COMPUTING SCIENCE

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TECHNICAL REPORT SERIES

No. CS-TR-1361 **November 2012**

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Printed and published by Newcastle University,
Computing Science, Claremont Tower, Claremont Road,
Newcastle upon Tyne, NE1 7RU, England.

Bibliographical details

KOUTNY, M., MIKULSKI, L., PIETKIEWICZ-KOUTNY, M.

A Taxonomy of Persistent and Nonviolent Steps
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Newcastle upon Tyne: Newcastle University: Computing Science, 2012.

(Newcastle University, Computing Science, Technical Report Series, No. CS-TR-1361)

Added entries

NEWCASTLE UNIVERSITY
Computing Science. Technical Report Series. CS-TR-1361

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Maciej Koutny is a Professor in the School of Computing Science, Newcastle University. His research interests centre on the theory of distributed and concurrent systems, including both theoretical aspects of their semantics and application of formal techniques to the modelling and verification of such systems; in particular, model checking based on net unfoldings. He has also investigated non-interleaving semantics of priority systems, and the relationship between temporal logic and process algebras. Recently, he has been working on the development of a formal model combining Petri nets and process algebras as well as on Petri net based behavioural models of membrane systems.

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Suggested keywords

PERSISTENCE
NONVIOLENCE
STEP SEMANTICS
PETRI NET
TAXONOMY
BEHAVIOUR
STRUCTURE

A Taxonomy of Persistent and Nonviolent Steps

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Abstract. A concurrent system is persistent if throughout its operation no activity which become enabled can subsequently be prevented from being executed by any other activity. This is often a highly desirable (or even absolutely necessary) property, in particular, if the system is to be implemented in hardware. Over the past 40 years, persistence has been investigated and applied in practical implementations assuming that each activity is a single atomic action which can be represented, for example, by a single transition of a Petri net used as a formal representation of a concurrent system. Recently, it turned out that such a notion of persistence is restricted and in dealing with the synthesis of GALS systems one also needs to consider activities represented by steps which are sets of simultaneously executed transitions. Moving into the realm of step based execution semantics creates a wealth of new fundamental problems and intriguing questions. In particular, there are different ways in which the standard notion of persistence could be lifted from the level of sequential semantics to the level of step semantics. Moreover, at a local level, one may consider steps which are persistent and cannot be disabled by other steps, as well as steps which are nonviolent and cannot disable other steps. In this paper, we aim at providing a classification of different types of persistence and nonviolence, both for steps and markings of PT-nets. We also investigate behavioural and structural properties of such notions both for the general PT-nets and their subclasses.
Keywords: persistence, nonviolence, step semantics, Petri net, taxonomy, behaviour, structure.

1 Introduction

A concurrent system is persistent [2–4, 7] if throughout its operation no activity which become enabled can subsequently be prevented from being executed by any other activity. This is often a highly desirable (or even absolutely necessary) property, in particular, if the system is to be implemented in hardware [5, 8]. Over the past 40 years, persistence has been investigated and applied in practical implementations assuming that each activity is a single atomic action which can be represented, for example, by a single transition of a Petri net used as a formal representation of a concurrent system. In other words, persistence was considered assuming the sequential execution semantics of concurrent systems.

Recently, in a companion paper [6] we argued that the notion of persistence is restricted and in dealing with the synthesis of GALS systems one also needs to consider activities represented by steps which are sets of simultaneously executed transitions. Moving into the realm of step based execution semantics creates a wealth of new fundamental problems and intriguing questions, some of which have been addressed in [6]. In particular, there are different ways in which the standard notion of persistence could be lifted from the level of sequential semantics to the level of step semantics. For example, if part of an enabled has been executed by another step, should we insist on the whole delayed step to be still enabled, or just its residual part? Moreover, at a local level, one may consider steps which are persistent and cannot be disabled by other steps, as well as steps which are nonviolent [1, 2] and cannot disable other steps. In this paper, we aim at providing a classification of different types of persistent and nonviolent steps taking PT-nets to be the system model in which the discussion is carried out. Moreover, we introduce and investigate persistence and nonviolence at the level of markings of PT-nets. We also investigate behavioural and structural properties of notions pertaining to persistence and nonviolence both for the general PT-nets and safe PT-nets.

The paper is organised as follows. In the next section, we present basic notions and notations used throughout. Section 3 introduces various types of persistent and nonviolent steps of transitions in PT-nets, and Section 4 provides their taxonomy. The following section extends the discussion and taxonomy of persistence and nonviolence to markings of PT-nets. In Section 6 we investigate the basic properties of persistent and nonviolent steps of transitions in PT-nets, and then, in Section 7, we focus specifically on the class of safe PT-nets.

2 Preliminaries

A PT-*net* is a tuple $N = (P, T, W, M_0)$, where P and T are finite disjoint sets of respectively *places* and *transitions*, $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is an arc weight function, and $M_0 : P \to \mathbb{N}$ is the *initial marking*. In general, any mapping $M : P \to \mathbb{N}$ is a *marking* of N, and if M' is a marking such that $M(p) \geq M'(p)$, for all $p \in P$, then we denote $M \geq M'$. We use the standard conventions concerning the graphical representation of nets.

A *step* α of N is a set of its transitions, $\alpha \subseteq T$. We will use $\alpha, \beta, \gamma, \dots$ to range over the set of steps. For every place $p \in P$, $W(p, \alpha) = \sum_{t \in \alpha} W(p, t)$ and $W(\alpha, p) = \sum_{t \in \alpha} W(t, p)$. Intuitively, $W(p, \alpha)$ gives the number of tokens that the firing of α removes from p, and $W(\alpha, p)$ is the total number of tokens inserted into p. The pre-places and post-places of a step α are respectively defined as ${}^\bullet\alpha = \{p \in P \mid W(p, \alpha) > 0\}$ and $\alpha^\bullet = \{p \in P \mid W(\alpha, p) > 0\}$. A singleton step $\alpha = \{t\}$ will often be denoted by t, and by a *non-singleton* step we will mean any step comprising at least two transitions.

A step α is *enabled* and may be *fired* at a marking M if $M(p) \geq W(p, \alpha)$, for every place $p \in P$. We denote this by $M[\alpha\rangle$. Firing such an enabled step leads to the marking M' defined by $M'(p) = M(p) - W(p, \alpha) + W(\alpha, p)$, for every place $p \in P$. We denote this by $M[\alpha\rangle M'$.

A *step sequence from a marking* M is a (possibly empty) sequence of steps $\sigma = \alpha_1 \dots \alpha_n$ such that there are markings $M_1, \dots M_{n+1}$ satisfying $M = M_1$ and $M_i[\alpha_i\rangle M_{i+1}$, for every $i \leq n$. We denote this by $M[\sigma\rangle$ and $M[\sigma\rangle M_{n+1}$. If $M = M_0$ then M_{n+1} belongs to the set $[M_0\rangle$ of *reachable* markings of N.

The *concurrent reachability graph* $CRG(N)$ of N is defined as a labelled directed graph $CRG(N) = ([M_0\rangle, A, M_0)$, where the reachable markings of N are vertices, the initial marking is the initial vertex, and the set of arcs is given by $A = \{(M, \alpha, M') \mid M \in [M_0\rangle \wedge M[\alpha\rangle M'\}$.

A PT-net N is *ordinary* if $W(P \times T \cup T \times P) \subseteq \{0, 1\}$, and *safe* if $M(P) \subseteq \{0, 1\}$, for every $M \in [M_0\rangle$. It can be seen that a PT-net without non-active transitions (i.e., transitions that are not enabled at any reachable marking) is ordinary.

