Determining L(2, 1)-Span in Polynomial Space

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Abstract

A k-L(2, 1)-labeling of a graph is a function from its vertex set into the set $\{0, \ldots, k\}$, such that the labels assigned to adjacent vertices differ by at least 2, and labels assigned to vertices of distance 2 are different. It is known that finding the smallest k admitting the existence of a k-L(2, 1)-labeling of any given graph is NP-Complete.

In this paper we present an algorithm for this problem, which works in time $O((9 + \epsilon)^n)$ and polynomial memory, where ϵ is an arbitrarily small positive constant. This is the first exact algorithm for L(2, 1)-labeling problem with time complexity $O(c^n)$ for some constant c and polynomial space complexity.

1 Introduction

A frequency assignment problem is the problem of assigning channels of frequency (represented by nonnegative integers) to each radio transmitter, so that no transmitters interfere with each other. Hale [12] formulated this problem in terms of so-called *T*-coloring of graphs.

According to [11], Roberts was the first who proposed a modification of this problem, which is called an L(2, 1)-labeling problem. It asks for such a labeling with nonnegative integer labels, that no vertices in distance 2 in a graph have the same label and labels of adjacent vertices differ by at least 2.

A k-L(2, 1)-labeling problem is to determine if there exists an L(2, 1)-labeling of a given graph with no label greater than k. By $\lambda(G)$ we denote an L(2, 1)span of G, which is the smallest value of k that guarantees the existence of a k-L(2, 1)-labeling of G.

The problem of L(2, 1)-labeling has been extensively studied (see [3, 7, 10, 20] for some surveys on the problem and its generalizations). A considerable attention has been given to bounding the value of $\lambda(G)$ by some function of G.

Griggs and Yeh [11] proved that $\lambda(G) \leq \Delta^2 + \Delta^{-1}$ and conjectured, that

 $^{^1\}Delta$ denotes the largest vertex degree in a graph

 $\lambda(G) \leq \Delta^2$ for every graph G. There are several results supporting this conjecture, for example Gonçalves [9] proved that $\lambda(G) \leq \Delta^2 + \Delta - 2$ for graphs with $\Delta \geq 3$. Have *et al.* [13] have settled the conjecture in affirmative for graphs with $\Delta \geq 10^{69}$. For graphs with smaller Δ , the conjecture still remains open. It is interesting to note that the Petersen and Hoffmann-Singleton graphs are the only two known graphs with maximum degree greater than 2, for which this bound is tight.

The second main branch of research in L(2, 1)-labeling was pointed to analyzing the problem from the complexity point of view. For $k \ge 4$, the k-L(2, 1)-labeling problem was proven to be NP-complete by Fiala *et al.* [6] (for $k \le 3$ the problem is polynomial). It remains NP-complete even for regular graphs (see Fiala and Kratochvíl [8]), planar graphs (see Eggeman *et al.* [4]) or series-parallel graphs (see Fiala *et al.* [5]).

An exact algorithm for the so called Channel Assignment Problem, presented by Král' [19], implies an $O^*(4^n)^2$ algorithm for the L(2, 1)-labeling problem. Havet *et al.* [14] presented an algorithm for computing L(2, 1)(G), which works in time $O^*(15^{\frac{n}{2}}) = O^*(3.8730^n)$. This algorithm has been improved [17, 18], achieving a complexity bound $O^*(3.2361^n)$. Recently, a new algorithm for L(2, 1)-labeling with a complexity bound $O^*(2.6488^n)$ has been presented [16].

All algorithms mentioned above are based on dynamic programming approach and use exponential memory. Havet *et al.* [14] presented a branching algorithm for k-L(2, 1)-labeling problem with a time complexity $O^*((k-2.5)^n)$ and polynomial space complexity. Until now, no algorithm for L(2, 1)-labeling with time complexity $O(c^n)$ for some constant c and polynomial space complexity has been presented. However, there are such algorithms for a related problem of classical graph coloring. The first one, with time complexity $O(5.283^n)$, was shown by Bodleander and Kratsch [2]. The best currently known algorithm for graph coloring with polynomial space complexity is by Björklund *et al.* [1], using the inclusion-exclusion principle. Its time complexity is $O(2.2461^n)$.

In this paper we present the first exact algorithm for the L(2, 1)-labeling problem with polynomially bounded space complexity. The algorithm works in time $O((9+\epsilon)^n)$ (where ϵ is an arbitrarily small positive constant) and is based on a divide and conquer approach.

2 Preliminaries

Throughout the paper we consider finite undirected graphs without multiple edges or loops. The vertex set (edge set) of a graph G is denoted by V(G) (E(G), respectively).

