Texts in Theoretical Computer Science
An EATCS Series

Springer-Verlag Berlin Heidelberg GmbH

Lane A. Hemaspaandra • Mitsunori Ogihara

The Complexity Theory Companion

With 43 Figures

Springer

Authors

Prof. Dr. Lane A. Hemaspaandra
Prof. Dr. Mitsunori Ogihara
Department of Computer Science
Rochester, NY 14627
USA
{lane,ogihara}@cs.rochester.edu

Library of Congress Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

Hemaspaandra, Lane A.:
The complexity theory companion/Lane A. Hemaspaandra; Mitsunori Ogihara. –
Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan;
Paris; Tokyo: Springer, 2002
(Texts in theoretical computer science)

ACM Computing Classification (1998): F.1, F.2.2, F.4

ISBN 978-3-642-08684-7   ISBN 978-3-662-04880-1 (eBook)
DOI 10.1007/978-3-662-04880-1

Originally published by Springer-Verlag Berlin Heidelberg New York in 2002.
Softcover reprint of the hardcover 1st edition 2002

Cover Design: KünkelLopka, Heidelberg
Typesetting: Camera-ready by the authors
Printed on acid-free paper   SPIN 10723529   45/3142SR - 5 4 3 2 1 0

This book is dedicated to our families—the best of companions.

Preface

Invitation

Secret 1 *Algorithms are at the heart of complexity theory.*

That is the dark secret of complexity theory. It is recognized by complexity theorists, but would be literally incredible to most others. In this book, we hope to make this secret credible. In fact, the real secret is even more dramatic.

Secret 2 *Simple algorithms are at the heart of complexity theory.*

A corollary of Secret 2 is that every practitioner of computer science or student of computer science already possesses the ability required to understand, enjoy, and employ complexity theory.

We realize that these secrets fly in the face of conventional wisdom. Most people view complexity theory as an arcane realm populated by pointy-hatted (if not indeed pointy-headed) sorcerers stirring cauldrons of recursion theory with wands of combinatorics, while chanting incantations involving complexity classes whose very names contain hundreds of characters and sear the tongues of mere mortals. This stereotype has sprung up in part due to the small amount of esoteric research that fits this bill, but the stereotype is more strongly attributable to the failure of complexity theorists to communicate in expository forums the central role that algorithms play in complexity theory.

Throughout this book—from the tree-pruning and interval-pruning algorithms that shape the first chapter to the query simulation procedures that dominate the last chapter—we will see that proofs in complexity theory usually employ algorithms as their central tools. In fact, to more clearly highlight the role of algorithmic techniques in complexity theory, *this book is organized by technique rather than by topic.* That is, in contrast to the organization of other books on complexity theory, each chapter of this book focuses on one technique—what it is, and what results and applications it has yielded.

The most thrilling times in complexity theory are when a new technique is introduced and sweeps like fire over the field. In addition to highlighting the centrality of algorithms in the proof arsenal of complexity theory, we feel that our technique-based approach more vividly conveys to the reader the flavor and excitement of such conflagrations. We invite the reader to come with us as we present nine techniques, usually simple and algorithmic, that burned away some of the field's ignorance and helped form the landscape of modern complexity theory.

Usage

We intend this book as a companion for students and professionals who seek an accessible, algorithmically oriented, research-centered, up-to-date guide to some of the most interesting techniques of complexity theory. The authors and their colleague Joel Seiferas have test-driven the book's approach in two different courses at the University of Rochester. We have used this technique-based approach in Rochester's one-semester basic complexity theory course, which is taken by all first-year computer science graduate students and also by those undergraduates specializing or specially interested in theoretical computer science, and in our second course on complexity theory, which is taken by all second-year graduate students as their theory "breadth" course.

We found in both these course settings that the technique-based approach allowed us to impart to students a significant amount of the feel and experience of complexity theory research and led to more student interest and involvement than occurred in earlier course incarnations using other texts. We expect that this will not only benefit the complexity theory students in the courses, but will also help all the course's students become prepared to do work that is theoretically aware, informed, and well-grounded.