Note that being a safe PT-net does not depend on the chosen semantics, i.e., the sequential semantics where only singleton steps are executed, or the full step semantics. In what follows, a step α of a PT-net:

- is *active* if there is a reachable marking which enables it.
- is *positive* if $W(\alpha, p) \geq W(p, \alpha)$, for every $p \in P$.
- is *disconnected* if $({}^\bullet t \cup t^\bullet) \cap ({}^\bullet t' \cup t'^\bullet) = \varnothing$, for all distinct transitions $t, t' \in \alpha$.
- *lies on self-loops* if $W(p, t) = W(t, p)$, for all $t \in \alpha$ and $p \in P$.

Clearly, if α lies on self-loops then it is also positive. We also have:

Fact 1 *If $M[\alpha\rangle$ and $M' \geq M$, then $M'[\alpha\rangle$.*

Fact 2 *If $M[\alpha\rangle$ and $\beta \subseteq \alpha$, then $M[\beta(\alpha \setminus \beta)\rangle$.*

Fact 3 *A step α is enabled at a reachable marking M of a safe* PT*-net iff α is disconnected and contains transitions enabled at M.*

3 Persistence and Nonviolence

In its standard form, persistence is stated as a property of nets executed according to the sequential semantics.

Definition 1 (persistent net, [7]). *A* PT*-net N is* persistent *if, for all transitions $t \neq t'$ and any reachable marking M of N, $M[t\rangle$ and $M[t'\rangle$ imply $M[tt'\rangle$.*

The above definition captures a property of the entire system represented by the PT-net. If one is interested in more fine-grained preservation of executability of actions, it is natural to re-phrase it in terms of individual transitions.

Definition 2 (nonviolent/persistent transition). *Let t be a transition enabled at a marking M of a* PT*-net N. Then:*

– t *is* locally nonviolent at M *if, for every transition* t' *enabled at* M*,*

$$t' \neq t \implies M[tt'\rangle \ .$$

– t *is* locally persistent at M *if, for every transition* t' *enabled at* M*,*

$$t' \neq t \implies M[t't\rangle \ .$$

Moreover, an active transition t *is* globally nonviolent *(or* globally persistent*) in* N *if it is locally nonviolent (resp. locally persistent) at every reachable marking of* N *at which it is enabled.*

The net-oriented and transition-oriented definitions are closely related as, due to the symmetric roles played by t and t' in Definition 1, we immediately obtain the following.

Proposition 1. *Let* N *be a* PT*-net. Then the following are equivalent:*

– N *is persistent.*
– N *contains only globally nonviolent transitions.*
– N *contains only globally persistent transitions.*

We will now introduce the central definitions of this paper, in which we lift the notions of persistence and nonviolence from the level of individual transitions to the level of steps.

Definition 3 (nonviolent step). *Let* α *be a step enabled at a marking* M *of a* PT*-net* N*. Then:*

– α *is* locally A-nonviolent at marking M *(or* LA*-nonviolent) if, for every step* β *enabled at* M*,*

$$\beta \neq \alpha \implies M[\alpha(\beta \setminus \alpha)\rangle \ .$$

– α *is* locally B-nonviolent at marking M *(or* LB*-nonviolent) if, for every step* β *enabled at* M*,*

$$\beta \cap \alpha = \varnothing \implies M[\alpha\beta\rangle \ .$$

– α *is* locally C-nonviolent at marking M *(or* LC*-nonviolent) if, for every step* β *enabled at* M*,*

$$\beta \neq \alpha = \varnothing \implies M[\alpha\beta\rangle \ .$$

Moreover, an active step α *is* globally A/B/C-nonviolent (or GA/GB/GC-nonviolent) in N *if it is respectively* LA/LB/LC*-nonviolent at every reachable marking of* N *at which it is enabled.*

Each of the three types of step nonviolence is a conservative extension of transition nonviolence introduced in Definition 2. Intuitively, type-A nonviolence requires that only the unexecuted part of a delayed step is kept enabled, and so it is 'protected' by α. Type-B and type-C nonviolence, however, insist on maintaining the enabledness of the whole delayed step.

Definition 4 (persistent step). *Let α be a step enabled at a marking M of a* PT*-net N. Then:*

- α *is* locally A-persistent at marking M *(or* LA*-persistent) if, for every step β enabled at M,*
$$\beta \neq \alpha \implies M[\beta(\alpha \setminus \beta)\rangle .$$
- α *is* locally B-persistent at marking M *(or* LB*-persistent) if, for every step β enabled at M,*
$$\beta \cap \alpha = \varnothing \implies M[\beta\alpha\rangle .$$
- α *is* locally C-persistent at marking M *(or* LC*-persistent) if, for every step β enabled at M,*
$$\beta \neq \alpha = \varnothing \implies M[\beta\alpha\rangle .$$

Moreover, an active step α is globally A/B/C-persistent (or GA/GB/GC-persistent) in N *if it is respectively* LA/LB/LC*-persistent at every reachable marking of N at which it is enabled.*

Again, each of the three types of step persistence is a conservative extension of transition persistence introduced in Definition 2. Type-A persistence requires that only unexecuted part of a delayed step is kept enabled, and in this case a persistent step can 'survive' only partially. Type-B and type-C persistence, however, insist on preserving the enabledness of the whole step. Note that in type-B of nonviolence and persistence, two steps are considered different if they are disjoint, whereas in the other two cases it is enough that they are different, and so they can have a nonempty intersection.

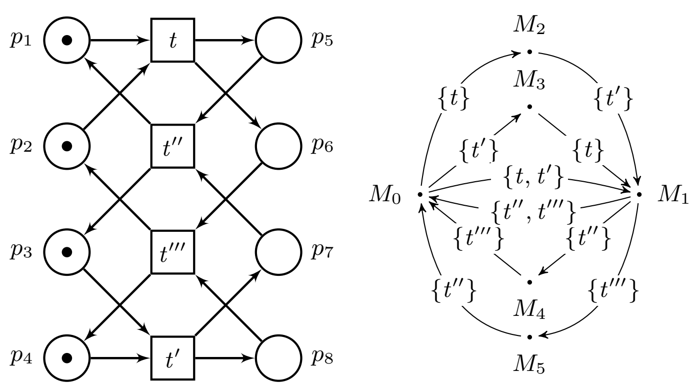

Fig. 1. A persistent safe PT-net and its concurrent reachability graph.

Moving from sequential to step semantics changes the way we perceive the persistence of PT-nets introduced by the standard Definition 1. In particular, in the sequential semantics, by Proposition 1, all transitions in a persistent net are both globally nonviolent and globally persistent. In the step semantics the situation is radically different. Consider, for example, the PT-net in Figure 1. It is persistent, and all of its active steps are both GA-persistent and GA-nonviolent. However, its nonempty steps fail to be LC-persistent or LC-nonviolent at some of the markings that enable them. More precisely, $\{t\}$, $\{t'\}$ and $\{t,t'\}$ are neither LC-persistent nor LC-nonviolent at M_0, while $\{t'''\}$, $\{t''\}$ and $\{t'',t'''\}$ are neither LC-persistent nor LC-nonviolent at M_1. This should not come as a surprise, as the type-C of persistence (or nonviolence) is a demanding property. Type-A of persistence and nonviolence, on the other hand, are close in spirit to their sequential counterparts.

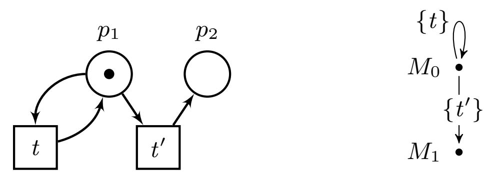

Fig. 2. A safe PT-net illustrating the duality of persistence and nonviolence.

