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Restricted-Orientation Convexity

With 74 Figures

Springer

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To Lena and Mary

Preface

Restricted-orientation convexity, also called $\mathcal{O}$-convexity, is the study of geometric objects whose intersections with lines from some fixed set are connected. This notion generalizes standard convexity and several types of non-traditional convexity. We explore this generalized convexity in multidimensional Euclidean space and identify the properties of standard convex sets that also hold for restricted-orientation convexity.

The purpose of the book is to present the current results on restricted-orientation convexity to the research community and discuss related open problems. The book requires only basic knowledge in geometry; the reader should be familiar with the notion of higher-dimensional Euclidean space and with basic objects in this space, such as lines, balls, and hyperplanes. We use geometric techniques in most proofs, which are accessible to all mathematics and computer-science researchers and graduate students.

$\mathcal{O}$-convexity: We begin with basic properties of $\mathcal{O}$-convex sets, and then introduce $\mathcal{O}$-connected sets, which are a subclass of $\mathcal{O}$-convex sets. We study restricted-orientation analogs of lines, flats and hyperplanes, and characterize $\mathcal{O}$-convex and $\mathcal{O}$-connected sets in terms of their intersections with hyperplanes. We also explore properties of $\mathcal{O}$-connected curves; in particular, we determine when the replacement of a segment of an $\mathcal{O}$-connected curve gives a new $\mathcal{O}$-connected curve, and when the catenation of several curvilinear segments gives an $\mathcal{O}$-connected segment. We use these results to characterize an $\mathcal{O}$-convex set in terms of $\mathcal{O}$-convex segments joining its points, and an $\mathcal{O}$-connected set in terms of $\mathcal{O}$-connected segments.

$\mathcal{O}$-halfspaces: We introduce $\mathcal{O}$-halfspaces, which are a generalization of standard halfspaces, defined as geometric objects whose intersection with every line from some fixed set is empty, a ray or a line. We give basic properties of $\mathcal{O}$-halfspaces and compare them with standard halfspaces; in particular, we show that $\mathcal{O}$-halfspaces may be disconnected and characterize them through

their connected components. We also characterize $\mathcal{O}$-halfspaces in terms of $\mathcal{O}$-convexity of their boundaries, and give a condition under which the complement of an $\mathcal{O}$-halfspace is an $\mathcal{O}$-halfspace.

Strong $\mathcal{O}$-convexity: We also introduce the notion of strong $\mathcal{O}$-convexity, which is an alternative generalization of convexity. We describe properties of strongly $\mathcal{O}$-convex flats and halfspaces, and establish the strong $\mathcal{O}$-convexity of the affine hull of a strongly $\mathcal{O}$-convex set. We then show that, for every point in the boundary of a strongly $\mathcal{O}$-convex set, there is a supporting strongly $\mathcal{O}$-convex hyperplane through it. Finally, we characterize strongly $\mathcal{O}$-convex sets in terms of the intersections of strongly $\mathcal{O}$-convex halfspaces.

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Eugene Fink
Derick Wood

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