Restricted Common Superstring and Restricted Common Supersequence

Raphaël Clifford¹, Zvi Gotthilf², Moshe Lewenstein² and Alexandru Popa¹

Department of Computer Science, University of Bristol, UK {clifford,popa}@cs.bris.ac.uk

Abstract. The shortest common superstring and the shortest common supersequence are two well studied problems having a wide range of applications. In this paper we consider both problems with resource constraints, denoted as the Restricted Common Superstring (shortly RCSstr) problem and the Restricted Common Supersequence (shortly RCSseq). In the RCSstr (RCSseq) problem we are given a set S of n strings, s_1, s_2, \ldots, s_n , and a multiset $t = \{t_1, t_2, \ldots, t_m\}$, and the goal is to find a permutation $\pi: \{1, \ldots, m\} \to \{1, \ldots, m\}$ to maximize the number of strings in S that are substrings (subsequences) of $\pi(t) = t_{\pi(1)}t_{\pi(2)}...t_{\pi(m)}$ (we call this ordering of the multiset, $\pi(t)$, a permutation of t). We first show that in its most general setting the RCSstr problem is NP-complete and hard to approximate within a factor of $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP. Afterwards, we present two separate reductions to show that the RCSstr problem remains NP-Hard even in the case where the elements of t are drawn from a binary alphabet or for the case where all input strings are of length two. We then present some approximation results for several variants of the RCSstr problem. In the second part of this paper, we turn to the RCSseq problem, where we present some hardness results, tight lower bounds and approximation algorithms.

1 Introduction

1.1 Motivation

In AI planning research it is very important to exploit the interactions between different parts of plans. This was observed early in the area [18, 23, 26]. One very important type of interaction is the *merging* of different actions to make the total plan more efficient.

In the general setting we have a set of goals (or tasks) which have to be accomplished and we want to find the most cost efficient plan which achieves all the goals. This problem is also known as the *shortest common superstring* in the case that every goal has to be done continuously or the *shortest common supersequence* if we can abandon a task and resume its process later. In both problems we assume that we have an unlimited set of resources and we want to achieve all our goals. Of course, in real life this is never the case: our resources are always limited.

² Department of Computer Science, Bar-Ilan University, Ramat Gan 52900, Israel {gotthiz,moshe}@cs.biu.ac.il

Therefore, a more realistic question is: given a fixed set of resources, how many goals can be achieved (continuously or not)?

It seems that most of the applications of the shortest common superstring and the shortest common supersequence problem, are more suitable for the case of limited resources. The main challenge for such applications is to find the best arrangement that will lead us to accomplish the maximum number of goals.

As an example, Wilensky [25] gives the scenario where John is planning to go camping for a week. He goes to the supermarket to buy a week's worth of groceries. John has to achieve a set of goals (i.e. to buy food for meals during the camping weekend) and he is able to merge some goals (i.e. to buy different products during a single trip to a supermarket) in order to make the plan more efficient.

Another application, from the computational biology area, is the case where only the set of amino acids can be determined and not their precise ordering. Here we want to know which ordering would maximize the number of short strings which can be substrings or subsequences of some ordering of the symbols in a given text.

1.2 Previous work

In the shortest common supersequence we are given a set S of n strings, s_1, s_2, \ldots, s_n and we want to find the shortest string that is a supersequence of every string in S. For arbitrary n the problem is known to be NP-Hard [11] even in the case of a binary alphabet [16]. However for fixed n a dynamic programming approach takes polynomial time and space. The shortest common supersequence problem has been studied extensively both from a theoretical point of view [9, 12, 15, 17], from an experimental point of view [1, 5] and from the perspective of its wide range of applications in data compression [21], query optimization in database systems [20] and text editing [19].

In the shortest common superstring problem we are given a set S of n strings, s_1, s_2, \ldots, s_n and we want to find the shortest string that is a superstring of every string in S. For arbitrary n the problem is known to be NP-Complete [7] and APX-hard [3]. Even for the case of binary alphabet Ott [13] presented lower bounds for the achievable approximation ratio. The best known approximation ratio so far is 2.5 [10, 22].

1.3 Our contributions

We consider the complexity and the approximability of two problems which are closely related to the well-known shortest common superstring and shortest common supersequence problems.