Let $dist_G(x, y)$ be the distance between vertices x and y in a graph G, which is the length of a shortest path joining x and y.

A set $X \subseteq V(G)$ is a 2-packing in G if and only if all its vertices are in distance at least 3 from each other $(\forall x, y \in X \ dist_G(x, y) > 2)$.

²In the O^* notation we omit polynomially bounded terms.

Let $N(v) = \{u \in V(G) : (u, v) \in E(G)\}$ denote the set of neighbors (the *neighborhood*) of a vertex v. The set $N[v] = N(v) \cup \{v\}$ denotes the closed *neighborhood* of v. The neighborhood of a set X of vertices in G is denoted by $N(X) = \bigcup_{v \in X} N(v)$ and its closed neighborhood is denoted by N[X] = $N(X) \cup X.$

For a subset $X \subseteq V(G)$, we denote the subgraph of G induced by the vertices in X by G[X]. A square of a graph G = (V, E) is the graph $G^2 = (V, \{uv \in V\})$ V^2 : $dist_G(u, v) \leq 2$ }).

Definition 1. For a graph G and sets $Y, Z, M \subseteq V(G)$, a $(k-1)-L_Z^M(Y)$ labeling of a graph G is a function $c: Y \to \{0, 1, \dots, k-1\}$, such that $c^{-1}(0) \cap$ $Z = c^{-1}(k-1) \cap M = \emptyset$, and for every $v, u \in Y$:

$$|c(v) - c(u)| \ge 2$$
 if $dist_G(u, v) = 1$
 $|c(v) - c(u)| \ge 1$ if $dist_G(u, v) = 2$.

A function $c: Y \to \mathbb{N}$ is an $L^M_Z(Y)$ -labeling of G if there exists $k \in \mathbb{N}$ such that c is a (k-1)- $L_Z^M(Y)$ -labeling of G

Definition 2. For $Y, Z, M \subseteq V(G)$ let $\Lambda^M_Z(Y, G)$ denote the smallest value of k admitting the existence of (k-1)- $L_Z^M(Y)$ -labeling of G. We define $\Lambda_Z^M(\emptyset, G) \stackrel{def.}{=} 0$ for all graphs G and sets $Z, M \subseteq V(G)$.

Any (k-1)- $L_Z^M(Y)$ -labeling of G with $k = \Lambda_Z^M(Y, G)$ is called *optimal*. We observe that even if c is an optimal $L_Z^M(Y)$ -labeling of G, then any of the sets $c^{-1}(0)$ and $c^{-1}(\Lambda_Z^M(Y, G) - 1)$ may be empty. In the extremal case, if Z = M = Y, then $c^{-1}(0) = c^{-1}(k-1) = \emptyset$ for all k and feasible (k-1)- $L_Z^M(Y)$ labelings c of G.

Notice that $\Lambda_{\emptyset}^{\emptyset}(V(G), G) = \lambda(G) + 1$ for every graph G.

Definition 3. For a graph G, a G-correct partition of a set $Y \subseteq V(G)$ is a triple (A, X, B), such that:

- 1. The sets $A, X, B \subseteq Y$ form a partition of Y
- 2. X is a nonempty 2-packing in G

3.
$$|A| \le \frac{|Y|}{2}$$
 and $|B| \le \frac{|Y|}{2}$

Algorithm 3

In this section we present a recursive algorithm for computing $\Lambda^M_Z(Y,G)$ for any graph G and sets $Y, Z, M \subseteq V(G)$. It is then used to find an L(2, 1)-span a graph G.

The algorithm is based on the divide and conquer approach. First, the algorithm exhaustively check if $\Lambda^M_Z(Y,G) \leq 3$. If not, the set Y is partitioned into three sets A, X, B, which form a G-correct partition of Y. The sets A and B are then labeled recursively.

The labeling of the whole Y is constructed from the labelings found in the recursive calls. The sets of labels used on the sets A and B are separated from each other by the label used for the 2-packing X. This allows to solve the subproblems for A and B independently from each other.

Iterating over all G-correct partitions of Y, the algorithm computes the minimum k admitting the existence of a $(k-1)-L_Z^M(Y)$ -labeling of G, which is by definition $\Lambda_Z^M(Y,G)$.