The duality of the nonviolent and persistent steps is illustrated in Figure 2, where:

- $\{t\}$ is both a GA-nonviolent and GC-nonviolent step, but neither LA-persistent nor LC-persistent at M_0.
- $\{t'\}$ is both a GA-persistent and GC-persistent step, but neither LA-nonviolent nor LC-nonviolent at M_0.

A step can be both nonviolent and persistent. For example, if we merge p_1 and p_2 in Figure 2, making both t and t' lie on self-loops, then $\{t\}$ and $\{t'\}$ become GA/GC-nonviolent/persistent.

4 Relating Persistent and Nonviolent Steps

Our aim in this section is to identify the expressiveness of different types of persistent and nonviolent steps. Directly from Definitions 3 and 4 we have the following.

Proposition 2. *Let α be a step enabled at a reachable marking M of a PT-net N. Then, respectively:*

1. *If α is GA/GB/GC-nonviolent in N, then α is LA/LB/LC-nonviolent at M.*
2. *If α is GA/GB/GC-persistent in N, then α is LA/LB/LC-persistent at M.*

We then obtain a number of inclusions between different types of persistent and nonviolent steps which all hold for general PT-nets.

Proposition 3. *Let α be a step and M be a marking of a* PT*-net N. Then:*

1. *α is* LA*-nonviolent at M iff α is* LB*-nonviolent at M.*
2. *α is* LA*-persistent at M iff α is* LB*-persistent at M (cf. [6]).*

Proof. Assume that α is enabled at M, and β is another step enabled at M.

(1) Suppose that α is LA-nonviolent at M and $\beta \cap \alpha = \varnothing$. Then $M[\alpha(\beta \setminus \alpha)\rangle$ and $\beta \setminus \alpha = \beta$. Hence $M[\alpha\beta\rangle$, and so α is LB-nonviolent at M.
Conversely, suppose α is LB-nonviolent at M. Then $M[\alpha(\beta \setminus \alpha)\rangle$ as $(\beta \setminus \alpha) \cap \alpha = \varnothing$ and $M[\beta \setminus \alpha\rangle$ (cf. Fact 2). Hence, α is LA-nonviolent at M.

(2) Suppose that α is LA-persistent at M and $\beta \cap \alpha = \varnothing$. Then $M[\beta(\alpha \setminus \beta)\rangle$ and $\alpha \setminus \beta = \alpha$. Hence $M[\beta\alpha\rangle$, and so α is LB-persistent at M.
Conversely, suppose that α is LB-persistent at M. Then $M[(\beta \setminus \alpha)\alpha\rangle$ as $(\beta \setminus \alpha) \cap \alpha = \varnothing$ and $M[\beta \setminus \alpha\rangle$ (cf. Fact 2). Hence, for every place $p \in P$:

$$M(p) - W(p, \beta \setminus \alpha) + W(\beta \setminus \alpha, p) \geq W(p, \alpha)$$

which implies:

$$M(p) - W(p, \beta) + W(p, \beta \cap \alpha) + W(\beta, p) - W(\beta \cap \alpha, p) \geq W(p, \alpha) \ .$$

As a result, we obtain that

$$\begin{aligned} M(p) - W(p, \beta) + W(\beta, p) &\geq W(p, \alpha) - W(p, \beta \cap \alpha) + W(\beta \cap \alpha, p) \\ &= W(p, \alpha \setminus \beta) + W(\beta \cap \alpha, p) \\ &\geq W(p, \alpha \setminus \beta) \end{aligned}$$

implying $M[\beta(\alpha \setminus \beta)\rangle$. Hence α is LA-persistent at M. ☐

Proposition 4. *Let α be a step of a* PT*-net N. Then:*

1. *α is* GA*-nonviolent in N iff α is* GB*-nonviolent in N.*
2. *α is* GA*-persistent in N iff α is* GB*-persistent in N (cf. [6]).*

Proof. Follows directly from Proposition 3. ☐

Proposition 5. *Let α be a step and M a marking of a* PT*-net N. Then:*

1. *If α is* LC*-nonviolent at M, then α is* LA*-nonviolent at M.*
2. *If α is* LC*-persistent at M, then α is* LA*-persistent at M.*

Proof. Since enabledness of steps is monotonic in PT-nets (see Fact 2), the two implications follow directly from Definitions 3 and 4, where the statements for LC-persistence and LC-nonviolence have stronger consequents. ☐

Proposition 6. *Let α be a step of a* PT*-net N. Then:*

1. *If α is* GC*-nonviolent in N, then α is* GA*-nonviolent in N.*
2. *If α is* GC*-persistent in N, then α is* GA*-persistent in N.*

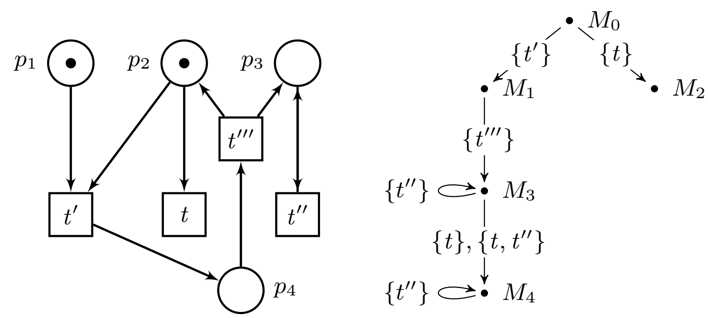

Fig. 3. A safe PT-net for (ii,v,vi) in Figure 7 and (i,ii) in Figure 9.

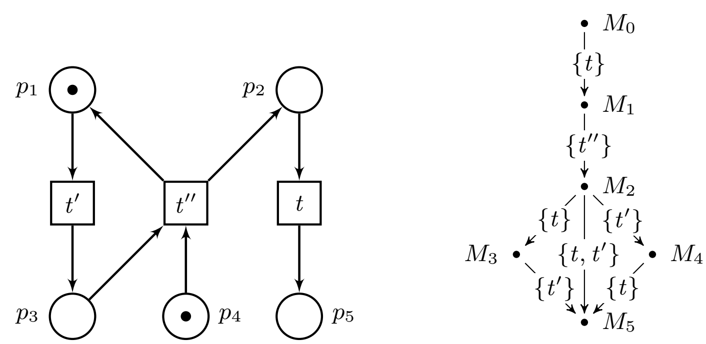

Fig. 4. A safe PT-net for (IV) in Figure 7.

Proof. Follows directly from Proposition 5. □

The implications in Propositions 5 and 2 (for type-A) cannot be reversed. A counterexample is provided in Figure 3, where $\{t\}$ is both LA-nonviolent and LA-persistent at M_3. However, it is neither LC-nonviolent nor LC-persistent at M_3 as well as it is neither GA-nonviolent nor GA-persistent (because of M_0).

The implications in Propositions 6 cannot be reversed. A counterexample is provided in Figure 3, where $\{t, t''\}$ is both GA-nonviolent and GA-persistent, but neither GC-nonviolent nor GC-persistent. As this step is only enabled at marking M_3, it fails to be LC-nonviolent or LC-persistent as well. Moreover, in Figure 3, $\{t'''\}$ is a step that is type-A and type-C globally nonviolent and persistent, because it is only enabled at one marking M_1, and no other nonempty step is enabled at M_1.

Figure 4 shows that a step $\{t\}$ may be GA-persistent, but only LC-persistent (at M_0 and M_4). Step $\{t\}$ is not GC-persistent in that reachability graph, because it is not LC-persistent at M_2. The same example can be used considering nonviolence.

