Optimal dimensioning of pipe networks: the new situation when the distribution and the transportation functions are disconnected

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Abstract

In [9], De Wolf and Smeers consider the problem of the optimal dimensioning of a gas transmission network when the topology of the network is known. The pipe diameters must been chosen to minimize the sum of the investment and operating costs. This two stage problem was solved by application of the bundle method for nonsmooth optimization.

This model does not reflect any more the current situation on the gas industry. Today, the transportation function and gas buying function are separated. This work considers the new situation for the transportation company. The objective for the transportation company is to determine the flows in the network that minimize the energy used for the gas transport. This corresponds to the minimization of the power used in the compressors. We introduce in the investment problem new decision variables, namely the maximal power of the compressor. We present here first results obtained on the belgian gas network and on a realistic network corresponding to a part of the french network.

1 Introduction

De Wolf and Smeers [9] consider the problem of the *optimal dimensioning of a gas transmission network* when the *topology of the network is known* as a two stages problem: *investment in pipe diameters* in the first stage, *network optimal operations* in the second stage.

The second stage problem considered by De Wolf and Smeers [10], namely the problem of the gas transportation through a network of pipelines, was formulated as a cost minimization subject to nonlinear flow-pressure relations, material balances and pressure bounds. This model does not reflect any more the current situation on the gas market. Today, the transportation and gas buying functions are separated. For example, on the Belgian gas industry, the transport is devoted to Fluxys company and several actors are in charge of gas supplying. This work considers the new situation for the transportation company.

In [6], the new situation for the exploitation model of the transportation company was presented. The objective for the transportation company is to determine the flows in the network that minimize the energy used for the gas transport. This corresponds to the minimization of the power used in the compressors. In order to reflect this new situation, a modelisation of the compressors was introduced in the exploitation model of De Wolf and Smeers [10].

We present here first results concerning the *optimal dimensioning model*. In [9], De Wolf and Smeers only consider the *pipe diameters* as *investment variables*. We add the *maximal power of compressors* as *investment variables*.

As in Andr et al [2], the new model presented in this paper consider the trade-off between the *minimization of capital expenditures* and the *minimization of operational expenditures*. In other terms, this model could balance any decrease in investment of pipelines with an increase of compressor power (and conversely) regarding the costs.

The paper is organized as follows. Section 2 presents the *new investment problem formulation* with the *new investment variables* (maximal power of compressors). Section 3 examines the *mathematical properties* of the problem. Section 4 presents the *solution procedure*. Section 5 presents the *numerical results* of the application to the belgian and french network. Finally, some conclusions are given in Section 6.

2 Problem formulation

We consider first the **investment problem.** The transportation company which must decide the *pipe diameter* for each new pipe, and the *maximal power* for each compression station, in order to minimize the sum of

- *investment cost* in diameters and compression power;
- *network operations costs*, namely the compressors used power.

netrep.eps scaled 1000

Figure 1: Network representation

Figure 1 gives the **network representation** through a graph $G = (N, A)$ where N is the *set of nodes*, and A is the *set of arcs* (pipelines, compressors or valves). We consider *three types of arcs.* The set of arcs A is thus divided in three subsets, namely:

- A_p, the subset of *passive arcs* corresponding to pipelines,
- A_c, the subset of *active arcs* corresponding to compressors,
- A_v, the subset of arcs corresponding to *valves.*

2.1 The investment problem

We use the following notation for the **investment variables:**

D_{ij}	notes the pipe diameter, $\forall (i,j) \in A_p$,
P_{ij}	notes the maximal power of compressor, $\forall (i,j) \in A_c$.