Problem 1. (Restricted Common Superstring (Supersequence)) The input consists of a set $S = \{s_1, s_2, \ldots, s_n\}$ of n strings over an alphabet Σ and a multiset $t = \{t_1, t_2, \ldots, t_m\}$ over the same alphabet. The goal is to find an ordering of the multiset t that maximizes the number of strings in S that are a substring (subsequence) of the ordered multiset. We denote this ordering by $\pi(t) = t_{\pi(1)}t_{\pi(2)}...t_{\pi(m)}$ (and we say that $\pi(t)$ is a permutation of t). If all the strings in S have length at most ℓ , we refer to the problem as $RCSstr[\ell]$ ($RCSseq[\ell]$). For simplicity of presentation, we assume throughout that all the input strings are distinct and every string $s_i \in S$ is a substring of at least one permutation $\pi(t)$.

Example 1. Let multiset $t = \{a, a, b, b, c, c\}$ and set $S = \{abb, bbc, cba, aca\}$ be an instance of RCSstr (and also of RCSstr[3]). In this example the maximum number of strings from S that can be a substring of a permutation of t is 3. One such possible permutation is $\pi(t) = acabbc$ which contains the strings aca, abb, bbc as substrings.

Example 2. Let multiset $t = \{a, a, b, c\}$ and set $S = \{ab, bc, cb, ca\}$ be an instance of RCSseq and also RCSseq[2]. In this example the maximum number of strings from S that can be a subsequence of a permutation of t is 3. One such possible permutation is $\pi(t) = abca$ which contains the strings ab, bc, ca as a subsequence.

The paper is organized as follows. In Section 2.1 we study the hardness of the RCSstr problem. We show first that in its most general setting the RCS problem is NP-complete and hard to approximate within a factor of less than $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP. Then, we show that even if all input strings are of length two (RCSstr[2]) and t is a set, i.e. no symbols are repeated, then the RCSstr problem is APX-Hard. Afterwards, we prove that the RCSstr problem remains NP-Hard even in the case of a binary alphabet.

In Section 2.2, we design approximation algorithms for several restricted variants of the RCSstr problem. We first present a 3/4 approximation algorithm for the RCSstr[2] problem where t is a set. Moreover, we give a $1/(\ell(\ell(\ell+1)/2-1))$ -approximation algorithm for $RCSstr[\ell]$, when ℓ is the length of the longest input string.

The RCSseq problem is studied in Section 3. In Section 3.1 we show that the hardness results for RCSstr hold also for RCSseq. Moreover, we show an approximation lower bound of $1/\ell!$ when ℓ is the length of the longest input string.

In Section 3.2, we present approximation algorithms for two variants of the RCSseq problem. The first is a $(1 + \Omega(1/\sqrt{\Delta}))/2$ approximation algorithm for RCSstr[2], where Δ is the number of occurrences of the most frequent character in S. Then, for RCSseq we show that a selection of an arbitrary permutation,

 $\pi(t)$, yields a $1/\ell!$ randomized approximation algorithm, thus matching the lower bound presented in Section 3.1.

2 RCSstr

2.1 Hardness of the RCSstr

In this section we present hardness results for several variants of the RCSstr problem.

We show here that RCSstr problem is NP-complete and hard to approximate within a factor better than $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP. To do so, we present an approximation-preserving reduction from the classical maximum clique problem.

Definition 1. (Maximum Clique) Given an undirected graph G = (V, E) the maximum clique problem is to find a vertex set $V' \subseteq V$ of maximum cardinality, such that for every two vertices in V', there exists an edge connecting the two.

The following seminal hardness result will be useful.

Theorem 1. [27] The maximum clique problem does not have an $n^{1-\epsilon}$ approximation, for any $\epsilon > 0$, unless P = NP.

We can now present our main hardness result of the RCSstr problem.

Theorem 2. RCSstr is NP-complete and hard to approximate within a factor of $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP.

Proof. We present an approximation-preserving reduction from the maximum clique problem to the RCSstr problem. Given an undirected graph G = (V, E), where $V = \{v_1, v_2, \ldots, v_n\}$, we construct an instance (S, t) of the RCSstr problem in the following way.

Set t to be $\{v_1^n, v_2^n, \dots, v_n^n\}$ and for each vertex $v_i \in V$ define a string $s_i \in S$ as follows. Set $d(v_i)$ to be the ordered sequence of the vertices not adjacent to v_i . Set s_i to be $v_i^n \cdot d(v_i)$, where \cdot denotes concatenation.