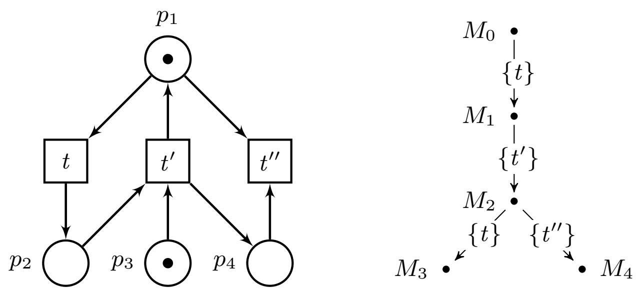

Fig. 5. A safe PT-net for (III) in Figure 7.

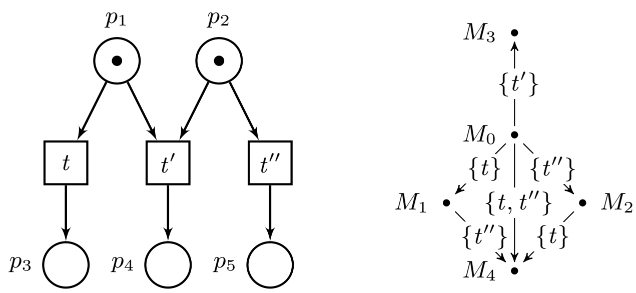

Fig. 6. A safe PT-net for (I) in Figure 7.

Figure 5 shows an example of a step, $\{t\}$, that is LC-nonviolent, LA-nonviolent, LC-persistent and LA-persistent at M_0, but neither GC-nonviolent nor GA-nonviolent nor GC-persistent nor GA-persistent.

There are steps in PT-nets that are neither persistent nor nonviolent according to any of the considered types; for example, $\{t, t''\}$ and $\{t'\}$ in Figure 6. They are enabled only at M_0, and fail to be persistent or nonviolent there.

Finally, there are PT-nets where all steps are neither persistent nor nonviolent whatever type (A or C) we choose. For example, take a safe PT-net with only two transitions, t and t', which share only one marked pre-place. Then, the only two steps in the concurrent reachability graph are $\{t\}$ and $\{t'\}$, and they prevent each other from being persistent and, as a consequence, they also fail to be nonviolent.

The relationships between different types of persistent and nonviolent steps are summarised in the diagram of Figure 7. As the relationships are the same for persistence or nonviolence, the diagram simply refers to different types of persistence or nonviolence.

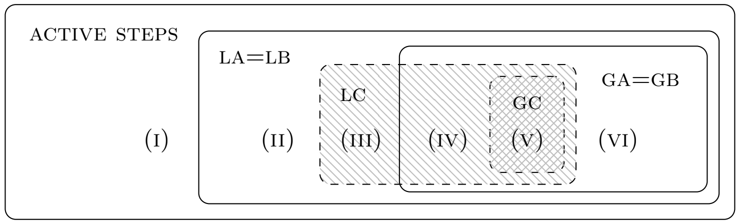

Fig. 7. A taxonomy of persistent and nonviolent steps. Examples of steps exhibiting the nonemptiness of the specific sets of steps in the diagram are as follows: $\{t, t''\}$ and $\{t'\}$ in Figure 6 for (I); $\{t\}$ in Figure 3 for (II); $\{t\}$ in Figure 5 for (III); $\{t\}$ in Figure 4 for (IV); $\{t'''\}$ in Figure 3 for (V); and $\{t, t''\}$ in Figure 3 for (VI).

5 Persistent and Nonviolent Markings

In this section we focus on steps enabled at a particular marking. A marking will be persistent (or nonviolent) according to some chosen type of persistence (or nonviolence) if all steps that it enables will satisfy appropriate definition of persistence (or nonviolence). Interestingly, in such markings, if all enabled steps are A (B or C) persistent they all are A (B or C) nonviolent, and vice versa. In some way, such markings create an environment where steps do not interfere with each other.

Definition 5 (nonviolent/persistent marking). *Let M be a reachable marking of a* PT*-net N. Then:*

- *M is* A/B/C-nonviolent in N *if every step enabled at M is respectively* LA/LB/LC*-nonviolent at M.*
- *M is* A/B/C-persistent in N *if every step enabled at M is respectively* LA/LB/LC*-persistent at M.*

Proposition 7. *A reachable marking of a* PT*-net is* A/B/C*-persistent iff it is* A/B/C*-nonviolent, respectively.*

Proof. By Definition 5, a reachable M is A-persistent in a PT-net N iff each step α enabled at M is LA-persistent at M. The latter in turn is equivalent to:

$$\begin{array}{rl} & \forall\alpha :\ M[\alpha\rangle \implies (\forall\beta :\ \alpha \neq \beta \wedge M[\beta\rangle \implies M[\alpha(\beta \setminus \alpha)\rangle) \\ \equiv & \forall\alpha,\beta :\ \alpha \neq \beta \wedge M[\alpha\rangle \wedge M[\beta\rangle \implies M[\alpha(\beta \setminus \alpha)\rangle \\ \equiv & \forall\alpha,\beta :\ \alpha \neq \beta \wedge M[\alpha\rangle \wedge M[\beta\rangle \implies M[\beta(\alpha \setminus \beta)\rangle \\ \equiv & \forall\alpha :\ M[\alpha\rangle \implies (\forall\beta :\ \alpha \neq \beta \wedge M[\beta\rangle \implies M[\beta(\alpha \setminus \beta)\rangle) \ . \end{array}$$

The last line is equivalent to stating that each step α enabled at M is LA-nonviolent at M. Hence, by Definition 5, M is A-nonviolent in N.

The proofs of the other two equivalences are similar. □

Proposition 8. *A reachable marking of a* PT*-net N is* A*-persistent (or* A*-nonviolent) in N iff it is* B*-persistent (resp.* B*-nonviolent) in N.*

Proof. Follows directly from Definitions 4 and 5, and Propositions 3 and 7. □

Proposition 9. *If a reachable marking of a* PT*-net N is* C*-persistent (or* C*-nonviolent) in N, then it is* A*-persistent (resp.* A*-nonviolent) in N.*

Proof. Follows directly from Definitions 4 and 5, and Propositions 5 and 7. □

The implications in Proposition 9 cannot be reversed, and a suitable counterexample is provided in Figure 3, where M_3 is both A-persistent and A-nonviolent marking but it is neither C-persistent nor C-nonviolent. Notice that all the steps enabled at M_3 (i.e., $\{t\}$, $\{t''\}$ and $\{t, t''\}$) are LA-persistent at this marking, making it A-persistent (and A-nonviolent, see Proposition 7). However, $\{t\}$ and $\{t, t''\}$ are neither LC-persistent nor LC-nonviolent at M_3.

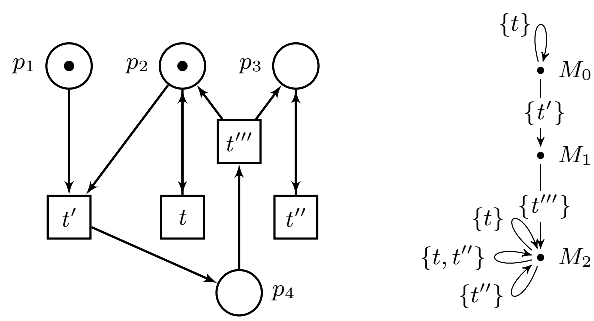

Fig. 8. A safe PT-net for (III) in Figure 9.

