Lecture Notes in Computational Science and Engineering 5

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Springer-Verlag Berlin Heidelberg GmbH

Dietmar Kröner Mario Ohlberger
Christian Rohde (Eds.)

An Introduction to Recent Developments in Theory and Numerics for Conservation Laws

Proceedings of the
International School on Theory and Numerics
for Conservation Laws,
Freiburg/Littenweiler, October 20–24, 1997

Springer

Editors

Dietmar Kröner
Mario Ohlberger
Christian Rohde

Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Straße 10
D-79104 Freiburg, Germany

e-mail:
dietmar@mathematik.uni-freiburg.de
mario@mathematik.uni-freiburg.de
chris@mathematik.uni-freiburg.de

The picture on the cover shows the time-evolved density distribution of a magneto-hydrodynamic Kelvin-Helmholts instability. The colors indicate an increasing density from blue to red.
Numerics: M. Wesenberg (Freiburg/Germany)
Visualization Software: GRAPE

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

An introduction to recent developments in theory and numerics for conservation laws; proceedings of the International School on Theory and Numerics for Conservation Laws, Freiburg/Littenweiler, October 20–24, 1997 / Dietmar Kröner... (ed.). – Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris; Singapore; Tokyo: Springer, 1999
(Lecture notes in computational science and engineering; 5)
ISBN 978-3-540-65081-2 ISBN 978-3-642-58535-7 (eBook)
DOI 10.1007/978-3-642-58535-7

Mathematics Subject Classification (1991): primary: 35L65, 76N15, 65M12
secondary: 76N10, 65M15, 76M10

ISBN 978-3-540-65081-2

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Originally published by Springer-Verlag Berlin Heidelberg New York in 1999

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Cover Design: Friedhelm Steinen-Broo, Estudio Calamar, Spain
Cover production: *design & production* GmbH, Heidelberg
Typeset by the editors using a Springer TEX macro package

SPIN 10653041 46/3143 – 5 4 3 2 1 0 – Printed on acid-free paper

Preface

The importance of hyperbolic conservation laws for scientific and industrial applications has led to a growing amount of research activity in this field. A variety of physical phenomena in fluid mechanics, astrophysics, groundwater flow, meteorology, reactive flow and several other areas can be effectively modeled by systems of conservation laws. In order to focus recent trends in theory and numerics in this research area and to stimulate further research in this field, we decided to organize the "International School on Theory and Numerics for Conservation Laws", which took place in Freiburg/Littenweiler, Germany, from 20 to 24 October 1997.
The school was sponsored by the DFG-Graduiertenkolleg "Nichtlineare Differentialgleichungen, Modellierung, Theorie, Numerik, Visualisierung" at the University of Freiburg. It was attended by about 60 young international researchers.

This volume contains the contributions of the five main lecturers of the school. Each article covers five hours of lectures on a specific new research area in the field of theory and numerics for conservation laws. Reviews of recent developments are given, accompanied by new research results of the authors.

The topics include a kinetic approach to conservation laws by Benoit Perthame, which can be used for the construction of approximate Riemann solvers for the full system of gas dynamics. Several ideas related to the stability and entropy analysis are discussed.
An introduction to the theory of non-classical shock waves is given by Philippe G. LeFloch. Non-classical shocks violate the classical entropy condition introduced by Lax and Liu, but they are sensitive to regularization mechanisms such as diffusion and dispersion. A theoretical approach to such shocks is given and a numerical approximation via finite difference schemes is investigated.
In the contribution of Athanasios E. Tzavaras the approximation of hyperbolic systems of conservation laws via viscosity and relaxation methods is discussed. In the context of zero-viscosity limits recent results on obtaining uniform BV estimates are presented. Furthermore the problem of construcing entropy weak solutions for hyperbolic conservation laws via relaxation is considered.
A major task for numerical approaches to hyperbolic problems is the development of *a posteriori* estimates and resulting adaptive strategies. Endre Süli presents an overview of recent developements in this area for finite element approximations of hyperbolic problems. The *a posteriori* analysis is based on the systematic use of hyperbolic duality arguments. The question

of computational implementation of the *a posteriori* bounds into adaptive finite element algorithms is discussed as well.
Finally Timothy J. Barth considers discretization and preconditioning algorithms for the Euler and Navier-Stokes equations on unstructured meshes. Some new results concerning congruence relationships for left or right symmetrized equations are presented. These results suggest new variants of existing finite volume, discontinuous Galerkin, Galerkin least–squares and fluctuation splitting schemes. The resulting schemes are computationally more efficient while retaining the pleasant theoretical properties achieved by entropy symmetrization.

The book is intended for students who work in the field of conservation laws and researchers who want to open up a new scope of work. Knowledge of modern methods in partial differential equations, specifically with basic concepts for hyperbolic equations, is a prerequisite.

Last but not least we thank M. Wesenberg for the support in the organization of the school and R. Axthelm and M. Werner for typesetting this book.

September 1998

Dietmar Kröner
Mario Ohlberger
Christian Rohde

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