Let us know explain the corresponding **investment costs.** For the *investment costs in compressors stations*, we suppose a *fixed installation cost*, noted k_C, and a *marginal cost proportional to the installed power, $k'_C P_{ij}$*. The *total investment cost in compression stations* is thus considered as a linear function of the installed powers:

$$C_{inv}(P) = \sum_{(i,j) \in A_c} (k_C + k'_C P_{ij}) \tag{1}$$

For the *investment costs for pipes,* we consider, following De Wolf and Smeers [9], the sum of three terms :

- the *pipe acquisition cost* which is proportional to the steel quantity:

$$k_A L_{ij} D_{ij}^2$$

- the *coating costs* which is proportional to the pipe diameter:

$$k_C L_{ij} D_{ij}$$

- *Pipe posing cost:* which has empirically the following form:

$$(k_P + k'_P D_{ij}^2) L_{ij}$$

The *total investment cost in pipes* is thus a quadratic function of the pipes diameters of the following form:

$$C_{inv}(D) = \sum_{(i,j)\in A_p} [k_P + k'_P D_{ij} + k''_P D_{ij}^2] L_{ij} \tag{2}$$

Let us know explains the **constraints** of the investment problem. We relax the constraint that the pipe diameters must be chosen in a set of discrete values to avoid the additional difficulty of solving a non linear non convex problem in integer variables. The same assumption is made on the maximal power of compression stations. We only impose a *maximal pipe diameter:*

$$0 D_{ij} \overline{D}$$

and an *upper bound on the maximal power of the compressors :*

$$0 P_{ij} \overline{P}$$

2.2 Formulation of second stage problem

The *second stage problem* was already formulated in Bakhouya and De Wolf [6]. We summarize here this formulation.

The **operation variables** are the following :

f_{ij}	note the gas flow in each arc $(i,j) \in A$,
W_{ij}	note the power dissipated in the compressor $(i,j) \in A_c$,
p_i	note the gas pressure at each node $i \in N$,
s_i	note the net gas supply in each node $i \in N$.

Note that the flow variables f_{ij} can be negative for pipes (a negative flow f_{ij} means that the flow $-f_{ij}$ goes from node j to node i). Note also that the net supply variables s_i can be negative (a negative net supply s_i means that there is a demand of $-s_i$ at node i).

The **objective function** corresponds to minimization of the *energy used in the compressors:*

$$\min Q_{op} = \alpha \sum_{(i,j)\in A_a} \frac{1}{0,9\eta_{therm}} W_{ij} \tag{3}$$

where α is the unitary energy price (in Euro/kW) and η_{therm}, the thermic efficacity.

The **constraints of the second stage problem** are the following. At a **supply node** i, the gas inflow s_i must remain within take limitations specified in the contracts:

$$\underline{s_i} \leq s_i \leq \overline{s_i}$$

At a demand node, the gas outflow $-s_i$ must be greater or equal to d_i, the demand at this node.

The gas transmission company cannot receive gas at a pressure higher than the one insured by the supplier at the entry point. Conversely, at each exit point, the demand must be satisfied at a minimal guaranteed pressure:

$$\underline{p_i} \leq p_i \leq \overline{p_i}$$

The flow conservation equation at node i (see Figure 2) insures the gas balance at node i:

$$\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i$$

Now, consider the constraints on the arcs. For an arc corresponding to a *pipe*, the relation between the flow f_{ij} in the arc and the pressures at the two supnode.eps scaled 1000

Figure 2: Supply node i

ends of the pipe p_i and p_j is of the following form (see O'Neill and al.[13]):

$$sign(f_{ij})f_{ij}^2 = C_{ij}^2(p_i^2 - p_j^2),\ \forall (i,j) \in A_p \tag{4}$$

where C_{ij} is a parameter depending on the *pipe length* L_{ij} and on the *pipe diameter*, D_{ij}:

$$C_{ij} = \pi \frac{D_{ij}^2}{4} \sqrt{\frac{D_{ij}}{\lambda R T L_{ij}}} = K_{ij} D_{ij}^{\frac{5}{2}} \tag{5}$$

For an *active arc* corresponding to a *compressor*, the following expression of the power used by the compressor can be found (See André et al [3], Babu et al [5] or Seugwon et al [12]):

$$W_{ij} = \gamma_1 f_{ij} \left(\left(\frac{p_j}{p_i} \right)^{\gamma_2} - 1 \right)\ \forall (i,j) \in A_a \tag{6}$$