We now prove that the optimal solution of the RCSstr instance (S,t) has size x if and only if the optimal solution of maximum clique problem on the graph G has size x.

Let π be a permutation on the multiset t and let $A \subseteq S$ be all the strings that are substrings of $\pi(t)$. Denote by A' the set of vertices in G corresponding to the set of strings A. We prove that the vertices in A' form a clique. Suppose that this is not true and there exist two vertices $v_i, v_j \in A'$ such that $(v_i, v_j) \notin E$. Note that, in any common superstring of the strings s_i and s_j either v_i or v_j must have

at least n+1 occurrences, since v_i is not present in the neighbors list of v_j and vice versa. This is a contradiction since the multiset t has only n copies of each character. Therefore the set of vertices A' forms a clique.

On the other hand, let $A' = \{v_1, \ldots, v_k\} \subseteq V$ be a clique and let $A = \{s_1, \ldots, s_k\} \subseteq S$ be the set of corresponding strings. We can find a permutation of t which contains all the strings in A as a substring by concatenating s_1, \ldots, s_k and appending the remaining characters arbitrarily at the end. No character is used more than n times since the vertices from A' form a clique and, therefore, $v_i \notin d(v_j)$ for any $v_i, v_j \in A'$.

Thus, the RCSstr problem is NP-complete and hard to approximate within a factor $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP.

We now show that the RCSstr[2] problem is APX-Hard even if t is a set, i.e. each character in t is unique. To do so, we present an approximation-preserving reduction from the classical $Asymmetric\ maximum\ TSP$ problem with edge weights of 0 and 1.

Definition 2. (Maximum Asymmetric Travelling Salesman Problem) Given a complete weighted directed graph G = (V, E) the Maximum Travelling Salesman Problem is to find a closed tour of maximum weight visiting all vertices exactly once.

Theorem 3. [6] For any constant $\epsilon > 0$, it is NP-Hard to approximate the Maximum Asymmetric Travelling Salesman with 0, 1 edge weights within $320/321+\epsilon$.

The hardness result for the RCSstr[2] problem is stated in the following theorem.

Theorem 4. There exists a constant $\beta > 0$, such that the RCSstr problem is NP-Hard to approximate within a factor of $1-\beta$, even if all the strings in S have length two and t is a set.

Proof. We present a gap-preserving reduction from the maximum asymmetric TSP to the RCSstr[2] problem where t is a set.

Given a complete directed graph G = (V, E), with |V| = n, |E| = n(n-1)/2 and edge weights of 0 and 1, we construct an instance (S, t) of the RCSstr[2] problem in the following way.

Set t = V and for each arc $(a, b) \in E$ with weight 1 set a string ab in S. Let OPT(G) be the length of the optimal tour on the graph G and let OPT(S, t) be the maximum number of strings from S which can be substrings of a permutation of t. In order to have an inapproximability factor less than 1, we also assume that n > 322.

We now prove that the reduction presented is a gap-preserving reduction. Specifically, we prove that:

$$OPT(G) = n \Rightarrow OPT(S, t) = n - 1$$

$$OPT(G) < (1 - \alpha)n \Rightarrow OPT(S, t) < (1 - \beta)(n - 1)$$

where $\alpha > 0$ and $\beta > 0$ are constants which are defined later. The permutation $v_1v_2...v_n$ corresponding to a tour of length n contains n-1 strings from S as substrings: $v_1v_2, v_2v_3, ..., v_{n-1}v_n$. Therefore, the first implication is true.

Suppose now that $OPT(G) < (1-\alpha)n$. Then, $OPT(S,t) < (1-\alpha)n$, since a permutation of t defines a path in the graph, which is shorter than a tour. We want to find a constant β such that $(1-\alpha)n \le (1-\beta)(n-1)$. The following inequality gives the desired.

$$\beta \le 1 - \frac{1 - \alpha}{1 - \frac{1}{n}}$$

Therefore, if the maximum ATSP problem does not admit a $1-\alpha$ approximation, then the RCSstr[2] problem (even in case that t is a set) does not admit a $1-\beta$ approximation (the reader may refer to [24] for a more detailed argument of this claim). From Theorem 3, we know that is hard to approximate the Maximum Asymmetric Travelling Salesman with 0, 1 edge weights within $320/321 + \epsilon$, for any $\epsilon > 0$. Therefore, our problem is inapproximable within $1-\beta \ge n(320/321+\epsilon)/(n-1)$, for any $\epsilon > 0$.