The relationships between different types of persistent and nonviolent markings are summarised in the diagram of Figure 9. As the relationships are the same for persistence or nonviolence, the diagram simply refers to different types of persistence or nonviolence.

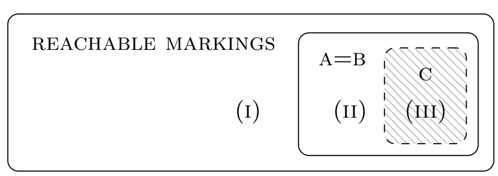

Fig. 9. A taxonomy of persistent and nonviolent markings. Examples of markings exhibiting the nonemptiness of the specific sets of markings in the diagram are as follows: M_0 in Figure 3 for (I); M_3 in Figure 3 for (II); and M_2 in Figure 8 for (III).

6 Persistent and Nonviolent Steps in PT-nets

In this section, we investigate general properties of persistent and nonviolent steps. The first question we address is whether persistence and nonviolence of steps can be 'inherited' by their substeps. For general PT-nets, the answer turns out to be positive only for local persistence.

Proposition 10. *Let $\gamma \subseteq \alpha$ be two steps and M be a reachable marking of a* PT*-net. Then:*

1. *If α is* LA*-persistent at M, then γ is* LA*-persistent at M.*
2. *If α is* LC*-persistent at M, then γ is* LC*-persistent at M.*

Proof. From $\gamma \subseteq \alpha$, $M[\alpha\rangle$ and Fact 2, we have $M[\gamma\rangle$. Hence γ is active. Moreover, we assume $\varnothing \neq \gamma \neq \alpha$ as the otherwise the results is obvious.

(1) Let $\beta \neq \gamma$ be a step enabled at M. If $\beta \neq \alpha$ then, as α is LA-persistent at M, we have $M[\beta(\alpha \setminus \beta)\rangle$. Hence, by $\gamma \setminus \beta \subseteq \alpha \setminus \beta$ and Fact 2, $M[\beta(\gamma \setminus \beta)\rangle$. If $\beta = \alpha$ then $M[\beta(\gamma \setminus \beta)\rangle$ as $M[\alpha\varnothing\rangle$ and $\gamma \setminus \beta = \varnothing$.

(2) Let $\beta \neq \gamma$ be a step enabled at M. If $\beta \neq \alpha$ then, as α is LC-persistent at M, we have $M[\beta\alpha\rangle$. Hence, by $\gamma \subseteq \alpha$ and Fact 2, $M[\beta\gamma\rangle$. If $\beta = \alpha$ and $\neg M[\alpha\gamma\rangle$, then we proceed as follows. By $M[\alpha\rangle$, there is $p \in P$ such that $M(p) - W(p,\alpha) + W(\alpha,p) < W(p,\gamma)$. On the other hand, since α is LC-persistent at M and $\gamma \neq \alpha$ is enabled at M, we have $M[\gamma\alpha\rangle$. Thus $M(p) - W(p,\gamma) + W(\gamma,p) \geq W(p,\alpha)$. As a result, $W(p,\alpha) + W(p,\gamma) - W(\gamma,p) < W(p,\gamma) + W(p,\alpha) - W(\alpha,p)$, and so $W(\gamma,p) > W(\alpha,p)$, yielding a contradiction. □

Proposition 10 does not hold for globally persistent steps and their substeps, whether we consider A-persistence or C-persistence. Figure 8 shows an example of a step, $\{t, t''\}$, which is both GA-persistent and GC-persistent, but its substep $\{t\}$ is neither GA-persistent nor GC-persistent, because of marking M_0. Furthermore, Proposition 10 extended to nonviolent steps does not hold, even for ordinary PT-nets. Figure 10 provides a counterexample, where $\{t, t''\}$ is both LA-nonviolent and LC-nonviolent at M_0 (in fact it is both GA-nonviolent and GC-nonviolent, as no other nonempty steps are enabled at M_0), but its substep $\{t\}$ is neither LA-nonviolent nor LC-nonviolent at M_0.

Type-C persistence and nonviolence are very demanding, and can only be satisfied by steps of a very particular kind. The presence of type-C persistent or nonviolent steps has therefore, some structural implications for nets and their reachability graphs. The next result gives sufficient conditions for being a globally nonviolent step.

Theorem 1. *Each active positive step of a* PT*-net is both* GC*-nonviolent and* GA*-nonviolent.*

Proof. Let $M[\alpha\rangle M'$ and $\beta \neq \alpha$ be a step enabled at M. From $M' \geq M$ (as α is positive) and Fact 1 it follows that $M'[\beta\rangle$. Hence $M[\alpha\beta\rangle$, and so α is GC-nonviolent. Moreover, by Proposition 6, α is also GA-nonviolent. □

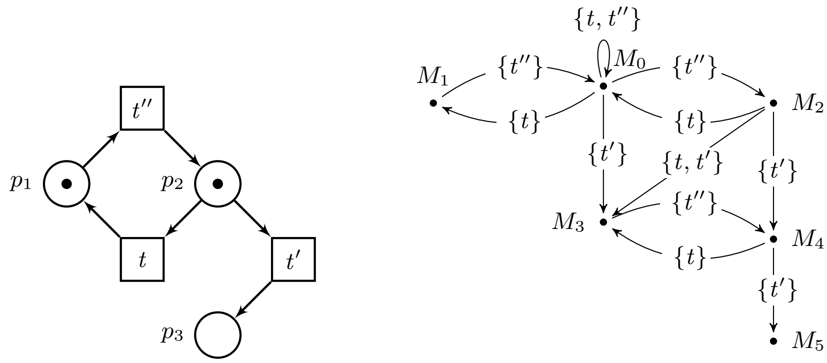

Fig. 10. An ordinary PT-net for the discussion of Proposition 10.

The next result gives necessary conditions for being a GC-persistent step. Intuitively, the intersection of a GC-persistent step with any other step enabled at the same marking consumes less resources (tokens) that it produces. This should not be a surprise, because in C-persistence the intersection of two enabled steps at a given marking must be able to fire twice in a row.

Proposition 11. *Let α be a* GC*-persistent non-singleton step enabled at a reachable marking M of a* PT*-net. Then, for every step $\beta \neq \alpha$ enabled at M, $\alpha \cap \beta$ is a positive step.*

Proof. Suppose that the step $\gamma = \alpha \cap \beta$ is not positive, and so there is $p \in P$ such that $W(p, \gamma) > W(\gamma, p)$. We consider two cases.

Case 1: $\alpha \not\subset \beta$. Since $M[\alpha\rangle$ and $\gamma \subseteq \alpha$, we have $M[\gamma\rangle$. Also, since $\alpha \not\subset \beta$, $\gamma \neq \alpha$. As α is GC-persistent, there exists a marking M_1 such that $M[\gamma\rangle M_1[\alpha\rangle$. We can repeat this construction, replacing M with M_1, as α is globally C-persistent. In fact, we can repeat this construction $k = M(p)+1$ times, obtaining $M[\gamma\rangle M_1[\gamma\rangle M_2[\gamma\rangle M_3 \ldots M_k[\alpha\rangle$.

We then observe that $M(p) - M_1(p) = W(p, \gamma) - W(\gamma, p) \geq 1$. Similarly, $M_i(p) - M_{i+1}(p) \geq 1$, for $i = 1, \ldots, k-1$. Hence $M(p) - M_k(p) \geq k = M(p) + 1$ and so $M_k(p) < 0$ which is obviously impossible, yielding a contradiction.

