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Sliding Modes in Control and Optimization

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Preface

The book is devoted to systems with discontinuous control. The study of discontinuous dynamic systems is a multifacet problem which embraces mathematical, control theoretic and application aspects. Times and again, this problem has been approached by mathematicians, physicists and engineers, each profession treating it from its own positions. Interestingly, the results obtained by specialists in different disciplines have almost always had a significant effect upon the development of the control theory. It suffices to mention works on the theory of oscillations of discontinuous nonlinear systems, mathematical studies in ordinary differential equations with discontinuous righthand parts or variational problems in nonclassic statements.

The unremitting interest to discontinuous control systems enhanced by their effective application to solution of problems most diverse in their physical nature and functional purpose is, in the author's opinion, a cogent argument in favour of the importance of this area of studies. It seems a useful effort to consider, from a control theoretic viewpoint, the mathematical and application aspects of the theory of discontinuous dynamic systems and determine their place within the scope of the present-day control theory. The first attempt was made by the author in 1975–1976 in his course on "The Theory of Discontinuous Dynamic Systems" and "The Theory of Variable Structure Systems" read to post-graduates at the University of Illinois, USA, and then presented in 1978–1979 at the seminars held in the Laboratory of Systems with Discontinous Control at the Institute of Control Sciences in Moscow.

First of all, the scope of problems to be dealt with should be outlined. The object of our attention will be a system in which the discontinuity of control in each control component is "prescribed" prior to the stages of selection of the criterion and design unlike, for instance, optimal systems where the need for jumpwise control variations at a finite or infinite frequency appears as a result of the solution of the variational problem.

Let us present major arguments showing why the class of discontinuous control systems provides such an effective tool for solving the entire family of control problems for complex dynamic plants. The discontinuity of control results in a discontinuity of the righthand part of the differential equations describing the system motions. If such discontinuities are deliberately introduced on certain surfaces in the system state space, then motions in a sliding mode may occur in the system. This type of motion features some attractive properties and has long since become applied in relay systems. Sliding modes are the basic motions in variable structure systems.

Consider the properties of sliding modes in some greater detail. First, the trajectories of the state vector belong to manifolds of lower dimension than that of the whole state space, therefore the order of differential equations describing sliding motions is also reduced. Second, in most of practical systems the sliding motion is control-independent and is determined merely by the properties of the control plant and the position (or equations) of the discontinuity surfaces. This allows the initial problem to be decoupled into independent lower dimension subproblems wherein the control is "spent" only on creating a sliding mode while the required character of motion over the intersection of discontinuity surfaces is provided by an appropriate choice of their equations. These properties turn to be quite essential for solving many application problems characterized by high order differential equations which prohibits the use of efficient analytic techniques and computer technology. A third specific feature of a sliding mode is that under certain conditions it may become invariant to variations of dynamic characteristics of the control plant which poses a central problem dealt with in the theory of automatic control. It is essential that, unlike continuous systems with nonmeasurable disturbances in which, the conditions of invariancy require the use of infinitely high gains, the same effect in discontinuous systems is attained by using finite control actions.

Finally, a purely technological aspect of using discontinuous control systems should be mentioned. To improve performance, electric inertialess actuators are increasingly employed now, built around power electronic elements which may operate in a switching mode only. Therefore even if we employ continuous control algorithms the control itself is shaped as a high frequency discontinuous signal whose mean value is equal to the desired continuous control. A more natural way then will be to employ such algorithms which are deliberately oriented toward the use of discontinuous controls.

Let us describe the problem faced in an attempt to employ the properties of sliding modes for the design of automatic control systems. Consider first the mathematical problems treated in Part 1. Discontinuous dynamic systems are outside the scope of the classical theory of differential equations and require the development of ad-hoc techniques to study their behaviour. Various publications on the matter have shown a diversity of viewpoints of their authors as to how the motion on a discontinuity boundary should be described, thus leading to diverse sliding mode equations. An approach followed by this author implies regularization through an introduction of a boundary layer which, on the one hand, allows the reason for ambiguity to be revealed and, on the other hand, outlines the class of systems whose sliding equations can be written quite unambiguously. Various methods of designing automatic control systems treated in this book are applied to exactly such class of systems.

As was already noted, the approach used in the book is oriented toward a deliberate introduction of sliding modes over the intersection of surfaces on which the control vector components undergo discontinuity. Realization of such approach obviously implies the knowledge of the conditions of the occurrence of sliding modes, rarely discussed in the literature. From the point of view of mathematics, the problem may be reduced to that of finding the area of attraction to the manifold of the discontinuity surfaces intersection. The solutions suggested are formulated in terms of the stability theory and are obtained via generalization of the classical Lyapunov theorems to discontinuous systems featuring not merely individual motions but, rather, a whole set of motion trajectories from a certain domain on the discontinuity surface.