We now show that even over a binary alphabet the RCSstr problem remains NP-Hard.

Theorem 5. If $|\Sigma| = 2$, then the RCSstr problem is NP-Hard.

Proof. Let $\Sigma = \{0,1\}$. We prove that if we can solve the *RCSstr* problem on the alphabet Σ in polynomial time, then we can solve in polynomial time the shortest common superstring problem on the alphabet Σ .

Consider a shortest common superstring instance S, where the longest string has length ℓ . It is easy to see that $s_1 \cdot s_2 \cdot \dots \cdot s_n$ is a superstring of all the strings in S. Hence, the solution is no longer than $n\ell$. We show that $O(n^2\ell^2)$ calls to RCSstr are sufficient to find the shortest common superstring of the given strings.

We name an RCSstr instance (S,t) complete, if all the strings of S are substrings of the optimal solution $\pi(t)$.

Note that there exists a string x with i 0's and j 1's that is a common superstring of all the strings in S if and only if the RCSstr instance $(S, 0^i 1^j)$ is complete. Therefore, we want to find the shortest string t such that the RCSstr instance (S,t) is complete. The shortest common superstring is given by the permutation $\pi(t)$ returned by calling the RCSstr on the instance (S,t). The

number of multisets $0^i 1^j$ where $i+j \leq n\ell$ is $O(n^2\ell^2)$. Therefore we can call the RCSstr on all of them and we can find the shortest common superstring on the given strings in polynomial time (note that this time can be improved somewhat by employing a binary search). The shortest common superstring problem is NP-Hard and the theorem follows.

2.2 Approximating RCSstr

Output: $\pi(t)$

In the this section we present approximation algorithms for two variants of the RCSstr problem.

We first present a 3/4-approximation algorithm for the RCSstr[2] problem where each character of t is unique. Our algorithm follows immediately from the NP-Hardness reduction presented in the previous section. Since each character in t is unique we can construct a complete directed graph G=(V,E), with $V=\Sigma$ as in the proof of Theorem 4. We then apply the 3/4 approximation algorithm for the $Maximum\ ATSP$ and we obtain a cycle $t_{\pi(1)}, t_{\pi(2)}, \ldots, t_{\pi(n)}, t_{\pi(1)}$ of total weight k, where $\pi:\{1,\ldots,n\}\to\{1,\ldots,n\}$ is a permutation.

If, for some $i < n, t_{\pi(i)}t_{\pi(i+1)} \notin S$, we output $t_{\pi(i+1)}t_{\pi(i+2)} \dots t_{\pi(n-1)}t_{\pi(n)}t_{\pi(1)}t_{\pi(2)}\dots t_{\pi(i)}$, that contains k strings from S as substrings (and yields an approximation ratio of 3/4). Otherwise, we output $t_{\pi(1)}t_{\pi(2)}\dots t_{\pi(n-1)}t_{\pi(n)}$ that contains exactly n-1 strings from S as substrings, which is optimal.

Here we present a simple $1/(\ell(\ell(\ell+1)/2-1))$ -approximation algorithm for $RCSstr[\ell]$.

The idea is output a concatenation of a maximal collection of strings from S. One can observe that each of the ℓ characters of a string in our solution cannot be used by more than $\ell(\ell+1)/2-1$ strings in the optimal solution. Therefore, the algorithm yields a $1/(\ell(\ell(\ell+1)/2-1))$ -approximation ratio. Formally, the algorithm is presented below.

Algorithm 1: A $1/(\ell(\ell(\ell+1)/2-1))$ approximation algorithm for $RCSstr[\ell]$ Find a maximal subset $S' = s'_1, s'_2, \ldots, s'_q \subset S$ of strings under the following constraint: there exists a permutation $\pi(t)$ of the multiset such that $s'_1 \cdot s'_2 \cdot \cdots \cdot s'_q$ is a prefix of $\pi(t)$.

Theorem 6. Algorithm 1 is a $1/(\ell(\ell(\ell+1)/2-1))$ -approximation algorithm for $RCSstr[\ell]$.