Algorithm 1: Find-Lambda **Input** : Graph G, Sets $Y, Z, M \subseteq V(G)$ 1 if $Y = \emptyset$ then return 0 2 foreach $c: Y \rightarrow \{0, 1, 2\}$ do for $k \leftarrow 1$ to 3 do 3 **if** c is a (k-1)- $L_Z^M(Y)$ -labeling of G **then return** k 4 5 $k \leftarrow \infty$ 6 foreach G-correct partition (A, X, B) of Y do if $A \neq \emptyset$ and $B \neq \emptyset$ then $k_X \leftarrow 1$ 7 if $A = \emptyset$ and $X \cap Z = \emptyset$ then $k_X \leftarrow 1$ 8 if $A = \emptyset$ and $X \cap Z \neq \emptyset$ then $k_X \leftarrow 2$ 9 if $B = \emptyset$ and $X \cap M = \emptyset$ then $k_X \leftarrow 1$ 10 if $B = \emptyset$ and $X \cap M \neq \emptyset$ then $k_X \leftarrow 2$ 11 $k_A \leftarrow \mathbf{Find-Lambda}(G, A, Z, N(X))$ $\mathbf{12}$ $k_B \leftarrow \mathbf{Find-Lambda}(G, B, N(X), M)$ $\mathbf{13}$ $k \leftarrow \min(k, k_A + k_X + k_B)$ 14 15 return k

Lemma 1. For a graph G and sets $Y, Z, M \subseteq V(G)$, if Y is a 2-packing in G, then $\Lambda_Z^M(Y,G) \leq 3$.

Proof. The labeling $c: Y \to \{0, 1, 2\}$ such that c(v) = 1 for every $v \in Y$ is a 2- $L_Z^M(Y)$ labeling of G.

Theorem 1. For any graph G and sets $Y, Z, M \subseteq V(G)$, the algorithm call *Find-Lambda*(G, Y, Z, M) returns $\Lambda_Z^M(Y, G)$.

Proof. If $Y = \emptyset$, the correct result is given in the line 1 (by the definition of $\Lambda_Z^M(\emptyset, G)$). If $\Lambda_Z^M(Y, G) \leq 3$, the result is found by the exhaustive search in the line 4. Notice that if $|Y| \leq 1$, then by Lemma 1 $\Lambda_Z^M(Y, G) \leq 3$.

Assume that the statement is true for all graphs G' and all sets $Y', Z', M' \subseteq V(G')$, such that |Y'| < n, where $n \ge 1$.

Let G be a graph and Y, Z, M be subsets of V(G) such that |Y| = n. We may assume that $\Lambda_Z^M(Y, G) > 3$. Let k be the value returned by the algorithm call **Find-Lambda**(G, Y, Z, M). First we prove that $k \ge \Lambda_Z^M(Y, G)$, i.e. there exists a (k-1)- $L_Z^M(Y)$ -labeling

First we prove that $k \ge \Lambda_Z^M(Y, G)$, i.e. there exists a $(k-1)-L_Z^M(Y)$ -labeling of G. Let us consider the G-correct partition (A, X, B) of Y, for which the value of k was set in the line 14. Since each of the sets A and B has less than n vertices, by the inductive assumption there exists a $(k_A - 1) - L_Z^{N(X)}(A)$ -labeling c_A of G and a $(k_B - 1) - L_{N(X)}^M(B)$ -labeling c_B of G.

One of the following cases occurs:

1. If $A \neq \emptyset$ and $B \neq \emptyset$, then in the line 7 the value of k_X is set to 1 and thus $k = k_A + k_B + 1$. The labeling c of Y, defined as follows:

$$c(v) = \begin{cases} c_A(v) & \text{if } v \in A \\ k_A & \text{if } v \in X \\ k_A + 1 + c_B(v) & \text{if } v \in B \end{cases}$$

is a (k-1)- $L_Z^M(Y)$ -labeling of G.

2. If $A = \emptyset$ and $X \cap Z = \emptyset$, then in the line 8 the value of k_X is set to 1 and thus $k = k_B + 1$. The labeling c of Y, defined as follows:

$$c(v) = \begin{cases} 0 & \text{if } v \in X \\ c_B(v) + 1 & \text{if } v \in B \end{cases}$$

is a (k-1)- $L_Z^M(Y)$ -labeling of G.

3. If $A = \emptyset$ and $X \cap Z \neq \emptyset$, then in the line 9 the value of k_X is set to 2 and thus $k = k_B + 2$. The labeling c of Y, defined as follows:

$$c(v) = \begin{cases} 1 & \text{if } v \in X \\ c_B(v) + 2 & \text{if } v \in B \end{cases}$$

is a (k-1)- $L_Z^M(Y)$ -labeling of G.

4. If $B = \emptyset$ and $X \cap M = \emptyset$, then in the line 10 the value of k_X is set to 1 and thus $k = k_A + 1$. The labeling c of Y, defined as follows:

$$c(v) = \begin{cases} c_A(v) & \text{if } v \in A \\ k_A & \text{if } v \in X \end{cases}$$

is a (k-1)- $L_Z^M(Y)$ -labeling of G.