At times, we stop the flow of a proof or discussion with a "Pause to Ponder." These are places at which we encourage the reader to pause for a moment and find his or her own solution to the issue raised. Even an unsuccessful attempt to craft a solution will usually make the proof/discussion that follows clearer and more valuable, as the reader will better understand the challenges posed by the hurdle that the proof/discussion overcomes.

With some exceptions due to result dependencies, the non-appendix chapters are generally ordered to put the easier chapters near the start of the book and the more demanding chapters near the end of the book.

Acknowledgments

We are extraordinarily indebted to the following people, who proofread one or more chapters, for their invaluable help, suggestions, corrections, and insights: Eric Allender, Russell Bent, Alina Beygelzimer, Matthew Boutell, Samuel Chen, Yin-He Cheng, Louis Deaett, Gregory Goldstein, Fred Green, Ulrich Hertrampf, Chris Homan, Gabriel Istrate, Jason Ku, David Lagakos, Andrew Learn, Tao Li, Ioan Macarie, Proshanto Mukherji, Kenneth Regan, William Scherer III, Alan Selman, D. Sivakumar, Howard Straubing, Robert Swier,

Mayur Thakur, Jonathan Tomer, Jacobo Torán, Leen Torenvliet, Dieter van Melkebeek, Heribert Vollmer, Julie Zhong, and Marius Zimand. We also thank the many other people who have helped us with advice, comments, corrections, literature pointers, most-recent-version information, and suggestions: Andris Ambainis, Vikraman Arvind, Richard Beigel, Nate Blaylock, Daniel Bovet, Jin-Yi Cai, Diane Cass, Stephen Fenner, Lance Fortnow, William Gasarch, Viliam Geffert, Oded Goldreich, Juris Hartmanis, Edith Hemaspaandra, Paul Ilardi, Sven Kosub, Richard Lipton, Alexis Maciel, Wolfgang Merkle, Christos Papadimitriou, Thanos Papathanasiou, Eduardo Pinheiro, Robert Rettinger, Jörg Rothe, Alex Samorodnitsky, Marcus Schaefer, Michael Schear, Uwe Schöning, Joel Seiferas, Samik Sengupta, Carl Smith, Scott Stoness, Madhu Sudan, Chunqiang Tang, Jun Tarui, Thomas Thierauf, Luca Trevisan, Chris Umans, Osamu Watanabe, Chee Yap, and Stathis Zachos. Any remaining errors are the responsibility of the authors.

We thank our thesis advisors, Juris Hartmanis and Kojiro Kobayashi; their insightful, dedicated, joyous approach to research has been a continuing inspiration to us.

We appreciate the grants—NSF-CCR-8957604, NSF-INT-9116781/JSPS-ENGR-207, NSF-CCR-9322513, NSF-INT-9513368/DAAD-315-PRO-fo-ab, NSF-CCR-9701911, NSF-CCR-9725021, NSF-INT-9726724, NSF-INT-9815095/DAAD-315-PPP-gü-ab NSF-DUE-9980943, DARPA-F30602-98-2-0133, NIA-R01-AG18231—that have supported our research programs during the planning and writing of this book.

For generous advice, help, and support, we thank the Springer-Verlag series editors and staff, namely, Wilfried Brauer, Grzegorz Rozenberg, Arto Salomaa, Alfred Hofmann, Frank Holzwarth, Ingeborg Mayer, Sherryl Sundell, and Hans Wössner. Our own department's technical and secretarial staff—especially Jill Forster, Elaine Heberle, and Jim Roche—was invaluable in keeping our computers up and our manuscript copied and circulating, and we much appreciate their help.

We are grateful to those colleagues—Peter van Emde Boas, Harald Hempel, Jörg Rothe, Alan Selman, Seinosuke Toda, Leen Torenvliet, Heribert Vollmer, Klaus Wagner, Osamu Watanabe, and Gerd Wechsung—who have generously hosted our research visits during the planning and writing of this book, and to the many colleagues, as cited in the Bibliographic Notes sections, who have collaborated with us on the research described in some sections of this book.

Above all, we thank Edith, Ellen, Emi, and Erica for their advice, love, and support.

Lane A. Hemaspaandra

Mitsunori Ogihara

Rochester, NY
October 2001

Contents