Case 2: $\alpha \subset \beta$. Then $\gamma = \alpha \cap \beta = \alpha$. As α is a non-singleton step, we can split it into two disjoint nonempty subsets: $\alpha = \gamma = \gamma' \uplus \gamma''$. Since $M[\alpha\rangle$ and $\gamma', \gamma'' \subseteq \alpha$, we have $M[\gamma'\rangle$ and $M[\gamma''\rangle$. Also, $\gamma' \neq \alpha$ and $\gamma'' \neq \alpha$. As α is GC-persistent, there exists a marking M' such that $M[\gamma'\rangle M'[\alpha\rangle$. Now, we can repeat this construction, for M' and step γ'' getting: $M[\gamma'\rangle M'[\gamma''\rangle M_1[\alpha\rangle$ or $M[\gamma'\gamma''\rangle M_1[\alpha\rangle$. We can repeat this construction, now starting at M_1, as α is globally C-persistent. In fact, we can repeat this construction $k = M(p) + 1$ times, obtaining $M[\gamma'\gamma''\rangle M_1[\gamma'\gamma''\rangle M_2[\gamma'\gamma''\rangle M_3 \ldots M_k[\alpha\rangle$.

We now observe that, by $\gamma = \gamma' \uplus \gamma''$, we have $W(p,\gamma) = W(p,\gamma') + W(p,\gamma'')$ and $W(\gamma,p) = W(\gamma',p) + W(\gamma'',p)$. The rest of the proof is then similar as in Case 1. $\square$

In Proposition 11, one cannot drop the assumption that α is a non-singleton step. Consider, for example, Figure 11 and take $\alpha = \{t'\}$ and $\beta = \{t,t'\}$, which are two different steps enabled at M_0. Although α is GC-persistent, the intersection $\alpha \cap \beta = \{t'\}$ is not a positive step, as $W(p_1,t') > W(t',p_1)$. Similarly, one cannot drop the assumption that α is GC-persistent. Consider, for example, Figure 13 and take $\alpha = \{t,t'\}$, and $\beta = \{t\}$, which are two different steps jointly enabled at several markings, such as M_0. Although α is a non-singleton step, the intersection $\alpha \cap \beta = \{t\}$ is not a positive step as $W(p_1, \alpha \cap \beta) > W(\alpha \cap \beta, p_1)$.

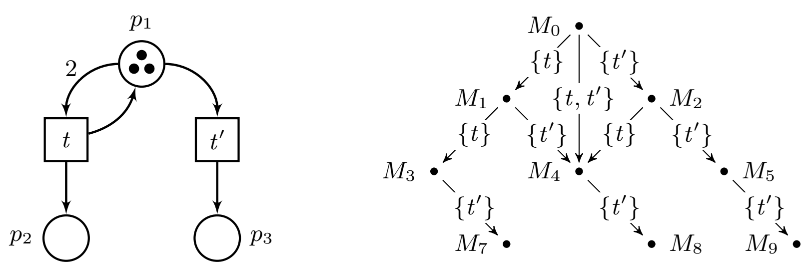

Fig. 11. A PT-net for the discussion of Proposition 11.

The implication in Proposition 11 cannot be reversed, and a counterexample is provided in Figure 12, where $\alpha = \{t,t'\}$ is a non-singleton step enabled (only) at M_0. There are two other nonempty steps enabled at M_0, viz. $\{t\}$ and $\{t'''\}$. Clearly, both $\alpha \cap \{t\} = \{t\}$ and $\alpha \cap \{t'''\} = \varnothing$ are positive steps. However, α is not GC-persistent as it is not enabled after the execution of t'''.

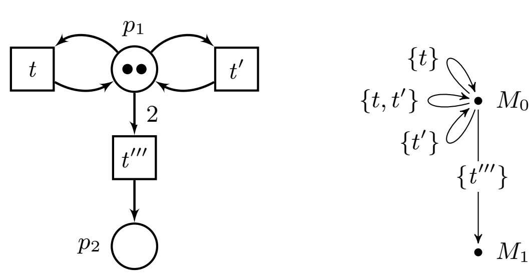

Fig. 12. A PT-net for the discussion of Proposition 11.

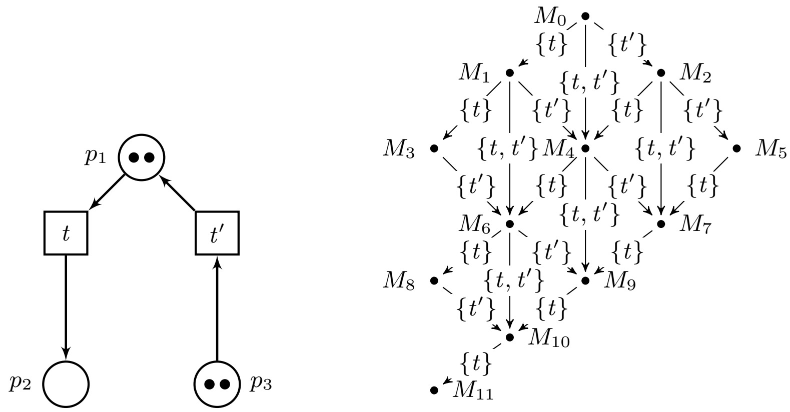

Fig. 13. A PT-net for the discussion of Proposition 11.

Finally, Proposition 11 cannot be re-stated for nonviolence, and a counterexample is provided in Figure 10, where $\alpha = \{t, t''\}$ is a GC-nonviolent non-singleton step, and $\beta = \{t\}$ is another step enabled together with α at M_0. However, $\alpha \cap \beta$ is not a positive step as $W(p_2, \alpha \cap \beta) > W(\alpha \cap \beta, p_2)$.

Theorem 2. *Let α be a* GC*-persistent non-singleton step of a* PT*-net N, and γ be a subset of α. Then:*

1. *For every reachable marking M enabling α, γ is* LC*-persistent at M.*
2. *γ is* GC*-nonviolent in N.*

Proof. (1) Follows directly from Proposition 10.

(2) As α is an active step, there is a reachable marking M enabling α. We now consider two cases.

Case 1: $\gamma \subset \alpha$. Then $\gamma \neq \alpha$ is a step enabled at M. As α is GC-persistent, we can use Proposition 11 to conclude that γ is positive.

Now, suppose that M' is a reachable marking enabling γ. We need to show that, for any step $\beta \neq \gamma$ enabled at M', we have $M'[\gamma\beta\rangle$. Let M'' be a marking such that $M'[\gamma\rangle M''$. This means $M''(p) = M'(p) - W(p, \gamma) + W(\gamma, p)$, for every $p \in P$. So, as γ is positive, we obtain that $M'' \geq M'$. Thus, from $M'[\beta\rangle$ and Fact 1, we have that $M''[\beta\rangle$, as was required.

Case 2: $\gamma = \alpha$. As α is a non-singleton step, we can represent it as a disjoint union of two nonempty subsets: $\alpha = \gamma' \uplus \gamma''$. From $M[\alpha\rangle$ and Fact 2, we have $M[\gamma'\rangle$ and $M[\gamma''\rangle$. Moreover, as α is GC-persistent, we can use Proposition 11 to conclude that both γ' and γ'' are positive steps. Therefore, γ is also positive. The rest of the proof is similar as in Case 1. □

In Proposition 11, the intersection of two different steps enabled at some reachable marking, $\alpha\cap\beta$, will be able to fire twice in a row (as α is GC-persistent).

As a result, $\alpha \cap \beta$ can be seen as a persistent step as well as a nonviolent step at markings that enable α (cf. Theorem 2(2)). As α can be covered by such intersections, GC-persistence of a non-singleton α implies its GC-nonviolence. In a way, in type-C case, the boundary between persistence and nonviolence blurs to some extent.