Proof. Note that, a single character can be used simultaneously in at most $\ell(\ell+1)/2-1$ strings of the optimal solution. Since for every $s_i \in S$, $|s_i| \leq \ell$, we can conclude that a single string in our solution can cause at most $\ell(\ell(\ell+1)/2-1)$

other strings of the optimal solution not to be chosen. Thus, the size of the optimal solution is at most $q(\ell(\ell(\ell+1)/2-1))$ and the approximation ratio follows. \Box

One tight example for the above analysis of Algorithm 1 is the following: $t = \{a, b, c, q, q, z, z, z, w, w, w, x, x, x\}$, and $S = \{abc, qa, az, wqa, qaz, azx, qb, bz, wqb, qbz, bzx, qc, cz, wqc, qcz, czx\}$. If we first select into the maximal collection the string abc, then we cannot add any other string to our solution. The optimal solution has size 15 and consists of all the other strings.

Observation 1. Given an $RCSstr[\ell]$ instance, if for every $s_i \in S$, s_i is not a substring of any other $s_i \in S$, then Algorithm 1 is an ℓ^2 -approximation algorithm.

Proof. Note that, a single character can be used simultaneously in at most ℓ strings of the optimal solution, thus, a single string in our solution can stop at most ℓ^2 other strings of the optimal solution from being placed.

One can notice that, in case that all input strings are of length ℓ the above observation must holds.

3 RCSseq

We now turn to the RCSseq problem. We first present hardness results and lower bound for several variants of the RCSseq problem and then we present two approximation algorithms.

3.1 Hardness of the *RCSseq* problem

In the following theorem we show that the hardness result for the general RCSstr holds also to the RCSseq.

Theorem 7. RCSseq is NP-complete and hard to approximate within a factor $n^{1-\epsilon}$, for any $\epsilon > 0$, unless P = NP.

Proof. Omitted (similar to the proof of Theorem 2).

Moreover, we state that even over a binary alphabet the *RCSseq* problem remains NP-Hard.

Theorem 8. If $|\Sigma| = 2$, then the RCSseq problem is NP-Hard.

Proof. Omitted (similar to the proof of Theorem 3).

We now prove that RCSseq is APX-Hard even if all the input strings are of length two and t is a set. To do so, we present an approximation-preserving reduction from the classical $maximum\ acyclic\ subgraph\ problem$.

Definition 3. (Maximum Acyclic Subgraph) Given a directed graph G = (V, E) the maximum acyclic subgraph problem is to find a subset A of the arcs such that G' = (V, A) is acyclic and A has maximum cardinality.

Theorem 9. [14] The Maximum Acyclic Subgraph problem is APX-Complete.

We can now present our hardness result.

Theorem 10. RCSseq is APX-Hard even if all the strings in S have length two and t is a set.

Proof. We present an approximation-preserving reduction from the maximum acyclic subgraph problem. Given a directed graph G = (V, E) we construct an instance (S, t) of the RCSseq problem as follows. Set t = V and for every arc $(a, b) \in E$ we add a string ab to S.

Let π be a permutation of the set t and let $A \subseteq S$ be all the strings that are subsequences of $\pi(t)$. The corresponding edge set A is an acyclic subgraph of G. On the other hand, let $A \subseteq E$ be an acyclic subgraph. Consider a topological ordering of (V, A). All strings corresponding to edges A are subsequences of $\pi(t)$ that corresponds to the topological ordering.

Note that the optimal solution of the RCSseq instance (S,t) has size x if and only if the optimal solution of maximum acyclic subgraph problem on the graph G has size x. Thus, the RCSseq problem is APX-Hard.

In [8] the following result is proven.

Theorem 11. The maximum acyclic subgraph problem is Unique-Games hard to approximate within a factor better than the trivial 1/2 achieved by a random ordering.

The maximum acyclic subgraph is a special case of permutation constraint satisfaction problem (permCSP). A permCSP of arity k is specified by a subset S of permutations on $\{1, 2, \ldots, k\}$. An instance of such a permCSP consists of a set of variables V and a collection of constraints each of which is an ordered k-tuple of V. The objective is to find a global ordering σ of the variables that maximizes the number of constraint tuples whose ordering (under σ) follows a permutation in S. In [4] Charikar, Guruswami and Manokaran prove the following result.

Theorem 12. For every permCSP of arity 3, beating the random ordering is Unique-Games hard.

Our problem corresponds a permCSP where S contains only the identical permutation. Therefore we can conclude the following.

Theorem 13. RCSseq[2] is Unique-Games hard to approximate within a factor better than 1/2.

Theorem 14. RCSseq[3] is Unique-Games hard to approximate within a factor better than 1/6.