5. If $B = \emptyset$ and $X \cap M \neq \emptyset$, then in line 11 the value of k_X is set to 2 and thus $k = k_A + 2$. The labeling c of Y, defined as follows:

$$c(v) = \begin{cases} c_A(v) & \text{if } v \in A \\ k_A & \text{if } v \in X \end{cases}$$

is a (k-1)- $L_Z^M(Y)$ -labeling of G (the label $k_A + 1$ is counted as used, but no vertex is labeled with it).

The case when $X = \emptyset$ is not possible, since the partition (A, X, B) is *G*-correct. The case when $A = B = \emptyset$ is not possible, since then Y = X is a 2-packing in *G* and by the Lemma 1 $\Lambda_Z^N(Y, G) \leq 3$, so the algorithm would finish in the line 4.

Now let us show that $k \leq \Lambda_Z^M(Y, G)$. Let c be an optimal $L_Z^M(Y)$ -labeling of

G. Let *l* be the smallest number, such that $|c^{-1}(0) \cup c^{-1}(1) \cup \cdots \cup c^{-1}(l)| \ge \frac{|Y|}{2}$. Let $A = c^{-1}(0) \cup \cdots \cup c^{-1}(l-1), X = c^{-1}(l)$ and $B = c^{-1}(l+1) \cup \cdots \cup c^{-1}(l+1) \cup \cdots \cup c^{-1}(l-1)$. $c^{-1}(\Lambda^M_Z(Y,G)-1)$. Notice that X is a 2-packing and $X \neq \emptyset$ by the choice of l. Hence we observe that the partition (A, X, B) is G-correct, so the algorithm considers it in one of the iterations of the main loop.

Let $c_A \colon A \to \mathbb{N}$ be a function such that $c_A(v) = c(v)$ for every $v \in A$ and $c_B \colon B \to \mathbb{N}$ be a function such that $c_B(v) = c(v) - (l+1)$ for every $v \in B$. Notice that c_A is an optimal $L_Z^{N(X)}(A)$ -labeling of G and c_B is an optimal $L_{X(X)}^M(B)$ -labeling of G, because otherwise c would not be an optimal.

Hence by the inductive assumption the call in the line 12 returns the number $k_A \leq \Lambda_Z^{N(X)}(A,G)$ and the call in the line 13 returns the number $k_B \leq$ $\Lambda^M_{N(X)}(B,G).$

Let k' be the value of $k_A + k_X + k_B$ in the iteration of the main loop when partition (A, X, B) is considered.

Let us consider the following cases:

- 1. $A, B \neq \emptyset$. In such a case the algorithm **Find-Lambda** sets $k_X = 1$ in the line 7 and $\Lambda_Z^M(Y,G) = \Lambda_Z^{N(X)}(A,G) + \underbrace{1}_{c^{-1}(l)=X} + \Lambda_Z^{N(X)}(B,G) \ge k_A + k_X + k_B = k'.$
- 2. $A = \emptyset$ and l = 0. In such a case $k_A = 0$ and $X \cap Z = \emptyset$ and the algorithm Find-Lambda sets $k_X = 1$ in the line 8 and $\Lambda_Z^M(Y,G) = \underbrace{\Lambda_Z^{N(X)}(A,G)}_{=0} + \underbrace{1}_{C^{-1}(0)=X} + \Lambda_Z^{N(X)}(B,G) \ge k_A + k_X + k_B = k'.$
- 3. $A = \emptyset$ and l = 1. In such a case $k_A = 0$ and $X \cap Z \neq \emptyset$. Otherwise c' defined by c'(v)=c(v)-1 for every $v\in Y$ would be a $L^M_Z(Y)\text{-labeling of }G$ using less labels than the optimal $L_Z^M(Y)$ -labeling c of G – contradiction. The algorithm **Find-Lambda** sets $k_X = 2$ in the line 9 and $\Lambda_Z^M(Y,G) = \underbrace{\Lambda_Z^{N(X)}(A,G)}_{=0} + \underbrace{1}_{c^{-1}(0)=\emptyset} + \underbrace{1}_{c^{-1}(1)=X} + \Lambda_Z^{N(X)}(B,G) \ge k_A + k_X + \underbrace{1}_{c^{-1}(0)=\emptyset} + \underbrace{1}_{c^{-1}(1)=X} + \underbrace{1}$ $k_B = k'$.
- 4. $B = \emptyset$ and $l = \Lambda_Z^M(Y, G) 1$. In such a case $k_B = 0$ and $X \cap M = \emptyset$, and the algorithm **Find-Lambda** sets $k_X = 1$ in the line 10 and $\Lambda_Z^M(Y,G) = \Lambda_Z^{N(X)}(A,G) + \underbrace{1}_{c^{-1}(\Lambda_M^N(Y,G)-1)=X} + \underbrace{\Lambda_Z^{N(X)}(B,G)}_{=0} \ge k_A + k_X + \underbrace{1}_{c^{-1}(\Lambda_M^N(Y,G)-1)=X} + \underbrace{1}_{c^{-1}(\Lambda_M^N(Y,G)$ $k_B = k'$.
- 5. $B = \emptyset$ and $l = \Lambda_Z^M(Y, G) 2$. In such a case $k_B = 0$ and $X \cap M \neq \emptyset$ and the algorithm **Find-Lambda** sets $k_X = 2$ in the line 11 and $\Lambda_Z^M(Y, G) = \Lambda_Z^{N(X)}(A, G) + \underbrace{1}_{c^{-1}(\Lambda_M^N(Y, G)-2)=X} + \underbrace{1}_{c^{-1}(\Lambda_M^N(Y, G))=\emptyset} + \underbrace{\Lambda_Z^{N(X)}(B, G)}_{=0} \ge$ $k_A + k_X + k_B = k'.$