Part 1 of the book is concluded with problems of robustness of discontinuous systems with respect to small dynamic imperfections disregarded in an idealized model yet always present in a real-life system due to small inertialities of measuring instruments, actuators, data processing devices, etc. In the theory of singularly perturbed differential equations, the control in a continuous system may be regarded as a continuous function of small time constants, provided the fast decaying motions are neglected; consequently, the same property is featured by solutions of the equations describing the motion in the system. This conclusion provides grounds for applying simplified models to solve various control problems in the class of continuous systems. In a discontinuous control system operating in a sliding mode small additional time constants neglected in the idealized model may cause a shift in the switching times thus making the controls in the idealized and real systems essentially different. As a result, the solution obtained may prove to be nonrobust to some insignificant changes in the model of our process which, in its turn, makes very doubtful the entire applicability of the control algorithms to the considered class of discontinuous systems. In this connection, a study of singularly perturbed discontinuous systems is carried out to show that despite the lack of any continuous correlation between the control and the small time constants, the systems with unambiguous equations of their sliding mode motions are robust with respect to small dynamic imperfections.

The major focus in the book is on control systems design methods, discussed in Part II. Despite the diversity of the control goals in the problems considered in this part and in their solution techniques, all the design procedures are built around a common principle: the initial system is decoupled into independent subsystems of lower dimension and sliding motions with required properties are designed at the final stage of a control process. The methods of the analysis of discontinuous dynamic systems suggested in the first part of the book serve as a mathematical background for the realization of this principle.

Part II of the book is aimed not only at stating and developing the results obtained in the sphere of discontinuous control system design, but also at presenting these results in close correlation with the basic concepts, problems and methods of the present-day control theory. This refers both to newly obtained results and to those which have appeared earlier in the literature.

Some problems traditionally posed in the linear control theory such as eigenvalue allocation and quadratic and parametric optimization have been treated with the use of the fundamental control theory concepts of controllability, stabilizability, observability and detectability. The analogs of the well-known theorems on the uniqueness and existence of solutions to such problems are given, and procedures of obtaining the solutions for systems with sliding modes are suggested.

The problem of invariance as applied to linear systems with vector-valued controls and disturbances is treated under the assumption that a dynamic model of disturbances with unknown initial conditions is available. However, in contrast to the well-known methods, the approach suggested does not require the knowledge of accurate values of the model parameters, for it is quite sufficient to know just the range of their possible variations.

Since all design methods are based on the idea of decoupling the system, it seems interesting to compare discontinuous systems with continuous ones realizing the same idea through the use of infinitely high gains. As is well known, the increase of the gain in an open-loop system decouples its overall motion into fast and slow components which may be synthesized independently. The comparison shows that systems with infinitely high gains are particular cases of singularly perturbed systems and their slow motions may be described by the same equations as used for sliding motions in discontinuous systems. The only difference is that in continuous systems these motions may be attained only asymptotically in tending the gains to infinity, while in discontinuous systems the same is attainable in a finite time and with a finite control.

Besides finite-dimensional control problems, a design method for a system with a mobile control in a distributed parameter control plant is discussed in the book. Most probably, this may be regarded as the first attempt to apply sliding modes to control plants of this class. The problem is in steering the controlled parameter to the required spatial distribution in the system with a mobile control subjected to uncontrollable disturbances. A deliberate introduction of sliding modes in the loops responsible for the control of the system motion and the intensity of the source permits such a problem to be solved using the information on the current state of the system, provided the magnitudes the spatially distributed disturbances and their velocities are bounded. The same statement is used to solve the problem in the case of a distributed or lumped control both for the control plants with a single spatial variable and for a set of interconnected distributed plants.

The problems of control and optimization under incomplete information on the operator and the system state as well as computational problems associated with the search for the optimal set of system parameters are considered in the final chapters of Part II. In solving these problems, continuous or discontinuous asymptotic observers and filters, nominal or adjustable models, search procedures, etc. are used wherein the dynamic processes of adaptation, identification or tending to an extremum are conducted in a sliding mode.

A sufficiently great deal of experience has been acquired up to the present in using sliding modes in various application problems. It seemed expedient to devote a separate section of the book (Part III) to applications and provide some examples, most convincing in the author's opinion. The first example dealing with the control of a robot arm is given to illustrate the whole idea. The use of sliding modes for this multivariable high order nonlinear design problem has made it possible to realize motions invariant to load torques and mechanical parameters which could be described by independent linear uniform differential equations with respect to each of the controlled parameters.