The power used in the compressor must be lower than the maximal power, noted P_{ij}:

$$W_{ij} \leq P_{ij},\ \forall (i,j) \in A_c$$

There is also an upper bound on the maximal pressure increase ratio:

$$\frac{p_j}{p_i} \leq \gamma_3\ \forall (i,j) \in A_c$$

For active arcs, the direction of the flow is fixed:

$$f_{ij} \geq 0,\ \forall (i,j) \in A_c$$

2.3 Two stages problem formulation

We can summarize the formulation as follows:

$$\left\{\begin{array}{l}
\min_{(D,P)} C_{inv}(D) + C_{inv}(P) + Q_{op}(D,P) \\
\text{s. t.} \left\{\begin{array}{l} 0 D_{ij} \overline{D} \\ 0 P_{ij} \overline{P} \end{array}\right. \\
C_{inv}(D) = \sum_{(i,j)\in A_p} [k_P + k'_P D_{ij} + k''_P D_{ij}^2] L_{ij} \\
C_{inv}(P) = \sum_{(i,j)\in A_a} (k_C + k'_C P_{ij}) \\
Q_{op}(D,P) = \min_{(f,s,p,W)} \alpha \sum_{(i,j)\in A_a} \frac{1}{0,9\eta_{therm}} W_{ij} \\
\text{s. t.} \left\{\begin{array}{ll}
\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i & \forall i \in N \\
sign(f_{ij}) f_{ij}^2 = K_{ij} D_{ij}^5 (p_i^2 - p_j^2) & \forall (i,j) \in A_p \\
W_{ij} = \gamma_1 f_{ij} \left(\left(\frac{p_j}{p_i} \right)^{\gamma_2} - 1 \right) & \forall (i,j) \in A_c \\
\underline{s_i} \le s_i \le \overline{s_i} & \forall i \in N \\
\underline{p_i} \le p_i \le \overline{p_i} & \forall i \in N \\
f_{ij} \ge 0 & \forall (i,j) \in A_c \\
\frac{p_j}{p_i} \le \gamma_3 & \forall (i,j) \in A_c \\
W_{ij} \le P_{ij} & \forall (i,j) \in A_c
\end{array}\right.
\end{array}\right. \quad (7)$$

3 Mathematical properties

To illustrate the *mathematical properties of the operations objective function* $Q_{op}(D, P)$, we consider the *simple network* illustrated at the Figure 3. All the data of the problem are also given on the Figure.

There is *one supply node*: the node 1. We note by s_1 the supply at this node, and the maximal pressure at this node is noted $\overline{p}_1$. There is also *one* power.eps scaled 1250

Figure 3: Simple network

compressor corresponding to arc $(1,2)$. We note by f_{12} the flow in this arc, and by W_{12} the used power. There is also *one pipe* corresponding to arc $(2,3)$. We note by f_{23} the flow in this arc, and by D_{23} the corresponding diameter. Finally, there is also *one demand node:* the node 3. We note the demand at this node by d_3. We consider a minimal pressure at this node : $\underline{p}_3$.

From the node balance equations, we can immediately deduce that:

$$s_1 \quad = \quad f_{12} \quad = \quad f_{23} \quad = \quad d_3 = 400$$

The equation corresponding to the compressor is of the following form:

$$W_{12} = 0,167 \left(\left(\frac{p_2}{p_1} \right)^{0,236} - 1 \right) f_{12} \tag{8}$$

since the equation corresponding to the pipe as the following expression:

$$f_{23}^2 = \frac{10^{-5}}{1,1} D_{23}^5 (p_2^2 - p_3^2) \tag{9}$$

Consider as **First case**, the case where D_{23} is *enough to avoid the use of the compressor.* The limit case corresponds thus to:

$$p_1 = \overline{p}_1 = 80 (= p_2) \text{ and } p_3 = \underline{p}_3 = 30$$

Equation (9) can thus be rewritten as:

$$400^2 = \frac{10^{-5}}{1,1} D_{23}^5 (80^2 - 30^2)$$

The resolution of this equation gives the minimal value of the diameter of the pipe that avoids the use of the compressor:

$$D_{23} = 20$$

Consider know as **second case**, a pipe diameter D_{23} lower than 20. In this case, the *compressor must be used.* We can also suppose that the gas will be delivered at node 3 at a pressure corresponding to the lower bound. Equation (9) can thus be rewritten as follows:

$$400^2 = \frac{10^{-5}}{1,1} D_{23}^5 (p_2^2 - 30^2)$$

Solving this equation with respect to p_2 variable gives the pressure at the end of the compressor:

$$p_2 = \sqrt{30^2 + \frac{400^2}{D_{23}^5} 1,1 \cdot 10^5}$$

We can suppose that the gas pressure at node 1 corresponds to the maximal pressure. By (8), we can compute the compressor used power as follows:

$$W_{12} = 0,167 \left(\left(\frac{p_2}{80} \right)^{0,236} - 1 \right) 400$$

Figure 4 plots the *compressor used power as a function of the pipe diameter*. We can conclude that the objective function of the *second stage problem* (the power used in compressor) is **not differentiable** as a function of the *first stage variables* (pipe diameters). In fact, at point $D_{23} = 20$, the right hand side and left hand side partial derivative differ.

plot.eps scaled 1200

Figure 4: Used power as a function of the pipe diameter

We shall now consider the resolution of the two stage problem formulated in (7).

4 Solution procedure

To solve the investment problem, we propose the following **solution procedure:**

1. We start from a **feasible solution** in terms of the *investment variables* D_{ij} and P_{ij}. For the two practical study cases (namely the belgian gas network and a realistic network corresponding to a part of the french

network), we start from the actual diameters increased by 20 % and from the actual maximal powers of compression stations increased by 20 %.

2. As explained in De Wolf et Bakhouya [6], we solve an *auxiliary convex problem* to achieve a **good starting point for the second stage problem**. This problem is inspired from Maugis [11]:

$$\begin{aligned} \min h(f,s) = \quad & \sum_{(i,j)\in A} \frac{|f_{ij}| f_{ij}^2}{3C_{ij}^2} \\ \text{s.t.} \quad & \sum_{j|(i,j)\in A} f_{ij} - \sum_{j|(j,i)\in A} f_{ji} = s_i \quad \forall i \in N \\ & \underline{s_i} \le s_i \le \overline{s_i} \qquad\qquad\qquad\qquad \forall i \in N \end{aligned} \tag{10}$$

It can be shown (See De Wolf and Smeers [8]) that problem (10) has a physical interpretation. Namely, its objective function is the mechanical energy dissipated per unit of time in the pipes. This implies that the point obtained by minimizing the mechanical energy dissipated in the pipes should constitute a good starting point for the complete problem.

3. We solve the *second stage problem* starting from the solution of the problem (10). We obtains thus a feasible solution for the complete problem.

4. Then, we solve the *complete problem* **all in one problem** from this feasible point. Namely, we replace in the two stages formulation of the investment problem (7), the operations function by its expression and we include all the constraints of the second stage problem in the first stage problem. We obtain thus the **one stage problem** given by (11).