Since those are all possible cases and k is the minimum over values of k' for all correct partitions, clearly $k \leq \Lambda_Z^M(Y, G)$.

Observation 1. By the definition of $\Lambda_Z^M(Y, G)$, the algorithm call **Find-Lambda** $(G, V(G), \emptyset, \emptyset)$ returns $\lambda(G) + 1$.

Lemma 2. Let G be a graph on n vertices, $Y, Z, M \subseteq V(G)$ and let y = |Y|. If G^2 is computed in advance, the algorithm **Find-Lambda** finds $\Lambda_Z^M(Y,G)$ in the time $O(C^{\log y}y^{3\log y}9^y)$ and polynomial space, where C is a positive constant.

Proof. Having G^2 computed, checking if any two vertices in V(G) are in distance at most 2 from each other in G takes a constant time. Hence verifying if a given $X \subseteq Y$ is a 2-packing in G can be performed in the time $O(y^2)$. Moreover, we can check if a given function $c: Y \to \mathbb{N}$ is an $L_Z^M(Y)$ -labeling of G in the time $O(y^2)$.

Let y = |Y| be the measure of the size of the problem. Let T(y) denote the running time of the algorithm **Find-Lambda** applied to a graph G and $Y, Z, M \subseteq V(G)$.

Ale algorithm **Find-Lambda** first checks in constant time if $Y = \emptyset$. Then it exhaustively checks if there exists a $(k-1)-L_Z^M(Y)$ -labeling of G for $k \in \{1, 2, 3\}$. There are 3^y functions $c: Y \to \{0, 1, 2\}$, so this step is performed in the time $O(y^2 \cdot 3^y)$.

Then for every G-correct partition of Y the algorithm is called recursively for two sets of size at most $\frac{y}{2}$. Notice that there are at most 3^y considered partitions. Checking if a partition of Y is G-correct can be performed in time $O(y^2)$. Hence we obtain the following inequality for the complexity (C_1 and C_2 are positive constants):

$$T(y) \le C_1 y^2 4^y + C_2 y^3 3^y 2 \cdot T\left(\frac{y}{2}\right)$$

Let $C = \max(C_1, 2C_2)$, then

$$T(y) \le Cy^2 4^y + Cy^3 3^y \cdot T\left(\frac{y}{2}\right)$$

It is not difficult to verify that $T(y) \leq D \cdot C^{\log y} y^{3 \log y} 9^y = O(C^{\log y} y^{3 \log y} 9^y)$, where D is a positive constant.

The space complexity of the algorithm is clearly polynomial.

Theorem 2. For a graph G on n vertices $\lambda(G)$ can be found in the time $O((9 + \epsilon)^n)$ and polynomial space, where ϵ is an arbitrarily small positive constant.

Proof. The square of a graph G can be found in the time $O(n^3)$. By the Observation 1 and Lemma 2, the algorithm **Find-Lambda** applied to G, Y = V(G) and $Z = M = \emptyset$ finds $\Lambda_{\emptyset}^{\emptyset}(V(G), G) = \lambda(G) - 1$ in the time $O(C^{\log n} n^{3\log n} 9^n) = O((9 + \epsilon)^n)$ and polynomial space.

Remark

We have just learned that results similar to those included in this paper were independently obtained (but not published) by Havet, Klazar, Kratochvíl, Kratsch and Liedloff [15].

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