Type-A persistence and nonviolence are different in nature. They follow most closely the ideas of persistence and nonviolence in the sequential case: they complement each other. The next result shows the complementarity of A-nonviolent and A-persistent steps at some reachable marking M.

Theorem 3. *Let M be a reachable marking of a* PT*-net N. If there are two disjoint steps α and β enabled at M such that every enabled step at M is a subset of their union, then the following holds:*

α is LA*-persistent at M implies β is* LA*-nonviolent at M.*

Proof. Let γ be a step enabled at M such that $\gamma \cap \beta = \varnothing$. This and $\gamma \subseteq \alpha \cup \beta$ implies $\gamma \subseteq \alpha$. By Propositions 10 and 3(2), the step γ is LB-persistent, and so $M[\beta\gamma\rangle$. Since γ is an arbitrary step, disjoint from β and enabled at M, we obtain that β is LB-nonviolent at M. Finally, by Proposition 3(1), β is LA-nonviolent at M. □

The implication in opposite direction does not hold, even in the case of ordinary nets. The counterexample is presented in Figure 10. Taking $\alpha = \{t'\}$ and $\beta = \{t, t''\}$ we see that β is a LA-nonviolent step at M_0, but α is not LA-persistent at M_0. We can, for example execute step $\{t\}$ at M_0, which leads us to marking M_1, where α is not enabled.

We end this section with a result that gives sufficient conditions for a step of an ordinary PT-net to be GC-nonviolent. It is a counterpart of Theorem 1 formulated for the general PT-nets. Although the conditions here are more restrictive than those in Theorem 1, they are linked to the structure of a net rather than the behaviour.

Theorem 4. *Let α be an active step of an ordinary* PT*-net N. If α lies on self-loops, then α is* GC*-nonviolent in N.*

Proof. Since all the transitions in α lie on self-loops, α is positive. Hence, by Theorem 1, α is GC-nonviolent in N. □

7 Persistent and Nonviolent Steps in Safe PT-nets

In the case of safe PT-nets, due to their specific properties, we can identify more interesting properties of persistent or nonviolent steps and, in particular, link them to the structure of the nets. We start with results are concerned with the structural properties related to C-persistence and C-nonviolence.

Proposition 12 (see [6]). *Let α be a step which is* LC*-persistent or* LC*-nonviolent at a reachable marking M of a safe* PT*-net N. Then ${}^\bullet(\alpha \cap \beta) = (\alpha \cap \beta)^\bullet$, for every step $\beta \neq \alpha$ enabled at M.*

Proof. The result for LC-persistent α was proven for [6]. We therefore assume that $\alpha = \{t_1, \ldots, t_n\}$ is LC-nonviolent.
Suppose that $p \in {}^\bullet(\alpha \cap \beta)$, for some step $\beta \neq \alpha$ enabled at M. Clearly, $M(p) = 1$. Moreover, by Fact 3, α is disconnected.

Since α is LC-nonviolent, there is a marking M' such that $M[\alpha\rangle M'[\beta\rangle$. As $M'[\beta\rangle$ and $p \in {}^\bullet(\alpha \cap \beta)$, we have $M'(p) = 1$. Hence $M'(p) = M(p) - W(p, \alpha) + W(\alpha, p)$, and so $W(t_1, p) + \cdots + W(t_n, p) = W(p, t_1) + \cdots + W(p, t_n)$. By N being safe, all the arc weights in this formula are 0 or 1. Moreover, α is disconnected. It therefore follows that $W(p, t_i) = W(t_i, p)$, for each t_i. Hence $p \in (\alpha \cap \beta)^\bullet$, and so ${}^\bullet(\alpha \cap \beta) \subseteq (\alpha \cap \beta)^\bullet$.

Suppose now that $p \in (\alpha \cap \beta)^\bullet \setminus {}^\bullet(\alpha \cap \beta)$. Then, by $M[\alpha \cap \beta\rangle$ and the safeness of N, $M(p) = 0$. Hence, by $M[\alpha\rangle$ and $M[\beta\rangle$, we must have $p \notin {}^\bullet\alpha \cup {}^\bullet\beta$. Consequently, since there is M'' such that $M[\alpha\beta\rangle M''$, we obtain $M''(p) \geq 2$, a contradiction with N being safe. Hence ${}^\bullet(\alpha \cap \beta) \supseteq (\alpha \cap \beta)^\bullet$, and so the result holds. □

As a result, we can link LC-persistence and LC-nonviolence with the structural property of lying on self-loops.

Theorem 5 (see [6]). *Let α be a non-singleton step which is* LC*-persistent or* LC*-nonviolent at a reachable marking M of a safe* PT*-net N. Then α lies on self-loops.*

Proof. The result for LC-persistent α was proven for [6]. We therefore assume that α is LC-nonviolent. Suppose that $t \in \alpha$. Since $\{t\} \neq \alpha$ is a step enabled at M, by Proposition 12, ${}^\bullet(\alpha \cap \{t\}) = (\alpha \cap \{t\})^\bullet$. Hence ${}^\bullet t = t^\bullet$. □

Corollary 1. *Let α be a non-singleton active step of a safe* PT*-net N. Then α is* GC*-nonviolent iff all the transitions of α lie on self-loops.*

Proof. Follows from Theorems 4 and 5. □

Theorem 6 ([6]). *Let α be an active step of a safe* PT*-net N. If all the transitions in α are globally persistent and lie on self-loops, then α is* GC*-persistent in N.*

Theorem 6 can be seen as a persistence counterpart of Theorem 4 which was proved for ordinary nets. (The latter is in fact stronger as we only need to assume that α lies on self-loops.) We note that the implication in Theorem 6 cannot be reversed, and a suitable counterexample is provided in Figure 8, where $\{t'\}$ is a GC-persistent step, but it does not lie on self-loops. Moreover, Theorem 6 cannot be lifted to the level of ordinary nets, and Figure 14 provides a counterexample, where $\alpha = \{t, t'\}$ is neither locally nor globally C-persistent step even though both t and t' are globally persistent transitions lying on self-loops.

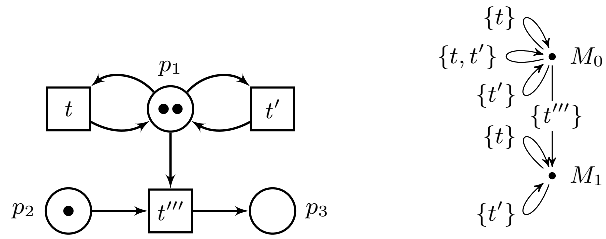

Fig. 14. An ordinary PT-net for the discussion of Theorem 6.

It is interesting to establish whether persistence or nonviolence are preserved by taking substeps. For general PT-nets we only had results concerning the LA, LC and GC persistent steps. Here we can obtain similar results about nonviolence. Moreover, for the type-C of nonviolence the result holds globally.

Proposition 13. *Let $\gamma \subseteq \alpha$ be two steps and M be a reachable marking of a safe* PT*-net N. Then:*

1. *If α is* LA*-nonviolent at M then γ is* LA*-nonviolent at M.*
2. *If α is* LC*-nonviolent at M then γ is* LC*-nonviolent at M.*
3. *If α is* GC*-nonviolent in N then γ is* GC*-nonviolent in N.*

Proof. As the case $\alpha = \gamma$ is obvious, below we assume that $\gamma \subset \alpha$. Also, we assume that γ is nonempty, as the empty step trivially satisfies Definitions 3 and 4.

