzy controller for the full operating region. According to the working conditions, the corresponding regional fuzzy controller is employed to control the nonlinear plant. On investigating the system stability, the regional information of the operating domain is employed for the relaxation of stability conditions.

In Chapter 7, a fuzzy combination control technique is employed to combine the state-feedback fuzzy controller and switching controller under the consideration of the system stability. A local and global fuzzy models are proposed to represent the nonlinear plant operating in local and full operating domains, respectively. A local fuzzy controller and global switching fuzzy controller are then proposed to stabilize the nonlinear plant based on the local and global fuzzy models. The nice properties of both types of controllers are integrated by fuzzy logic to offer an effective control scheme. It forms a theoretical background to support further research on fuzzy combination of various types of controllers.

In Chapter 8, the system stability of time-delay FMB control systems with system states subject to time delay is considered. Time delay is one of the causes for system instability that can be found in most domestic and

industrial applications. Time-delay independent/dependent analysis approaches are adopted to investigate the system stability. It offers a systematic approach to study the nonlinear systems with time delay under the consideration of the system stability. LMI-based stability conditions for both analysis approaches are given.

In Chapter 9, the model reference tracking control for sampled-data FMB control systems is considered. A sampled-data fuzzy controller is proposed to drive the system states of the nonlinear plant to following those of a stable reference model. As the sampled-data fuzzy controller can be implemented by a microprocessor or a digital computer that is available at low cost, the implementation cost and time can be reduced. However, due to the zero-order-hold unit, control signal is kept constant during the sampling period. It causes discontinuity introduced by the sampling activity that complicates the system dynamics and thus makes the analysis difficult. A Lyapunov-based analysis approach is proposed to handle the discontinuity under the consideration of system stability. The tracking performance is guaranteed by applying the $H_\infty$ control theory.

From Chapter 2 to Chapter 9, the system stability analyses reported are for type-1 FMB control systems only. In Chapter 10, we extend the stability analysis approach to interval type-2 FMB control systems. By using the interval type-2 fuzzy logic, the parameter uncertainties of the nonlinear plant can be captured and the system dynamics can be described with some simple fuzzy rules. An interval type-2 TS fuzzy model is then proposed to describe the system dynamics of the nonlinear plant. An interval type-2 fuzzy controller is proposed to close the feedback loop. Some MFSI and MFSD matrices are introduced to facilitate the stability analysis and controller synthesis. LMI-based stability conditions are derived to guarantee the stability of the interval type-2 FMB control system.

Hak-Keung Lam
Frank Hung-Fat Leung

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H.K. Lam

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# Acronyms

BMI Bilinear matrix inequality
LMI Linear matrix inequality
FLF Fuzzy Lyapunov function
FMB Fuzzy-model-based
IT2 Interval type-2
MFB Membership-function boundary
MFSD Membership-function-shape-dependent
MFSI Membership-function-shape-independent
MRAC Model reference adaptive control
PDC Parallel distributed compensation
PDLF Parameter-dependent Lyapunov function
PILF Parameter-independent Lyapunov function
PLF Piecewise Lyapunov function
RS Regional switching
SWLF Switching Lyapunov function
SMF Staircase membership function
TD Time delay
TS Takagi-Sugeno
TSK Takagi-Sugeno-Kang