Perhaps the most suitable for the application of sliding modes proved to be one of the basic engineering problems, that of control of electrical machines and, in particular, of electrical motors. Increasingly dominating nowadays are AC motors built around power electronic elements operating in a switching mode, which puts the problem of the algorithmic supply in the forefront. The technological aspect of this problem has already been discussed and the considerations speaking in favour of the control algorithms oriented toward introduction of sliding modes. The principles of designing multivariable discontinuous control systems suggested in Part II may be easily interpreted in terms of electrical engineering: serving as the control vector components in this case are discontinuous voltages at the output of power semiconductor converters which are then fed to the motor phases and to the excitation winding (if any); the motion differential equations for all types of motors which are generally nonlinear turn to be linear with respect to the control (in which case the sliding equations are written unambiguously); only some components of the state vector are to be controlled (for instance, angular position of the motor shaft, angular velocity and torque, magnetic flux, power coefficient, currents, etc.) Wide-scale experimental tests have testified to the efficiency of sliding mode control for any types of electrical motors, including induction motors, which are known to be the most reliable and economic yet the hardest to control.

Considered in Chapter 17 of Part III is a set of problems associated with control algorithms for DC, induction and synchronous motors and with methods of acquiring information on the controlled process state.

The closing chapter of this part presents the results obtained in implementing sliding mode control for electrical drives of metal-cutting machine tools and transport vehicles utilizing both independent and external power sources. The potential of sliding mode systems is demonstrated for physical processes most diverse in their nature such as chemical fibre production, metal melting in electric arc furnaces, processes in petroleum refining and petrochemical industries, automation in fishery, stabilization of the resonant frequency of an accelerator intended for physical experiments.

It is obvious that all feasible applications of systems with sliding modes could not be possibly covered in one book. The examples given in Part III have made a stress upon the algorithms not touching the problems of their technical implementation. Consideration of such problems would somehow fall out of the general theoretical line of the book. Nonetheless, the author believes that it is some specific examples that makes the "Mathematical Tools" and "Design" parts of the book appropriately completed.

The author takes a chance to express his deep gratitude to all of his colleagues who have contributed, directly or indirectly, to the book. Most fruitful have been his discussions of the mathematical and technical aspects of the theory of singularly perturbed systems with Prof. A. Vasilyeva and Prof. P. Kokotovic. The results of these discussions have formed a framework of Chapters 5 and 11 where a comparison is made between singularly perturbed systems, high gain systems and discontinuous control systems. The results of Section 3 in Chapter 4 are a "direct corollary" of a discussion with Prof. A. Filippov of the problem of existence of multidimensional sliding modes. Prof. D. Siljak has drawn the author's attention to a possibility of constructing an upper bound for the Lyapunov function using the comparison principle (Section 6, Chapter 4). The statement of the problem of mobile control for a distributed system has been suggested by Prof. A. Butkovsky with whom the author have subsequently solved this problem in a joint effort.

The book has gained from the results published in recent years by Dr. K. Young (Section 1, Chapter 13 and Chapter 15) who had started enthusiastic work in this field while a post-graduate student.

In the works conducted at the Institute of Control Sciences in the design of control algorithms for electrical machines, the basic contribution was made

by Dr. D. Izosimov who helped the author to write Chapter 17. The control principles developed in these studies have been extended by post-graduate student S. Ryvkin to synchronous electrical motors (Section 4, Chapter 17).

Unquestionably, it was a good luck for the author that the Editorial Board of the Publishing House made a decision to send the book for review to Prof. A. Filippov, who studied it with a mathematician's rigour and made a number of useful remarks and advices to improve the manuscript.

The appearance of this book could be hardly possible without a permanent help from N. Kostyleva throughout all of its stages starting with the idea to write it and finishing with the preparation of the final version of the manuscript. The seminars held in the Laboratory of Discontinuous Control Systems of the Institute of Control Sciences and discussions with his co-workers have significantly helped the author shape his ideas on the methodology of presenting this material and its correlation with various parts of the control theory.

The author is indebted to L. Govorova who has faultlessly accomplished a hard task of typing the manuscript.

In preparing the English version of the book, the author has gratefully used a chance given by Springer-Verlag to revise the book, and to include some results obtained in the theory of sliding modes and its applications after the publication of the book in Russian in 1981. These refer to stochastic regularization of discontinuous dynamic systems (Section 5, Chapter 2), control of distributed systems (Sections 2 through 4, Chapter 12), the method of system parameter identification employing discontinuous dynamic models (Section 2, Chapter 13), robustness of sliding modes to dynamic discrepancies between the system and its model (Sections 3 and 4, Chapter 14). Part III was enriched by a new Chapter 18 describing direct application of the sliding mode control to a large class of technological systems.

The project of publishing the book in the English translation could hardly ever be realized without constant friendly support and help from Prof. M. Thoma, to whom the author is sincerely indebted.

Moscow, May 1991 Vadim I. Utkin

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