$$
\begin{aligned}
&\min_{(D,P,f,s,p,W)} \sum_{(i,j)\in A_p} [k_P + k'_P D_{ij} + k''_P D_{ij}^2] L_{ij} \\
&+ \sum_{(i,j)\in A_a} (k_C + k'_C P_{ij}) + \alpha \sum_{(i,j)\in A_a} \frac{1}{0,9\eta_{therm}} W_{ij} \\
&\text{s. t.} \left\{
\begin{array}{ll}
0 D_{ij} \overline{D} & \forall (i,j) \in A_p \\
0 P_{ij} \overline{P} & \forall (i,j) \in A_a \\
\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i & \forall i \in N \\
sign(f_{ij}) f_{ij}^2 = K_{ij} D_{ij}^5 (p_i^2 - p_j^2) & \forall (i,j) \in A_p \\
W_{ij} = \gamma_1 f_{ij} \left(\left(\frac{p_j}{p_i} \right)^{\gamma_2} - 1 \right) & \forall (i,j) \in A_a \\
\underline{s_i} \leq s_i \leq \overline{s_i} & \forall i \in N \\
\underline{p_i} \leq p_i \leq \overline{p_i} & \forall i \in N \\
f_{ij} \geq 0 & \forall (i,j) \in A_c \\
\frac{p_j}{p_i} \leq \gamma_3 & \forall (i,j) \in A_c \\
W_{ij} \leq P_{ij} & \forall (i,j) \in A_c
\end{array}
\right.
\end{aligned}
\tag{11}
$$

The *solver* we have used is GAMS/CONOPT with used of DNLP subroutine to take into account the non-differentiability. Let us now come to the practical study cases.

5 Numerical results

Our **first study case** is the *belgium gas transport network*. The main characteristics of the belgian gas network (See Figure 5) are the following:

- there are *24 passive arcs*, and *2 compressors*,
- there are *9 demand nodes*, *6 supply nodes*, and *20 single nodes*,
- there are *no cycle* on this network.

belnet.eps scaled 900

Figure 5: Belgian gas transmission network

As previously said, we use as **starting point,** the actual network and for the *passive arcs,* we increase the *current diameter* by 20 %, since for *active arcs,* we increase the *maximal power* by 20 %,

The resolution of problem (11) for the *belgian gas network* by GAMS/CONOPT gives the following conclusions:

- The solver *increases some diameters* in order to reduce the use of the two compressors.
- The solver *keeps the maximal power of compression stations unchanged.*

We can conclude that the **model prefers to increase the capacity of the passive arcs**, namely to increase the pipe diameters, **in place of increasing the capacity of the active arcs,** namely in place of increasing of the maximal power of compression stations. This is the **first important result** of this work.

Our **second study case** concerns a *realistic cycled network which corresponds to a part of the french network.* The main characteristics are the following:

- there are *41 passive arcs*, *7 compressors*, and *10 valves*,
- there are *19 demand nodes*, *6 supply nodes*, and *56 single nodes*,
- there are *3 cycles* on this network.

For this second case study, we have received the data from Gaz de France for a period from 2006 to 2021. Two cases must be distinguished:

- *Case of years 2006 to 2011:* for these years, the actual capacities are enough to transport all the demand. The same conclusions can be established as for the belgian gas network. Namely, the solver *increases some diameters* in order to reduce the use of the compressors but it *keeps the maximal power of compression stations unchanged.*
- *Case of years 2012 to 2021:* for these years, a reinforcement of the capacities of the network is needed. In this case also, the **model prefers to increase the capacity of the passive arcs** (the pipe diameters) **in place of increasing the capacity of the active arcs** (the maximal power of compression stations). But the simulations for these

years have emphasized the /em role of another important design variable, namely the **maximal flow in the compression stations**. The maximal power was never increased but we have to increase the maximal flow in the compression station to take into account the increase of demand. This is the **second main result** of this work.

6 Conclusions

We have formulated the *optimal dimensioning problem* for a gas transport company as a **two stage problems**: *investment in pipe diameters* and in *maximal power of compression stations* in the first stage, *operations of the network* in the second stage.

We have solved this problem for two practical study cases: the *belgian gas network* and *a part of the french gas transmission network*. The mains results of theses simulations are the following.

First, the **model prefers to increase the capacity of the passive arcs** (the pipe diameters) **in place of increasing the capacity of the active arcs** (the maximal power of compression stations).

Secondly, *another important design variable* is the **maximal flow in the compression stations** which must be increased in accordance with the increase of the demand.

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