Currently there is an unpublished result by Charikar, Håstad and Guruswami stating that every k-ary permCSP is approximation resistant. This implies that $RCSseq[\ell]$ cannot have an approximation algorithm better than $1/\ell!$.

3.2 Approximating RCSseq

In the this subsection we present a $(1 + \Omega(1/\sqrt{\Delta}))/2$ approximation algorithm for the RCSseq[2] problem where Δ is the number of occurrences of the most frequent character in S. We also present a simple randomized approximation algorithm which achieves an approximation ratio of $1/\ell!$.

Theorem 15. [2] The maximum acyclic subgraph problem is approximable within $(1 + \Omega(1/\sqrt{\Delta}))/2$, where Δ is the maximum degree of a node in the graph.

Given a multiset t, let P' be the set of characters that have a single occurrence in t and let P be $\Sigma \backslash P'$, where Σ is the alphabet of t. We define Q to be the following multiset. For every $\sigma \in P$, if σ has r occurrences in t, then σ has r-2 occurrences Q.

Algorithm 2: A $(1 + \Omega(1/\sqrt{\Delta}))/2$ approximation algorithm for *RCSseq2*

- 1. Given a multiset t, construct a graph G = (V, E) such that: $v_i \in V$ iff $v_i \in P'$ and $(a, b) \in E$ iff $a, b \in P'$ and $ab \in S$.
- 2. Apply the $(1 + \Omega(1/\sqrt{\Delta}))/2$ approximation algorithm for the maximum acyclic subgraph to the graph G. Denote the output subgraph by G'(V, E').
- 3. Let F' be a topological order of the vertices of G'. Let F and F'' be an arbitrary ordering of P and Q respectively.
- 4. Output $F \cdot F' \cdot F \cdot F''$.

Figure 1 is an example of Algorithm 2. In the first stage we construct a graph according to the first two steps, note that $P = \{e\}$, $P' = \{a, b, c, d\}$ and $Q = \emptyset$. Then we present an acyclic directed subgraph and we output $F \cdot F' \cdot F \cdot F''$, where F = e and F' = cadb.

Theorem 16. Algorithm 2 is a $(1 + \Omega(1/\sqrt{\Delta}))/2$ approximation algorithm for the RCSseq[2] problem, where Δ is the maximum number of occurrences of a character in the set S.

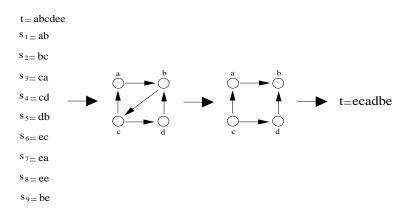


Fig. 1. Algorithm 2 example.

Proof. Given a string $ab \in S$. If $a \in P$ or $b \in P$ (or both), then ab is always a subsequence of $F \cdot F' \cdot F$. Otherwise, if both a and b appear only once in t, then ab is a subsequence of $F \cdot F' \cdot F$ if only if the edge (a,b) is selected in the arc set of the maximum acyclic subgraph. Since the maximum acyclic subgraph problem has an approximation ratio of $(1 + \Omega(1/\sqrt{\Delta}))/2$, the same approximation ratio holds for RCSseg2 problem.

We now deal with $RCSseq[\ell]$ instances. We show that selecting an arbitrary permutation $\pi(t)$ achieves an expected approximation ratio of $\frac{1}{\ell!}$.

We define by $P(s_i, \pi(t))$ the probability that a string $s_i \in S$ is a subsequence of a permutation $\pi(t)$.

Note that, $P(s_i, \pi(t)) \geq \frac{\binom{|t|}{\ell}(|t|-\ell)!}{|t|!} = \frac{1}{\ell!}$. Therefore, the expected number of strings from S to be subsequences of an arbitrary permutation $\pi(t) \geq \frac{|S|}{\ell!}$. Thus, selecting an arbitrary permutation $\pi(t)$ achieves an expected approximation ratio of at least $\frac{|S|}{|S|\ell!} = \frac{1}{\ell!}$.

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 A1
 a b c a b c a b a

 A2
 b c a a b c c a b

 B1
 a a c

 B2
 c a b b

 A1
 a b c a b c a b a

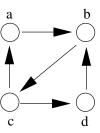
 A2
 b c a a b c c a b

 B1
 c b c b

 B2
 b c a a

c b a

В3





t = abcd

 $s_1 = ab$

 $s_{2}=bc$

 $s_3 = ca$

 $s_4 = cd$

 $s_{5} = db$