(1) From $M[\alpha\rangle$ and $\gamma \subset \alpha$, we have $M[\gamma\rangle$. Let $\beta \neq \gamma$ be a step enabled at M. We need to prove that $M[\gamma\rangle M''[\beta \setminus \gamma\rangle$, for some marking M''. We consider two cases.

Case 1: $\beta \neq \alpha$. Since α is LA-nonviolent at M, we have $M[\alpha(\beta \setminus \alpha)\rangle$. Hence, for every place $p \in {}^\bullet(\beta \setminus \alpha)$, $p \in {}^\bullet\alpha$ implies $p \in \alpha^\bullet$. Furthermore, since α is disconnected (by Fact 3), we have that, for every place $p \in {}^\bullet(\beta \setminus \alpha)$ and transition $t \in \alpha$, $p \in {}^\bullet t$ implies $p \in t^\bullet$. As a result, for every place $p \in {}^\bullet(\beta \setminus \alpha)$, $p \in {}^\bullet\gamma$ implies $p \in \gamma^\bullet$. Hence, by $M[\beta\rangle$, we obtain that $M''[\beta \setminus \alpha\rangle$. We further observe that, by Facts 2 and 2, we get $M''[(\alpha \setminus \gamma) \cap \beta\rangle$. It therefore follows that all the transitions in $(\beta \setminus \alpha) \cup (\alpha \setminus \gamma) \cap \beta = \beta \setminus \gamma$ are enabled at M''. Moreover, as $\beta \setminus \gamma \subset \beta$ and $M[\beta\rangle$, we obtain from Fact 3 that the step $\beta \setminus \gamma$ is disconnected. Hence, again by Fact 3, $M''[\beta \setminus \gamma\rangle$.

Case 2: $\beta = \alpha$. Since $M[\alpha\rangle M'$ and $\gamma \subset \alpha$, by Fact 2, $M[\gamma\rangle M''[\alpha \setminus \gamma\rangle$.

(2) Since $\varnothing \neq \gamma \subset \alpha$, α is a non-singleton step. Thus, by Theorem 5, α lies on self-loops. Hence γ also lies on self-loops, and so we have $M[\gamma\rangle M[\beta\rangle$ as required.

(3) Since α is GC-nonviolent, it is LC-nonviolent at some reachable marking M. Proceeding similarly as in (2), we get that α lies on self-loops and, consequently, that γ lies on self-loops. Then, from Theorem 1, proved for general PT-nets, we obtain that γ is GC-nonviolent. □

Proposition 13 does not hold for ordinary PT-nets, and Figure 10 shows a counterexample. The step $\{t, t''\}$ there is both GA and GC-nonviolent as well as LA and LC-nonviolent at M_0 (the only marking which enables it), but its substep $\{t\}$ is neither LA-nonviolent nor LC-nonviolent at M_0 (as once it is executed, the previously enabled step $\{t'\}$ becomes disabled). Also, Proposition 13(1) cannot be generalised to GA-nonviolent steps, an a suitable counterexample is provided in Figure 3, where $\{t, t''\}$ is a GA-nonviolent step, but its substep $\{t\}$ is not GA-nonviolent (as after executing $\{t\}$ at M_0, an enabled step $\{t'\}$ becomes disabled).

Theorem 7. *Let α be a* GC*-nonviolent step of a safe* PT*-net N. Then all the transitions in α are globally nonviolent in N.*

Proof. Let $t \in \alpha$. Consider a marking M enabling α, and so $M[t\rangle$. Then, from $\{t\} \subseteq \alpha$, the fact that α is GC-nonviolent and Proposition 13(3), we obtain that $\{t\}$ is GC-nonviolent in N. This means in particular that, for any reachable marking M of N enabling $\{t\}$, if $\{t'\} \neq \{t\}$ is a step enabled at M, we have $M[\{t\}\{t'\}\rangle$. We can therefore conclude that t, as a transition (rather than a step), is globally nonviolent (see Definition 2). $\square$

The above result does not hold for ordinary PT-nets, and a suitable counterexample is provided in Figure 10 which we used to demonstrate that Proposition 13 does not hold for ordinary PT-nets. In the latter case, we took a singleton substep of a GC-nonviolent step of an ordinary PT-net and showed that it disables another singleton step. The two singleton steps can be treated as transitions here.

The next two results give sufficient conditions for a step to be GA-persistent or GA-nonviolent in terms of their constituent transitions.

Theorem 8 (see [6]). *Let α be an active step of a safe* PT*-net N. If all the transitions in α are globally persistent (nonviolent) in N, then α is* GA*-persistent (resp.* GA*-nonviolent) in N.*

Proof. In the case of persistence, the result was proven for [6]. We therefore assume that all the transitions in α are globally nonviolent. Let M be a reachable marking and $\beta \neq \alpha$ be a step in N such that $M[\alpha\rangle$ and $M[\beta\rangle$. Note that, by Fact 3, β is disconnected. We need to show that $M[\alpha(\beta \setminus \alpha)\rangle$.

Assume that $\alpha = \{t_1, \ldots, t_m\}$ and $\beta \setminus \alpha = \{u_1, \ldots, u_k\}$. From $M[\alpha\rangle$ and Fact 2, we have $M[t_1 \ldots t_m\rangle M'$. Now, for each transition u_i, since $M[u_i\rangle$ and every t_i is globally nonviolent, we have that $M'[u_i\rangle$. Thus, by Fact 3, $M'[\beta \setminus \alpha\rangle$, and so $M[\alpha(\beta \setminus \alpha)\rangle$. $\square$

The two implications in Theorem 8 cannot be reversed, and and a suitable counterexample is provided in Figure 3, where a GA-nonviolent and GA-persistent step $\{t, t''\}$ contains a transition t that is neither globally nonviolent nor globally persistent (because of the marking M_0).

The last result concerning persistence and nonviolence in safe PT-nets, shows how they can complement each other. It is a counterpart of Theorem 3, but here the result holds in both directions due to Proposition 13, which was not available for the general nor ordinary PT-nets.

Theorem 9. *Let M be a reachable marking of a safe* PT*-net N. If there are two disjoint steps α and β enabled at M such that every enabled step at M is a subset of their union, then the following holds:*

$$\alpha \text{ is LA-persistent at } M \text{ iff } \beta \text{ is LA-nonviolent at } M.$$

Proof. The ($\Longrightarrow$) implication follows from Theorem 3. To show the ($\Longleftarrow$) implication, let γ be a step enabled at M such that $\gamma \cap \alpha = \varnothing$. Since $\gamma \subseteq \alpha \cup \beta$, we have $\gamma \subseteq \beta$. As β is LA-nonviolent at M we have, from Proposition 13(1), that γ is LA-nonviolent at M. By Proposition 3(1), we have that γ is also LB-nonviolent at M, and so $M[\gamma\alpha\rangle$. Since γ is an arbitrary step, disjoint from α and enabled at M, we get that α is LB-persistent at M. Using Proposition 3(2), we can then conclude that α is LA-persistent at M. □

8 Conclusions

In this paper we initiated a comprehensive investigation of different notions of persistence and nonviolence in the step based semantics of concurrent systems. Among the problems and issues we plan to investigate in future are the phenomenon of confusion on the level of steps, persistence and nonviolence in the context of whole processes, and some less restrictive notions of persistence and nonviolence, especially a guarantee of disabling retrieve known as k-persistence.

Acknowledgement

This research was supported by a fellowship funded by the "Enhancing Educational Potential of Nicolaus Copernicus University in the Disciplines of Mathematical and Natural Sciences" Project POKL.04.01.01-00-081/10, and the EPSRC GAELS and UNCOVER projects.

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