

# Texts in Theoretical Computer Science

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Wan Fokkink

# Introduction to Process Algebra

With 11 Figures and 11 Tables



Springer

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# Preface

Computer software and network protocols are increasingly important in daily life. At the same time the complexity of software has rocketed, so that its correctness is at stake. New methodologies and disciplines are being developed to bring structure to the ever growing jungle of computer technology. (Semi-)automated manipulation has become an important means in discovering flaws in software and hardware systems. Process algebra is a mathematical framework in which system behaviour is expressed in the form of algebraic terms, enhancing the available techniques for manipulation.

Concurrency is omnipresent in system behaviour, and in a large part responsible for its complexity: even simple behaviours become wildly complicated when they are executed in parallel. In order to study such systems in detail, it is imperative that they are dissected into their concurrent components. Fundamental to process algebra is a parallel operator, to break down systems into their concurrent components. A set of equations is imposed to derive whether two terms are behaviourally equivalent. In this framework, non-trivial properties of systems can be established in an elegant fashion. For example, it may be possible to equate an implementation to the specification of its required input/output relation. In recent years a variety of automated tools have been developed to facilitate the derivation of such properties.

Applications of process algebra exist in diverse fields such as safety critical systems, network protocols, and biology. In the educational vein, process algebra has been recognised to teach skills to deal with complex concurrent systems, by representing and reasoning about such systems in a mathematically clear and precise manner.

This text developed from an undergraduate course on process algebra at the computer science department of the University of Wales Swansea during the autumn of 1997 and of 1998. Chapters 2-7 contain sufficient material for more than twenty hours of lecturing; a set of slides and further material to support such a course are available from my homepage (currently at <http://www.cwi.nl/~wan>). It is recommended to use a tool set based on process algebra, such as the  $\mu$ CRL tool set or the Concurrency Workbench Edinburgh, to enliven the course. Appendices A and B provide useful background information; they are not intended to be included in the course.

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I am grateful to John Tucker for his encouragement to further develop a raw set of lecture notes, and to Judi Romijn for her support. Over the years I have benefited from discussions with Jan Bergstra, Rob van Glabbeek, Jan Friso Groote, Frits Vaandrager, Alban Ponse, Chris Verhoef, Jaco van de Pol, Jos Baeten, Luca Aceto, Jos van Wamel, Steven Klusener, Bas Luttik, Dennis Dams, and many others.

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*Wan Fokkink*

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